B.Math. (Hons.) Admission Test 2010 Solution Paper

1. The product of the first 100 positive integers ends with

(a) 21 zeros
(b) 22 zeros
(c) 23 zeros
(d) 24 zeros.

Solution: (D)

Number of zeros = $[100/5] + [100/5^2] = 20 + 4 = 24$.

Where, [x] denotes the largest integer less than or equal to x.

2. Given four 1-gm stones, four 5-gm stones, four 25-gm stones and four 125-gm stones each, it is possible to weigh material of any integral weight up to

(a) 600 gms
(b) 625 gms
(c) 624 gms
(d) 524 gms

Solution: (C)

Any integer ≡ 1, 2, 3, 4, 5 (mod 5)

Now, it is clear that we can measure any weight up to 24 with four 1-gm and four 5-gm stones.

We can measure 25.

Similarly, we can measure weights up to 50-gm, 75-gm, 100-gm, 124-gm.

Now, 125-gm is available.

So, we can measure up to 150-gm, 175-gm, 200-gm, 225-gm, 250-gm, 500-gm, 525-gm, 550-gm, 575-gm, 600-gm, 624-gm.

3. The function $f(x) = |x| + \sin(x) + \cos^3(x)$ is

- **(a) Continuous but not differentiable at x = 0**
- **(b) Differentiable at x = 0**
- **(c) A bounded function which is not continuous at x = 0**
- **(d) A bounded function which is continuous at x = 0.**

Solution: (A)

$f(x) = x + \sin(x) + \cos^3(x)$ for $x \geq 0$ f(x)

$= -x + \sin(x) + \cos^3(x)$ for $x < 0$

Now, lim f(x) as x -> 0+ = lim $\{x + \sin(x) + \cos^3(x)\}$ as x -> 0+ = 1

And, lim f(x) as x -> 0- = lim $\{-x + \sin(x) + \cos^3(x)\}$ as x -> 0- = 1

And, f(0) = 1

As, lim f(x) as x -> 0+ = lim f(x) as x -> 0- = f(0) = 1,

so f(x) is continuous at x = 0

Now, lim $[\{f(x) - f(0)\}/(x - 0)]$ as x -> 0+

$= \lim [\{x + \sin(x) + \cos^3(x) - 1\}/x]$ as x -> 0+

$= \lim [\{1 + \cos(x) - 3\cos^2(x)\sin(x)\}/1$ as x -> 0+ (Applying L'Hospital)

$= (1 + 1 - 0)/1 = 2$

And, lim $[\{f(x) - f(0)\}/(x - 0)]$ as x -> 0-

$= \lim [\{-x + \sin(x) + \cos^3(x) - 1\}/x]$ as x -> 0-

$= \lim [\{-1 + \cos(x) - 3\cos^2(x)\sin(x)\}/1\}$ as x -> 0- (Applying L'Hospital rule)

$= (-1 + 1 - 0)/1 = 0$

As, lim $[\{f(x) - f(0)\}/(x - 0)]$ as x -> 0+

$\neq$ lim $[\{f(x) - f(0)\}/(x - 0)]$ as x -> 0-,

so f(x) is not differentiable at x = 0.

Now, f(x) is not bounded above or below as f(x) -> ∞ when x -> ∞ .

4. The sum of the first n terms of an arithmetic progression whose first term is a (not necessarily positive) integer and common difference is 2, is known to be 153. If n> 1, then number of possible values of n is

(a) 2
(b) 3
(c) 4
(d) 5

Solution: (D) Let the first term of the arithmetic progression is a.

We have, $(n/2)\{2a + (n - 1)\times 2\} = 153$

⇨ $n(a + n - 1) = 153$

⇨ $n^2 + n(a - 1) - 153 = 0$

⇨ $n = [-(a - 1) \pm \sqrt{\{(a - 1)^2 + 4\times 1\times 153\}}]/2$

Let, $(a - 1)^2 + 4\times 1\times 153 = p^2$

⇨ $p^2 - (a - 1)^2 = 2^2\times 3^2\times 17$

⇨ $\{p + (a - 1)\}\{p - (a - 1)\} = 2^2\times 3^2\times 17$

Now, $p + (a - 1) = 2\times 3\times 17$ and $p - (a - 1) = 2\times 3$

$p = 54$ and $a - 1 = 48$ gives $n = 3$.

Let, $p + (a - 1) = 2\times 3$ and $p - (a - 1) = 2\times 3\times 17$

$p = 54$ and $a - 1 = -48$ gives $n = 51$.

Let, $p + (a - 1) = 2\times 17$ and $p - (a - 1) = 2\times 3^2$

⇨ $p = 26$ and $a - 1 = 8$ gives $n = 9$

Let, $p + (a - 1) = 2\times 3^2$ and $p - (a - 1) = 2\times 17$

⇨ $p = 26$ and $a - 1 = -8$ gives $n = 17$

Let, $p + (a - 1) = 2\times 3^2\times 17$ and $p - (a - 1) = 2$

⇨ $p = 154$ and $a - 1 = 152$ gives $n = 1$ (not possible as $n > 1$) Let,

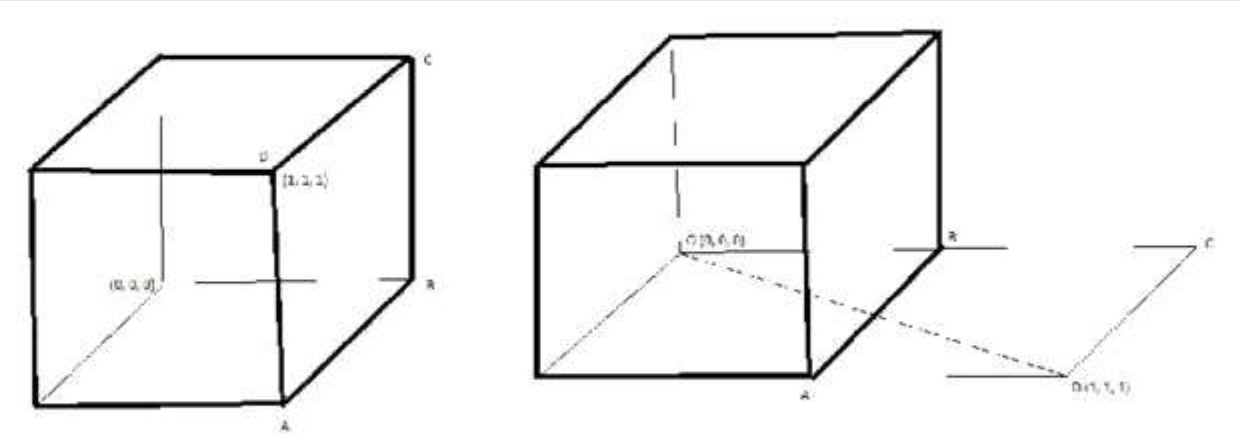
$p + (a - 1) = 2$ and $p - (a - 1) = 2\times 3^2\times 17$

⇨ $p = 154$ and $a - 1 = -152$ gives $n = 153$

No other combination is possible as $(a - 1)$ and p are either both odd or both even. So, possible values of n are 3, 51, 9, 17 and 153.

5. **Consider a cubical box of 1 m side which has one corner placed at (0, 0, 0) and the opposite corner placed at (1, 1, 1). The least distance that an ant crawling from the point (0, 0, 0) to the point (1, 1, 1) must travel is**
   (a) $\sqrt{6}$ m
   (b) $\sqrt{5}$ m
   (c) $2\sqrt{3}$ m
   (d) $1 + \sqrt{3}$ m.

Solution: (B)



We have opened the side ABCD of the cube and clearly OD is the shortest path to reach D from O.

Clearly, $OD = \sqrt{(OC^2 + CD^2)} = \sqrt{(2^2 + 1^2)} = \sqrt{5}$.

6. **Let $x_1 < -1$ and $x_{n+1} = x_n/(1 + x_n)$ for all $n \geq 1$. Then**
   (a) $\{x_n\} \to -1$ as $n \to \infty$
   (b) $\{x_n\} \to 1$ as $n \to \infty$
   (c) $\{x_n\} \to 0$ as $n \to \infty$
   (d) $\{x_n\}$ diverges.

Solution: (C)

Now, $x_1 < -1$

⇨ $1 + x_1 < 0$

⇨ $x_2 = x_1/(1 + x_1) > 0$ as both numerator and denominator < 0

⇨ All $x_i > 0$ except x_1

Therefore, option (a) cannot be true.

Now, $x_3 = x_2/(1 + x_2) < x_2$

Similarly, $x_4 < x_3$ and in general we can write $x_i < x_j$ if $i > j$

Option (d) cannot be true.

Also, $x_3 = x_2/(1 + x_2) < 1$

Similarly all terms of the sequence $\{x_n\} < 1$ except x_2

So, $0 < \{x_n\} < 1$ for $n > 2$

And $\{x_n\}$ is approaching towards 0 as $n \to \infty$

7. The number of perfect cubes among the first 4000 positive integers is
- **(a) 16**
- **(b) 15**
- **(c) 14**
- **(d) 13**

Solution: (B)

Clearly $15^3 = 3375$ and $16^3 = 4096$.

8. The roots of the equation $x^4 + x^2 = 1$ are
- **(a) All real and positive**
- **(b) Never real**
- **(c) 2 positive and 2 negative**
- **(d) 1 positive, 1 negative and 2 non-real.**

Solution: (D)

Now, $x^4 + x^2 - 1 = 0$

⇨ $x^2 = [-1 \pm \sqrt{\{1^2 - 4\times1(-1)\}}]/2\times1 = (-1 \pm \sqrt{5})/2$

Now, for $x^2 = (-1 - \sqrt{5})/2$ two roots are non-real

Now, we consider, $x^2 = (\sqrt{5} - 1)/2$

⇨ $x = \pm\sqrt{\{(\sqrt{5} - 1)/2\}}$

⇨ one is positive and another is negative.

9. If $\{(x + a)/(x - a)\}^x$ -> 9 as x -> ∞ then a =
- **(a) 3^e**
- **(b) log(3)**
- **(c) log(9)**
- **(d) 3**

Solution: (B)

Now, $\{(x + a)/(x - a)\}^x = (1 + a/x)^x/(1 - a/x)^x$

Now, lim $(1 + a/x)^x/(1 - a/x)^x$ as x -> ∞ = $e^a/(1/e^a) = e^{2a} = 9$ (given)

⇨ 2a = log(9)

⇨ 2a = 2log(3)

⇨ a = log(3)

10. The number of multiples of 4 among all 10 digit numbers is
- **(a) 25×10^8**
- **(b) 25×10^7**
- **(c) 225×10^7**
- **(d) 234×10^7**

Solution: (C)

It's an arithmetic progression with first term = 10^9 and common difference = 4 and last term = $10^{10} - 4$.

Let number of terms = n

Then, $10^9 + (n - 1) \times 4 = 10^{10} - 4$

⇨ $25\times10^7 + n - 1 = 25\times10^8 - 1$

⇨ $n = 25\times10^8 - 25\times10^7 = 10^7(250 - 25) = 225\times10^7$

11. The larger diagonal of a parallelogram of area 8 must have length

- **(a) At least 4**
- **(b) Equal to 8**
- **(c) At most 4**
- **(d) Equal to √8.**

Solution: (A)

Area of triangle ABC = 8/2 = 4

Clearly point C can rotate among point B and length of the diagonal gets changed.

So, any particular value cannot be an answer.

Option (b) and (d) cannot be true.

Now, let AB = 4 and perpendicular from C to AB = 2.

Then area of triangle ABC = (1/2)×4×2 = 4

Now, AC > $\sqrt{(4^2 + 2^2)}$ (As perpendicular from C to AB will be stretched portion of AB)

AC > 4

Option (d) cannot be true.

12. Let $\{a_n\}$ be a sequence of positive real numbers such that $\lim 2^n(a_n + a_{n+1})$ as n -> ∞ is finite. Then

(a) $\{a_n\}$ converges to 1
(b) $\{a_n\}$ converges to 0
(c) $\{a_n\}$ converges to ½
(d) $\{a_n\}$ converges to $1/\sqrt{2}$.

Solution: (B)

13. Consider the following two statements about a positive integer n and choose the correct option below.

(I) n is perfect square.
(II) The number of positive integer divisors of n is odd.

(a) I and II are equivalent
(b) I implies II but not conversely
(c) II implies I but not conversely
(d) Neither statement implies the other.

Solution: (A)

Any perfect square number has odd number of factors or divisors.

14. For triangles ABC and PQR, it is given that AB = PQ, BC = QR and the angle ACB equals the angle QRP. Then the triangles ABC and PQR

(a) Are equivalent
(b) Cannot be congruent
(c) Need not be congruent but must be similar
(d) Need not be similar, if they are, then they must be congruent

Solution: (D)

If 2 sides of two triangles and the angle between the sides are equal then the two triangles are similar.

Here AB = PQ, BC = QR and the angle between AB and BC is angle ABC and also the angle between PQ and QR is angle PQR. The relation angle ABC = angle PPQR is not given. So, the triangles need not to be similar. If they are they are congruent because the rest sides must be equal.

15. A particle starts at the origin and travels along the positive x-axis. For the first one second, its speed is 1 m/sec. Therefore, its speed at any time t is at the most $(9/10)^{th}$ of its speed at time t – 1. Then

(a) The particle reaches any point x > 0 at some finite time
(b) The particle must reach x = 10
(c) The particle may or may not reach x = 9 but it will never reach x = 10
(d) Nothing of the above nature can be predicted without knowing the exact speed.

Solution: (C)

So, in infinite time the particle reaches at most $1 + (9/10) + (9/10)^2 + \ldots.. = 1/(1 - 9/10) = 10$. So, the particle cannot reach x = 10 in finite time.

16. Let $f(x) = \min(e^x, e^{-x})$ for any real number x. Then

(a) f has no maximum
(b) f attains its maximum at a point where f'(x) = 0
(c) f attains its maximum at a point where it is not differentiable
(d) M := max(f(x) : x real) < ∞ but there is no number x_0 such that $f(x_0) = M$.

Solution: (C)

17. If ϴ is an acute angle, the maximum value of 3sinϴ + 4cosϴ is

(a) 4
(b) 5
(c) 5√2
(d) 3(1 + √3/2)

Solution: (B)

Now, 3sinϴ + 4cosϴ

$= \sqrt{(3^2 + 4^2)}[\{3/\sqrt{(3^2 + 4^2)}\}\sin\theta + \{4/\sqrt{(3^2+4^2)}\}\cos\theta]$

$= 5\{\cos\alpha\sin\theta + \sin\alpha\cos\theta\}$,

Where, $\cos\alpha = 3/\sqrt{(3^2 + 4^2)}$ and $\sin\alpha = 4/\sqrt{(3^2 + 4^2)}$

⇨ $3\sin\theta + 4\cos\theta = 5(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = 5\sin(\theta + \alpha)$

Now, maximum value of $\sin(\theta + \alpha) = 1$

⇨ Maximum value of 3sinϴ + 4cosϴ is 5.

18. I sold 2 books for Rs. 30 each. My profit on one was 25% and the loss on the other was 25%. Then on whole, I

(a) Lost Rs. 5
(b) Lost Rs. 4
(c) Gained Rs. 4
(d) Neither gained nor lost.

Solution: (B)

Let, the buying prize of first book is Rs. x.

Gain in first book = 25x/100

Selling prize = x + 25x/100 = 125x/100

Now, 125x/100 = 30

⇨ x = 24.

Let, the buying prize of another book is Rs. y.

Loss in that book = 25y/100

Selling prize = y – 25y/100 = 75y/100

Now, 75y/100 = 30

⇨ y = 40.

Gain in first book = (30 – 24) = Rs. 6

Loss in another book = (40 – 30) = Rs. 10

So, I lost (10 – 6) = Rs. 4.

19. Suppose x is an irrational number and a, b, c, d are non-zero rational numbers. If (ax + b)/(cx + d) is rational, then we must have

(a) a = c = 0
(b) a = c, b = d
(c) ad = bc
(d) a + d = b + c.

Solution: (C)

(ax + b)/(cx + d) = p/q

aqx + bq = cpx + dp

(aq – cp)x = dp – bq

x = (dp – bq)/(aq – cp)

20. If a, b, c are real numbers s.t. $x^3 + ax^2 + bx + c = (x^2 + 1)g(x)$ for some polynomial g, then

(a) b = 1, a = c
(b) b = 0 = c
(c) a = 0
(d) None of the above.

Solution: (A)

We have, $x^3 + ax^2 + bx + c = (x^2 + 1)g(x)$

Putting x = i we get, $i^3 + ai^2 + bi + c = (i^2 + 1)g(i)$

⇨ $-i - a + bi + c = (-1 + 1)g(i)$ $(i^2 = -1)$

⇨ (c − a) + i(b − 1) = 0

Equating the real and imaginary part of both sides we get, c = a and b = 1

21. The average scores of 12 students in a test is 74. The highest score is 79. Then, the minimum possible lowest score must be

(a) 25
(b) 12
(c) 19
(d) 28

Solution: (C)

Let all the students except the student who has got lowest mark has got 79 each.

Sum of scores of 12 students = 12×74

Sum of scores of 11 students = 79×11

Minimum possible lowest score = 12×74 − 79×11 = 19

22. If x > y are positive integers such that 3x + 11y leaves a remainder 2 when divided by 7 and 9x + 5y leaves a remainder 3 when divided by 7, then the remainder when x − y is divided by 7, equals

(a) 3
(b) 4
(c) 5
(d) 6

Solution: (D)

(9x + 5y) − (3x + 11y) ≡ 3 − 2 (mod 7)

⇨ 6(x − y) ≡ 1 (mod 7)

Now, if x − y ≡ 3 (mod 7), then 6(x − y) ≡ 4 (mod 7)

So, option (a) is not true.

If x − y ≡ 4 (mod 7(, then 6(x − y) ≡ 3 (mod 7)

So, option (b) is not true.

If $x - y \equiv 5 \pmod 7$ then $6(x - y) \equiv 2 \pmod 7$

So, option (c) is not true.

If $(x - y) \equiv 6 \pmod 7$, then $6(x - y) \equiv 1 \pmod 7$

23. The set of all real numbers which satisfy $(x^2 - 2x + 3)/\sqrt{(x^2 - 2x + 2)} \geq 2$ is

(a) The set of all integers
(b) The set of all rational numbers
(c) The set of all positive real numbers
(d) The set of all real numbers.

Solution: (D)

Now, $x^2 - 2x + 2 = (x - 1)^2 + 1 > 0$

Let, $x^2 - 2x + 2 = a$

Now, the equation becomes, $(a + 1)/\sqrt{a} \geq 2$

⇨ $(a + 1)/2 \geq \sqrt{a}$

Which is true for any $a > 0$ as AM ≥ GM.

We have shown that $a > 0$ always.

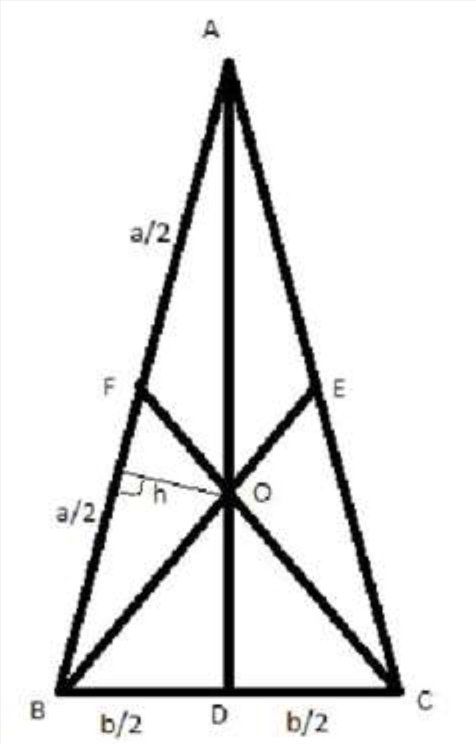
24. Let ABC be a triangle such that the three medians divide it into six parts of equal area. Then, the triangle

(a) Cannot exist.
(b) Can be any triangle
(c) Must be equilateral
(d) Need not be equilateral but must be isosceles.

Solution: (B)

If the triangle is equilateral then it must have all the areas equal.

If it is an isosceles triangle as shown below in figure, then also it can be true.



Now, it is clear that area of triangle BOF = area of triangle AOF (as base is same = a/2 and perpendicular from O to the base is same = h) = area of triangle COE = area of triangle AOE

Also, area of triangle BOD = area of triangle COD.

Now, we will see if area of triangle BOD = area of triangle BOF

Then we have the required condition i.e. 6 triangles have same area.

Now, area of triangle BOF = (1/2)*h)*(a/2)

AD = $\{\sqrt{(a^2 - b^2)}\}/2$

⇨ OD = $(1/3)\{\sqrt{(a^2 - b^2)}\}/2$

⇨ Area of triangle BOD = $(1/2)(1/3)(b/2)\{\sqrt{(a^2 - b^2)}\}/2$

Equating both the area, we get, h = $(b/6a)\sqrt{(a^2 - b^2)}$

Which is very much possible.

⇨ It can be true for equilateral as well as isosceles triangle.

⇨ Option (a), (c), (d) cannot be true.

25. From a bag containing 10 distinct objects, the number of ways one can select an odd number of objects is

(a) 2^{10}
(b) 2^{9}
(c) 10!
(d) 5.

Solution: (B)

Total number of selection = ${}^{10}C_0 + {}^{10}C_1 + \dots + {}^{10}C_{10} = 2^{10}$

Now, number of ways to select an odd number of objects = number of ways to select an even number of objects.

⇨ Number of ways to select an odd number of objects = $2^{10}/2 = 2^9$.

26. Consider the two statements :

(I) Between any two rational numbers, there is an irrational number.

(II) Between any two irrational numbers, there is a rational number.

Then,

(a) Both (I) and (II) are true.
(b) (I) is true but (II) is not
(c) (I) is false but (II) is not
(d) Both (I) and (II) are false.

Solution: (A)

27. **Let $f(x) = ax^2 + bx + c$ where a, b, c are real numbers. Suppose $f(x) \neq x$ for any real number x. Then the number of solutions of $f(f(x)) = x$ in real numbers x is**

(a) 4
(b) 2
(c) 0
(d) Cannot be determined.

Solution: (C)

Now, $f(f(x)) = x$

⇨ $f(ax^2 + bx + c) = x$

⇨ $a(ax^2 + bx + c)^2 + b(ax^2 + bx + c) + c = x$

$a(a^2x^4 + b^2x^2 + c^2 + 2abx^3 + 2acx^2 + 2bcx) + b(ax^2 + bx + c) + c = x$

$a^3x^4 + 2a^2bx^3 + (ab^2 + 2a^2c + ab)x^2 + (2abc + b^2 - 1)x + (ac^2 + bc + c) = 0$

28. Let, S = Σne^{-n} where the summation runs from n = 1 to n = ∞. Then

- **(a) $S \le 1$**
- **(b) $1 < S < \infty$**
- **(c) S is infinite**
- **(d) S = 0.**

Solution: (B)

$S = 1e^{-1} + 2e^{-2} + 3e^{-3} + 4e^{-4} + ...$

$Se^{-1} = \quad 1e^{-2} + 2e^{-3} + 4e^{-4} + ...$

Subtracting we get, $S(1 - e^{-1}) = e^{-1} + e^{-2} + e^{-3} + e^{-4} + ...$

⇨ $S(1-e^{-1}) = e^{-1}/(1-e^{-1})$

29. Let $f(x) = ax^3 + bx^2 + cx + d$ be a polynomial of degree 3 where a, b, c, d are real. Then

- **(a) f(x) -> ∞ as x -> ∞**
- **(b) f is 1-1 as well as onto**
- **(c) The graph of f(x) meets the x-axis in one or three points**
- **(d) f must be onto but need not be 1-1.**

Solution: (D)

Definition of onto function: A function f from A to B is called onto if for all b in B there is an a in A such that f (a) = b. All elements in B are used.

Now, f(x) can take any value.

⇨ f is onto.

Now, f(x) is of degree. 3. So it has either 1 or 3 real root. If it has 3 real roots then f(x) = 0 for 3 values of x.

⇨ f(x) need not be 1-1

Further, putting x = 1 we get, f(1) = a + b + c + d

Putting x = -1 we get f(-1) = -a + b – c + d

Now, f(1) = f(-1) if a + b + c + d = -a + b − c + d i.e. if a + c = 0 which is possible.

f(x) need not be 1 − 1.

30. Let $a_1, \ldots, a_n$ be arbitrary integers and suppose $b_1, \ldots, b_n$ is a permutation of the a_i's. Then the value of $|a_1 - b_1| + |a_2 - b_2| + \ldots + |a_n - b_n|$

- **(a) Is less than or equal to n**
- **(b) Can be an arbitrary positive integer**
- **(c) Can be any even nonnegative integer**
- **(d) Must be 0.**

Solution: (C)

Let, $(a_1, a_2, \ldots, a_n) = (1, 2, \ldots, n)$ and $(b_1, b_2, \ldots, b_n) = (n, n-1, \ldots, 1)$
Now, $|a_1 - b_1| + |a_2 - b_2| + \ldots + |a_n - b_n| = (n - 1) + (n - 2) + \ldots > n$

Option (a) and (d) cannot be true.

Let, $(b_1, b_2, \ldots, b_n) = (2, 3, \ldots, n, 1)$

Now, $|a_1 - b_1| + |a_2 - b_2| + \ldots + |a_n - b_n|$
$= 1 + 1 + \ldots$ (n-1) times + (n− 1)
$= 2(n - 1)$

And in the first case $|a_1 - b_1| + |a_2 - b_2| + \ldots + |a_n - b_n|$

$$> (n - 1) + (n - 2) + (n - 3) = 3(n - 2)$$

Now, $3(n - 2) > 2(n - 1)$

Option (b) cannot be true.