1. area, $A = \sqrt{s(s-a)(s-b)(s-c)}$

AM-GM inequality,

$$(s-a)(s-b)(s-c) \le \left(\frac{s-a+s-b+s-c}{3}\right)^3 = \frac{s^3}{27}$$

$$\text{so } A^2/s = (s-a)(s-b)(s-c) \text{ will be maximum if } a=b=c$$

2. $\tan(\alpha+\beta) = 50/d = \dfrac{10/d + \tan\beta}{1 - 10/d \tan\beta}$

solving $\tan\beta$ and maximizing w.r.t d you'll find $d = 10\sqrt{5}$ feet and corresponding to a maximum $\beta = 41.81°$

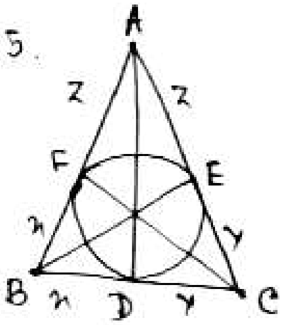
3.

4 $S \in P$; $T \in Q$) are tangency points

C, K are midpoints of AB, MN

$\Delta PAC \cong \Delta PQT$ and $\Delta QMK \cong \Delta QPS$

$\frac{1}{2}AB = AC = PA \cdot QT/PQ = \dfrac{PS \cdot QM}{PQ} = MK = \frac{1}{2}MN$ QED

5. 

$s = (x+y+z)$

$\Delta = \sqrt{(x+y+z)(x+y+z-x-y)(x+y+z-y-z)(x+y+z-x-z)}$

$= \sqrt{(x+y+z)xyz}$

$\Delta = \frac{1}{2} r \left((x+y)+(y+z)+(x+z)\right) = r(x+y+z)$

$= \sqrt{(x+y+z)xyz}$

$\therefore r^2 = (xyz)/(x+y+z)$

6 $x = \lim\limits_{n\to\infty} \dfrac{1}{2n} \log \dfrac{2n!}{(n!)^2}$

$$= \lim_{n\to\infty} \frac{1}{2n} \lim_{n\to\infty} \log \frac{\dfrac{(n+1)(n+2)\ldots(n+n)}{n^n}}{\dfrac{1.2.3\ldots n}{n^n}}$$

$$= \lim_{n\to\infty} \frac{1}{2n} \left(\log\left(1+1/n\right)\left(1+2/n\right)\left(1+3/n\right)\cdots\left(1+n/n\right) - \log\left(1/n + 2/n + \ldots + n/n\right)\right)$$

$$= \frac{1}{2}\int_0^1 \ln(1+x) - \ln x \, dx = \frac{1}{2}\left[x\log(x+1)\Big|_0^1 - \int_0^1 \frac{x\,dx}{1+x}\right] - [x\ln x - x]_0^1$$

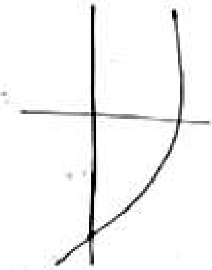
$= \frac{1}{2}\left((\ln 2 + \ln 2 - 1) - (-1)\right) = \frac{1}{2}\ln 4$

7. a) $f(x) = x^5 + x - 10$

$f'(x) = 4x^4 + 1 > 0$

b) $f(1) = -8$; $f(2) = 24$

$f(1)\,f(2) < 0$



c) let $x = p/q$; $(p/q)^5 + (p/q) = 10$

$\Rightarrow p^5/q = ~~10q~~\ 10q - p$

fraction = integer   contradiction!

8. Number of pairs w/o any kind of war $= \binom{8}{2} = 28$

The pairs that harms = 5 ([(2,4), (2,6), (4,6), (6,8), ((2,4), (6,8)))

$28 - 5 = 23$

9. $(a*b)$ lcm = km ; gcd = m ; $a = km$; $b = kn$; $c = kp$

$(a*b) = \frac{kmn}{k} = mn$; $b*c = \frac{knp}{k} = np$

$(a*b)*c = kmnp/1 = kmnp$

$(a*(b*c)) = kmnp/1 = kmnp$

$a = km$ $i = kp$

$a*i = kmp/k = mp \neq a$

10. 'any two similar figures have an isomerism and homothetic relation which takes one figure to the other'

a) have segment $AB (A \neq B)$ and $CD (C \neq D)$. Have point $P \in AB$ with $PA/AB = \lambda \in [0,1]$; associate to it the point $Q \in CD$ with $QC/CD = \lambda$

b) have circle γ (of the centre m and radius $r > 0$) and Γ (of centre M and radius $R > 0$). Have point $P \in \gamma$ making angle $\theta \in [0, 2\pi)$ with the horizontal; associate to it point $Q \in \Gamma$ making an angle θ with the horizontal