# B. Math. Admission Test 2009 Solution Paper

**1. The domain of definition of $f(x) = -\log(x^2 - 2x - 3)$ is**

**(a) $(0, \infty)$**
**(b) $(-\infty, -1)$**
**(c) $(-\infty, -1)\cup(3, \infty)$**
**(d) $(-\infty, -3)\cup(1, \infty)$**

Solution: (C)

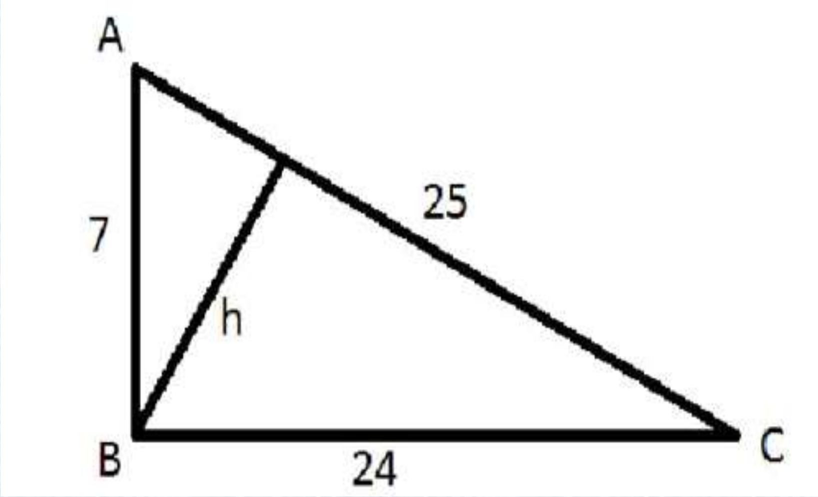
$x^2 - 2x - 3 > 0$

- $(x - 1)^2 > 4$
- $x - 1 > 2$ or $x - 1 < -2$
- $x > 3$ or $x < -1$

**2. ABC is a right-angled triangle with the right angle at B. If AB = 7 and BC = 24, then the length of the perpendicular from B to AC is**

**(a) 12.2**
**(b) 6.72**
**(c) 7.2**
**(d) 3.36**

Solution: (B)




Clearly $AC^2 = AB^2 + BC^2$

⇨ AC = 25

Let the length of the perpendicular from B to AC is h.

Area of triangle ABC = $(1/2)\times 7\times 24 = (1/2)\times h\times 25$

⇨ $h = 6.72$.

**3. If the points $z_1$ and $z_2$ are on the circles $|z| = 2$ and $|z| = 3$ respectively and the angle included between these vectors is 60°, then $|(z_1 + z_2)/(z_1 - z_2)|$ equals**

**(a) $\sqrt{(19/7)}$**
**(b) $\sqrt{19}$**
**(c) $\sqrt{7}$**
**(d) $\sqrt{133}$**

Solution: (A)

Clearly, $|z_1| = 2$ and $|z_2| = 3$.

Let, $z_1 = 2e^{i\Theta}$ and $z_2 = 3e^{i(\Theta + \pi/3)}$

Now, $(z_1 + z_2)/(z_1 - z_2) = (2e^{i\Theta} + 3e^{i(\Theta + \pi/3)})/(2e^{i\Theta} - 3e^{i(\Theta + \pi/3)})$

⇨ $(z_1 + z_2)/(z_1 - z_2) = (2 + 3e^{i\pi/3})/(2 - 3e^{i\pi/3})$

⇨ $|(z_1 + z_2)/(z_1 - z_2)|^2 = [\{2 + 3\cos(\pi/3)\}^2 + 3\sin^2(\pi/3)]/[\{2 - 3\cos(\pi/3)\}^2 + 3\sin^2(\pi/3)]$

⇨ $|(z_1 + z_2)/(z_1 - z_2)|^2 = \{13 + 12\cos(\pi/3)\}/\{13 - 12\cos(\pi/3)\} = \frac{19}{7}$

⇨ $|(z_1 + z_2)/(z_1 - z_2)| = \sqrt{\frac{19}{7}}$.

**4. Let a, b, c and d be positive integers such that $\log_a(b) = 3/2$ and $\log_c(d) = 5/4$. If $a - c = 9$, then $b - d$ equals**

**(a) 55**
**(b) 23**
**(c) 89**
**(d) 93**

Solution: (D)

$\log_a(b) = 3/2$

$b = a^{3/2}$

Similarly, $d = c^{5/4}$

b and d are both integers.

So, a must be a square number and c must be a $4^{th}$ power number.

Let, $a = x^2$ and $c = y^4$

Now, $a - c = 9$

⇨ $x^2 - y^4 = 9$

⇨ $(x + y^2)(x - y^2) = 9$

Now, 9 can be divided in two ways viz. 3×3 or 9×1.

Now, 3×3 is not possible otherwise y = 0 i.e. c = 0. But c is positive integer.

⇨ $x + y^2 = 9$ and $x - y^2 = 1$

⇨ $x = 5$ and $y^2 = 4$

⇨ $a = 25$ and $c = 16$.

⇨ $b = 125$ and $d = 32$

⇨ $b - d = 125 - 32 = 93$

**5. $1 - x - e^{-x} > 0$ for :**

**(a) All $x \in R$.**

**(b) No $x \in R$.**

**(c) $x > 0$.**

**(d) $x < 0$.**

Solution: (B)

Clearly, for $x \geq 1$, $1 - x - e^{-x} < 0$.

So, option (a) and (c) cannot be true.

Let us take x = -1.

For x = -1, $1 - x - e^{-x} = 1 - (-1) - e = 2 - e < 0$ (As $2 < e < 3$)

So, Option (d) cannot be true.

**6. If $P(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + bx + c$ where $ac \neq 0$, then the equation $P(x)Q(x) = 0$ has :**

**(a) Only real roots.**

**(b) No real roots.**

**(c) At least two real roots.**

**(d) Exactly two real roots.**

Solution: (C)

Discriminant of P(x) = $b^2 - 4ac$ and discriminant of Q(x) = $b^2 + 4ac$.

If $4ac > b^2$ then roots of Q(x) are real and roots of P(x) is not real.

If $4ac < b^2$ then roots of P(x) are real but nothing can b said about roots of Q(x) i.e. both roots of Q(x) may be real or both may not be real.

If $4ac = b^2$ then both the roots of P(x) and Q(x) are real.

So, we attend the conclusion that at least two of the roots are real of the equation P(x)Q(x) = 0.

**7. $\lim|\sqrt{(x^2 + x)} - x|$ as $x \to \infty$ is equal to**

- **(a) ½**
- **(b) 0**
- **(c) ∞**
- **(d) 2**

Solution: (A)

Now, $\sqrt{(x^2 + x)} - x = x/\{\sqrt{(x^2 + x)} + x\}$

Let, z = 1/x. As $x \to \infty$, $z \to 0$.

The above expression becomes, $(1/z)/\sqrt{\{(1/z^2 + 1/z) + (1/z)\}} = 1/\{\sqrt{(1 + z)} + 1\}$

Now, $\lim[1/\{\sqrt{(1 + z)} + 1\}]$ as $z \to 0$ is ½.

**8. $\lim (\pi/2^n)\Sigma\sin(j\pi/2^n)$ where j runs from 1 to $2^n$ as $n \to \infty$ is equal**

- **(a) 0**
- **(b) Π**
- **(c) 2**
- **(d) 1**

Solution: (C)

**9. Let f: R -> R is given by f(x) = x(x – 1)(x + 1). Then,**

- **(a) f is 1 – 1 and onto**
- **(b) f is neither 1 – 1 nor onto**
- **(c) f is 1 – 1 but not onto**
- **(d) f is onto but not 1 – 1**

Solution: (D)

Clearly f(0) = f(1) = f(-10) = 0
So, f cannot be 1 – 1.

Definition of onto function: A function f from A to B is called onto if for all b in B there is an a in A such that f (a) = b. All elements in B are used.

Clearly f(x) can take any real value.

So, f(x) is onto.

**10. The last digit of $22^{22}$ is :**
**(a) 2**
**(b) 4**
**(c) 6**
**(d) 0**

Solution: (B)

$2^{10} = 1024$

⇨ $2^{11} = 2048$
⇨ $2^{11} \equiv 8 \pmod{10}$
⇨ $2^{22} \equiv 64 \pmod{10}$
⇨ $2^{22} \equiv 4 \pmod{10}$

**11. The average scores of 10 students in a test is 25. The lowest score is 20. Then the highest score is at most**
**(a) 100**
**(b) 30**
**(c) 70**
**(d) 75**

Solution: (C)

Let all the 9 students except the student who has got highest mark have got 20 each which is the lowest score.

Then sum of their scores = 20×9 = 180

Now, sum of the scores of 10 students = 25×10 = 250.

So, highest mark can be at most 250 – 180 = 70.

**12. The coefficient of $t^3$ in the expansion of $\{(1 - t^6)/(1 - t)\}^3$ is**

**(a) 10**
**(b) 12**
**(c) 8**
**(d) 9**

Solution: (A)

Now, $1 - t^6 = (1 - t^3)(1 + t^3) = (1 - t)(1 + t + t^2)(1 + t^3)$

⇨ $(1 - t^6)/(1 - t) = (1 + t + t^2)(1 + t^3)$

⇨ $\{(1 - t^6)(1 - t)\}^3 = (1 + t + t^2)^3(1 + t^3)^3$

⇨ $\{(1 - t^6)(1 - t)\}^3 = \{1 + 3(t + t^2) + 3(t + t^2)^2 + (t + t^2)^3\}(1 + 3t^3 + 3t^6 + t^9)$

⇨ $\{(1 - t^6)(1 - t)\}^3 = \{1 + 3t + 3t^2 + 3(t^2 + 2t^3 + t^4) + t^3 + 3t^4 + 3t^5 + t^6\}(1 + 3t^3 + 3t^6 + t^9)$

⇨ $\{(1 - t^6)(1 - t)\}^3 = (1 + 3t + 6t^2 + 7^3 + ...)(1 + 3t^3 + ..)$

⇨ $\{(1 - t^6)(1 - t)\}^3 = (1 + 3t^3 + 3t + 3t^4 + 6t^2 + 18t^5 + 7t^3 + ...)$

Clearly, coefficient of $t^3$ is 3 + 7 = 10.

**13. Let $p_n(x)$, $n \geq 0$ be polynomials defined by $p_0(x) = 1$, $p_1(x) = x$ and $p_n(x) = xp_{n-1}(x) - p_{n-2}(x)$ for $n \geq 2$. Then $p_{10}(x)$ equals**

**(a) 0**
**(b) 10**
**(c) 1**
**(d) -1**

Solution: (D)

$p_{10}(0) = 0 \times p_9(0) - p_8(0)$

implying $p_{10}(0) = - p_8(0) = - \{- p_6(0)\} = p_6(0) = - p_4(0) = - \{- p_2(0)\}$

$= p_2(0) = - p_0(0) = -1$

**14. Suppose A, B are matrices satisfying AB + BA = 0. Then $A^2B^5$ is equal to**

**(a) 0**
**(b) $B^2A^5$**
**(c) $-B^2A^5$**
**(d) AB**

Solution: (B)

AB + BA = 0

- ➢ AB = -BA

Now, $A^2B^5 = (AB)^2B^3 = (-BA)^2B^3 = B^2(AB)^2B = B^2(-BA)^2B = B^4A^2B$
$= B^4A(AB) = B^4A(-BA) = B^4(-AB)A = B^4(BA)A = B^5A^2$

**15. The number of terms in the expansion of $(x + y + z + w)^{2009}$ is**

**(a) $^{2009}C_4$**
**(b) $^{2013}C_4$**
**(c) $^{2012}C_3$**
**(d) $(2010)^4$**

Solution: (C)

Number of terms = $^{2009 + 4 - 1}C_{4 - 1} = {}^{2012}C_3$.

**16. If a, b, c are positive real numbers satisfying ab + bc + ca = 12, then the maximum value of abc is**

**(a) 8**
**(b) 9**
**(c) 6**
**(d) 12**

Solution: (A)

Now, ab + bc + ca = 12

⇨ (1/a) + (1/b) + (1/c) = 12/abc

Now, $\{(1/a) + (1/b) + (1/c)\}/3 \geq 1/(abc)^{1/3}$ (As AM ≥ GM)

- ➢ $4/abc \geq 1/(abc)^{1/3}$
- ➢ $(abc)^{2/3} \leq 4$
- ➢ $abc \leq 8$

**17. If at least 90 percent students in a class are good in sports, and at least 80 percent are good in music and at least 70 percent are good in studies, then the percentage of students who are good in all three is at least**

**(a) 25**
**(b) 40**
**(c) 20**
**(d) 50**

Solution: (B)

At least 90 percent students are good in sports.

At most 10 percent students are not good in sports.

Similarly, at most 20 percent students are not good in music and at most 30 percent students are not good in studies.

If all the students in the above record are different then at most (10 + 20 + 30) = 60 percent students are not good in sports, music and studies.

At least 100 – 60 = 40 percent students are good in all three.

**18. If $\cot\{\sin^{-1}\sqrt{(13/17)}\} = \sin(\tan^{-1}\theta)$, then $\theta$ is**

**(a) $2/\sqrt{17}$**
**(b) $\sqrt{(13/17)}$**
**(c) $\sqrt{(2/\sqrt{13})}$**
**(d) 2/3**

Solution: (D)

Let, $\sin^{-1}\sqrt{(13/17)} = A$

$\sin A = \sqrt{(13/17)}$

⇨ $\cot A = 2/\sqrt{13}$

⇨ $\sin(\tan^{-1}\theta) = 2/\sqrt{13}$

Let, $\tan^{-1}\theta = B$

⇨ $\tan B = \theta$

⇨ $\sin B = \theta/\sqrt{(\theta^2 + 1)}$

⇨ $\theta/\sqrt{(\theta^2 + 1)} = 2/\sqrt{13}$

⇨ $\theta^2/(\theta^2 + 1) = 4/13$

⇨ $13\theta^2 = 4\theta^2 + 4$

⇨ $9\theta^2 = 4$

$\theta = 2/3.$

**19. Let f(t) = (t + 1)/(t – 1). Then f(f(2010)) equals**

**(a) 2011/2009**
**(b) 2010**
**(c) 2010/2009**
**(d) None of the above**

Solution: (B)

f(2010) = (2010 + 1)/(2010 – 1) = 2011/2009

f(f(2010)) = f(2011/2009) = {(2011/2009) + 1}/{)2011/2009) – 1}
= (2011 + 2009)/(2011 – 2009) = 4020/2 = 2010

**20. If each side of a cube is increased by 60%, then the surface area of the cube increased by**

**(a) 156%**
**(b) 160%**
**(c) 120%**
**(d) 240%**

Solution: (A)

Surface area = S = $6a^2$ where a is each side of the cube.

Now, each side is increased by 60%

New side length = 1.6a

Let new surface area = $S_1 = (1.6a)^2 = 2.56a^2$

Percentage increase in surface area = $\{(S_1 - S)/S\}\times 100\%$
$= \{(2.56a^2 - a^2)/a^2\}\times 100\% = 156\%$

**21. If a > 2, then**

**(a) $\log_e(a) + \log_a(10) < 0$**
**(b) $\log_e(a) + \log_a(10) > 0$**
**(c) $e^a < 1$**
**(d) None of the above is true.**

Solution: (B)

Now, $e^a < 1$

⇨ $\log(e^a) < \log(1)$

⇨ $a < 0$

⇨ Option (c) is not correct.

Now, $\log_e(a) + \log_a(10) = \log_e(a) + \log_e(10)/\log_e(a)$

Now, $\log_e(a)$ and $\log_e(10)$ both > 0

So, $\log_e(a) + \log_a(10) > 0$

22. **The number of complex numbers w such that |w| = 1 and imaginary part of $w^4$ is 0, is**
    - **(a) 4**
    - **(b) 2**
    - **(c) 8**
    - **(d) Infinite**

Solution: (C)

Let $w = e^{i\Theta}$

⇨ $w^4 = e^{i4\Theta} = \cos4\Theta + i\sin4\Theta$

Now, $\sin4\Theta = 0$

⇨ $4\Theta = n\pi$

Now, this will given 8 distinct results for n = 0, 1,...,7 and then it will run into loop.

23. **Let f(x) = csin(x) for all x Є R. Suppose $f(x) = \sum f(x + k\pi)/2^k$ (summation is running from k = 1 to k = ∞) for all x Є R. Then**
    - **(a) c = 1**
    - **(b) c = 0**
    - **(c) c < 0**
    - **(d) c = -1**

Solution: (B)

Now, $f(x + k\pi) = c\sin(x + k\pi) = c(-1)^k \times \sin(x)$

We have, $f(x) = \sum f(x + k\pi)/2^k$ (summation is running from k=1 to k=∞)

⇨ $c\sin(x) = c\sin(x)\sum\{(-1)^k/2^k\}$ (summation is running from k=1 to k=∞)

⇨ $c\sin(x) = c\sin(x)[(-1/2)/\{1 - (-1/2)\}]$

(infinite GP series with first term -1/2 and common ratio -1/2)

⇨ $c\sin(x) - c\sin(x)(-1/3) = 0$

⇨ $c\sin(x)(4/3) = 0$

⇨ $c\sin(x) = 0$ So, c = 0.

**24. The number of points at which the function f(x) = max(1 + x, 1 – x) if x < 0 and f(x) = min(1 + x, 1 + $x^2$) if x ≥ 0 is not differentiable, is**

**(a) 1**
**(b) 0**
**(c) 2**
**(d) None of the above.**

Solution: (C)

Clearly f(x) = 1 – x if x < 0

Now, 1 + $x^2$ ≤ 1 + x
⇨ x(x – 1) ≤ 0
⇨ x < 1 as x ≥ 0
⇨ f(x) = 1 + $x^2$ if 0 ≤ x < 1

Now, 1 + x < 1 + $x^2$
⇨ x(x – 1) > 0
⇨ x > 1 as x ≥ 0

f(x) = 1 + x if x ≥ 1

Now, f'(0-) = lim(f(x) – f(0))/(x – 0) as x -> 0-
= lim(1 – x – 1)/x as x -> 0- = -1

Now, f'(0+) = lim(f(x) – f(0))/(x – 0) as x -> 0+
= lim(1 + $x^2$ – 1)/x as x -> 0+ = 0

So, the function is not differentiable at x = 0.

Now, f'(1-) = lim(f(x) – f(1))/(x – 1) as x -> 1-
= lim(1 + $x^2$ – 2)/(x – 1) as x -> 1-
= lim(x + 1)(x – 1)/(x – 1) as x -> 1-
= lim(x + 1) as x -> 1-= 2.

Now, f'(1+) = lim(f(x) – f(1))/(x – 1) as x -> 1+
= lim(1 + x – 2)/(x – 1) as x -> 1+
= lim(x – 1)/(x – 1) as x -> 1+ = 1
⇨ f(x) is not differentiable at x = 1.

**25. The greatest value of function f(x) = $\sin^2(x)\cos(x)$**

**(a) 2/3√3**
**(b) √(2/3)**
**(c) 2/9**
**(d) √2/3√3**

Solution: (A)

Now, $f(x) = \sin^2(x)\cos(x)$

⇨ $f'(x) = 2\sin(x)\cos(x)\cos(x) + \sin^2(x)\{-\sin(x)\}$

⇨ $f'(x) = 2\sin(x)\cos^2(x) - \sin^3(x)$

Now, $f'(x) = 0$ gives, $2\sin(x)\cos^2(x) - \sin^3(x) = 0$

⇨ $\sin(x) = 0$ or $2\cos^2(x) - \sin^2(x) = 0$ i.e. $\sin(x) = \pm\sqrt{(2/3)}$

Now, $f''(x) = 2\cos(x)\cos^2(x) + 2\sin(x)2\cos(x)\{-\sin(x)\} - 3\sin^2(x)\cos(x)$

$= 2\cos^3(x) - 4\sin^2(x)\cos(x) - 3\sin^2(x)\cos(x)$

$= \cos(x)\{2\cos^2(x) - 4\sin^2(x) - 3\sin^2(x)\}$

$= \cos(x)\{2 - 9\sin^2(x)\} < 0$ for $\cos(x)$

$= 1/\sqrt{3}$ ( $\sin^2(x) = 2/3$ i.e. $\cos^2(x) = 1/3$ i.e. $\cos(x) = \pm 1/\sqrt{3}$)

So, $f(x)_{max} = (2/3)(1/\sqrt{3}) = 2/3\sqrt{3}$

26. **Let g(t) = $\int (x^2 + 1)^{10}dx$ (integration running from -10 to t) for all t ≥ -10. Then**

**(a) g is not differentiable.**
**(b) g is constant.**
**(c) g is increasing in (-10, ∞).**
**(d) g is decreasing in (-10, ∞).**

Solution: (C)

Now, $g'(t) = (t^1 + 1)^{10} > 0$

⇨ g is increasing.

27. **Let p(x) be a continuous function which is positive for all x and ∫p(x)dx = c∫p{(x+4)/2}dx (first integration is running from 2 to 3 and second integration running from 0 to 2). Then**
   - **(a) c = 4**
   - **(b) c = ½**
   - **(c) c = ¼**
   - **(d) c = 2**

Solution: (B)

Now, ∫p{(x + 4)/2}dx (integration running from 0 to 2)

Putting (x + 4)/2 = z we get, dx = 2dz

Now, ∫p{(x + 4)/2}dx (integration running from 0 to 2)
= 2∫p(z)dz (integration running from 2 to 3)
= 2∫p(x)dx (integration running from 2 to 3) (change of variable)

⇨ ∫p(x)dx = 2c∫p(x)dx (both integration running from 2 o 3)

⇨ 2c = 1

⇨ c = ½

28. **Let f : [0, 1] -> (1, ∞) be a continuous function. Let g(x) = 1/x for x > 0. Then, the equation f(x) = g(x) has**
   - **(a) No solution.**
   - **(b) All points in (0, 1] as solutions.**
   - **(c) At least one solution.**
   - **(d) None of the above.**

Solution: (B)

Clearly, the intersection of domain of definition of f(x) and g(x) is (0, 1].

Hence f(x) = g(x) should be defined for all (0, 1]

29. **Let 0 ≤ ϴ, Φ < 2п be two angles.**
**Then the equation sinϴ + sinΦ = cosϴ + cosΦ**
   - **(a) Determines ϴ uniquely in terms of Φ**
   - **(b) Gives two value of ϴ for each value of Φ**
   - **(c) Gives more than two values of ϴ for each value of Φ**
   - **(d) None of the above.**

Solution: (B)

We have, sinϴ + sinΦ = cosϴ + cosΦ

⇨ sinΘ - cosΘ = cosΦ – sinΦ
⇨ $(\sin\Theta - \cos\Theta)^2 = (\cos\Phi - \sin\Phi)^2$
⇨ $\sin^2\Theta + \cos^2\Theta - 2\sin\Theta\cos\Theta = \cos^2\Phi + \sin^2\Phi - 2\cos\Phi\sin\Phi$
⇨ 1 – sin2Θ = 1 – sin2Φ
⇨ sin2Θ = sin2Φ
⇨ 2Θ = 2Φ or 2Θ = π - 2Φ or 2Θ = 2π - 2Φ or 2Θ = 4π - 2Φ
⇨ Θ = Φ or Θ = π/2 – Φ or Θ = π – Φ or Θ = 2π – Φ

Now, out of these 4 relations only 2 satisfies the given equation viz. Θ = Φ and Θ = π/2 – Φ

⇨ We are getting two values of Θ for each Φ.

**30. Ten players are to pay a tennis tournament. The number of pairings for the first round is**

**(a) $10!/2^5 5!$**
**(b) $2^{10}$**
**(c) $^{10}C_2$**
**(d) $^{10}P_2$**

Solution: (C)