B.Math. (Hons.) Admission Test 2011 Solved Paper

1. The equation of the circle of smallest radius which passes through the points (-1, 0) and (0, -1) is:

(a) $x^2 + y^2 + 2xy = 0$

(b) $x^2 + y^2 + x + y = 0$

(c) $x^2 + y^2 - x - y = 0$

(d) $x^2 + y^2 + x + y + 1/4 = 0$

Solution: (B)

Clearly the circle of option (b) passes through the given two points. No other option satisfies this condition.

2. The function $f(x) = x^2e^{-|x|}$ defined on entire real line is

(a) **Not continuous at exactly one point**

(b) **Continuous everywhere but not differentiable at exactly one point**

(c) **Differentiable everywhere**

(d) **Differentiable everywhere.**

Solution: (C)

Now, $f(x) = x^2e^{-|x|}$

⇨ $f(x) = x^2e^{-x}$ if $x > 0$

⇨ $f(x) = x^2e^{x}$ if $x < 0$

⇨ $f(x) = 0$ if $x = 0$

Now, lim f(x) as x -> 0+

= lim x^2e^{-x} as x -> 0+

= 0 lim f(x) as x -> 0-

= lim x^2e^{x} as x -> 0-

= 0

f(0) = 0

f(x) is continuous everywhere.

Now, lim [{f(x) – f(0)}/(x – 0)] as x -> 0+
= lim {(x^2e^{-x} – 0)/x} as x -> 0+
= lim ($2xe^{-x}$ – x^2e^{-x})/1 as x -> 0+ (Applying L'Hospital rule)
= 0
lim [{f(x) – f(0)}/(x – 0)] as x -> 0-
= lim {(x^2e^{x} – 0)/x} as x -> 0-
= lim ($2xe^{e}$ + x^2e^{x})/1 as x -> 0- (Applying L'Hospital rule)
= 0
So, f(x) is differentiable everywhere.

3. Let c_1 and c_2 be positive real numbers. Consider the function
f(x) = c_1x $\quad 0 \le x < 1/3$
f(x) = $c_2(1 - x)$ $\quad 1/3 \le x \le 1$
If f is continuous and ∫f(x)dx = 1 (integration running from 0 to 1), the value of c_2 is
(a) 2
(b) 1
(c) 3
(d) ½

Solution: (C)

Now, lim f(x) as x -> 1/3-

= lim(c_1x) as x -> 1/3-

= $c_1/3$

lim f(x) as x -> 1/3+

= lim{$c_2(1 - x)$} as x -> 1/3+

= $2c_2/3$ And, f(1/3)

= $2c_2/3$

f is continuous.

⇨ $c_1/3 = 2c_2/3$

⇨ $c_1 = 2c_2$

Now, ∫f(x)dx = 1 (integration running from 0 to 1)

∫f(x)dx (integration running from 0 to 1/3) + ∫f(x)dx (integration running from 1/3 to 1) = 1.
∫$c_1x$dx (integration running from 0 to 1/3 + ∫$c_2(1 - x)$dx (integration running from 1/3 to 1) = 1

⇨ $c_1x^2/2$ (0 to 1/3) – $c_2(1 – x)^2/2$ (1/3 to 1) = 1
⇨ $c_1/18 + 4c_2/18 = 1$
⇨ $2c_2/18 + 4c_2/18 = 1$ (Putting $c_1 = 2c_2$)
⇨ $6c_2/18 = 1$
⇨ $c_2 = 3$

4. Mr. Gala purchased 10 plots of land in the year 2007, all plots costing the same amount. He made a profit of 25 percent on each of the 6 plots which he sold in 2008. He had a loss of 25 percent on each of the remaining plots when he sold them in 2009. If he ended with a total profit of Rs. 2 crores in this project, his total purchase price was
(a) 8 crores
(b) 40 crores
(c) 10 crores
(d) 20 crores.

Solution: (B)
Let purchase price of each plot is x crores.

In 2008 he made profit = x(25/100)×6 = 3x/2

In 2009 he made loss = x(25/100)×4 = x

Net profit = 3x/2 – x = x/2

According to question, x/2 = 2 crores
⇨ x = 4 crores.
⇨ 10x = 40 crores.

5. Let f(x) = xsin(1/x) for x > 0. Then
(a) f is unbounded
(b) f is bounded
(c) lim f(x) as x -> ∞ = 1
(d) lim f(x) as x -> ∞ = 0.

Solution: (C)

lim f(x) as x -> ∞
= lim {xsin(1/x)} as x -> ∞

Let, z = 1/x. As z -> 0 as x -> ∞

So, the limit is lim(sinz/z) as z -> 0 = 1.

6. Let

$f(x) = x^2 \sin(1/x)$, $x \neq 0$
$f(x) = 0$, $x = 0$. Then

(a) f is continuous at x = 0.
(b) f is differentiable, and f′ is continuous.
(c) f is not differentiable at x = 0.
(d) f is differentiable at every x but f′ is discontinuous at x = 0.

Solution: (D)

Now, lim f(x) as x -> 0+ = lim $x^2\sin(1/x)$ as x -> 0+

Let z = 1/x
z -> ∞ as x -> 0

The limit is, lim $(\sin z)/z^2$ as z -> ∞ = 0 (as $-1 \leq \sin z \leq 1$)

Similarly, lim f(x) as x -> 0- = 0

f(x) is continuous everywhere.

Now, lim [{f(x) – f(0)}/(x – 0)] as x -> 0+
= lim $\{x^2\sin(1/x)\}/x$ as x -> 0+
= xsin(1/x) as x -> 0+
= 1

Similarly, lim [{f(x) – f(0)}/(x – 0)] as x -> 0- = 1

f(x) is differentiable everywhere.

Now, $f'(x) = 2x\sin(1/x) + x^2\cos(1/x)(-1/x^2)$
$= 2x\sin(1/x) - \cos(1/x)$

lim f′(x) as x -> 0 = lim {2xsin(1/x) – cos(1/x)} as x -> 0
= does not exist

7. Let $P(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$ be a polynomial of degree n with real coefficients a_i. Suppose that there is a constant c > 0 and an integer k ≥ 0 such that $|P(x)| < cx^k$ for all x > 0. Then

(a) n must be equal to k
(b) The given information is not sufficient to say anything about n
(c) n ≥ k
(d) n ≤ k.

Solution: (D)

Let, n = 1 and so, $P(x) = a_0 + a_1x$

Now, let k = 0

Then $-C < a_0 + a_1x < C$

⇨ $x < (C - a_0)/a_1$

⇨ This is not defined for all x > 0

⇨ $k \geq n$

8. Let f be a strictly increasing function on R, that is f(x) < f(y) whenever x < y. Then

(a) f is a continuous function

(b) f is a bounded function

(c) f is an unbounded function

(d) The given information is not sufficient to say anything about continuity or boundedness of f.

Solution: (D)

9. The minimum value of $x^2 + y^2$ subject to x + y = 1 is

(a) 0

(b) ½

(c) ¼

(c) 1

Solution: (B)

Now, $(x^2 + y^2)/2 \geq \{(x + y)/2\}^2$

⇨ $x^2 + y^2 \geq ½$

10. The number 2532645918 is divisible by

(a) 3 but not 11

(b) 11 but not 3

(c) Both 3 and 11

(d) Neither 3 nor 11.

Solution: (C)

(8 + 9 + 4 + 2 + 5) – (1 + 5 + 6 + 3 + 2) = 28 – 17 = 11

So, divisible by 11.

Now, 28 + 17 = 45

So, divisible by 3 as well.

11. Let p > 3 be a prime number. Which of the following is always false?

- **(a) p + 2 is a prime number.**
- **(b) p + 4 is a prime number.**
- **(c) Both p + 2 and p + 4 are prime numbers.**
- **(d) Neither p + 2 nor p + 4 are prime numbers.**

Solution: (C)

Now, p is a prime> 3.

⇨ $p \equiv 1, -1 \pmod 3$

At first, let $p \equiv 1 \pmod 3$

⇨ $p + 2 \equiv 1 + 2 = 3 \equiv 0 \pmod 3$

Now, $p \equiv -1 \pmod 3$

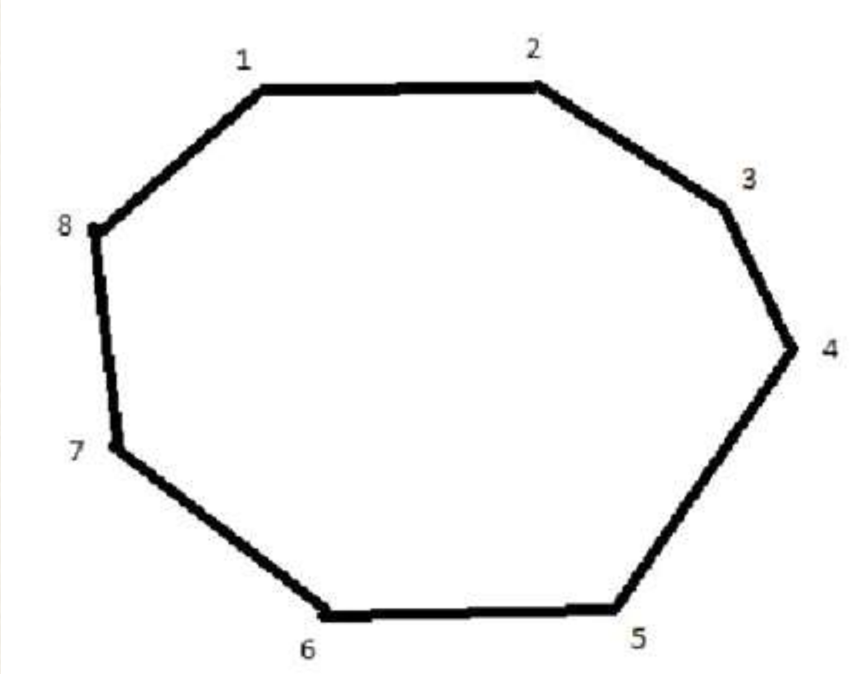
⇨ $p + 4 \equiv -1 + 1 = 0 \pmod 3$

⇨ Either p + 2 or p + 4 i divisible by 3 and both together cannot be prime when p is prime.

12. By a diagonal of a convex polygon, we mean a line segment between any two non-consecutive vertices. The number of diagonals of a convex polygon of 8 sides is:

- **(a) 15**
- **(b) 20**
- **(c) 28**
- **(d) 35.**

Solution: (A)



The diagonals are,

From 1, (1, 3); (1, 4); (1, 5); (1, 6); (1, 7) = 5

From 2, (2, 4); (2, 5); (2, 6); (2, 7); (2, 8) = 5

From 3, (3, 5); (3, 6); (3, 7); (3, 8) = 4 (3, 1 is already taken) From 4, (4, 6); (4, 7); (4, 8) = 3 ((4, 1); (4, 2) are already taken) From 5, (5, 7); (5, 8) = 2 ((5, 1); (5, 2); (5, 3) are already taken) From 6, (6, 8) = 1 ((6, 1); (6, 2); (6, 3); (6, 4) are already taken) Therefore, total number of diagonals = 5 + 5 + 4 + 3 + 2 + 1 = 20.

13. The coefficients of three consecutive terms in the expansion of $(1 + t)^n$ are 120, 210, 252. Then, n must be
(a) 10
(b) 12
(c) 14
(d) 16

Solution: (A)

Let, $^nC_r = 120$, $^nC_{r+1} = 210$ and $^nC_{r+2} = 252$
⇨ $n!/\{(n - r)! \times r!\} = 120$,

⇨ n!/{(n – r – 1)!×(r + 1)!} = 210 and
⇨ n!/{(n – r – 2)!×(r + 2)!} = 252

Now, [n!/{(n – r – 1)!×(r + 1)!}]/[n!/{(n – r)!×r!}] = 210/120

⇨ (n – r)/(r + 1) = 7/4
⇨ 4n – 4r = 7r + 7
⇨ 4n – 11r = 7 ......(A)

Now, [n!/{(n – r – 2)!×(r + 2)!}]/[n!/{(n – r – 1)!×(r + 1)!}] = 252/210

⇨ (n – r – 1)/(r + 2) = 126/105
⇨ 105n – 105r – 105 = 126r + 252
⇨ 105n – 231r = 357 ....(B)

From (A), 84n – 231r = 147 (multiplying both sides by 11)

Now, subtracting from (B) we get,

105n – 84n = 357 – 147

⇨ 21n = 210
⇨ n = 10

14. If 2sec(2α) = tan(β) + cot(β), then α + β can have the value
(a) π/2
(b) π/3
(c) π/4
(d) 0.

Solution: (C)

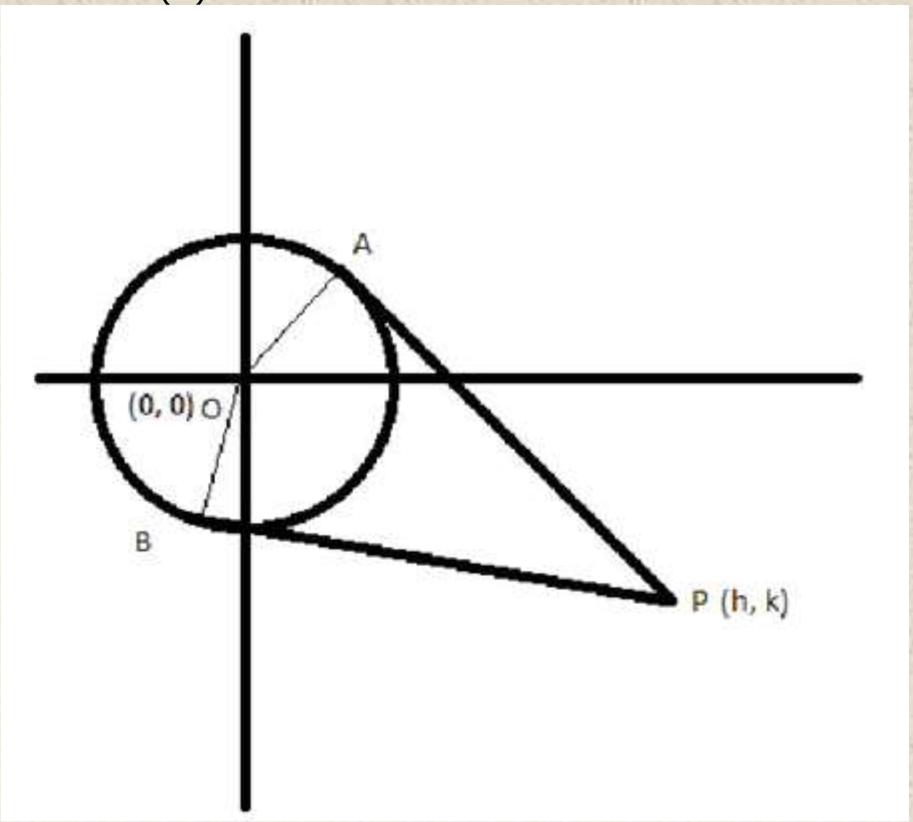
Now, 2sec(2α) = tan(β) + cot(β)

⇨ 2sec(2α) = sin(β)/cos(β) + cos(β)/sin(β)
⇨ 2sec(2α) = $\{\sin^2(\beta) + \cos^2(\beta)\}/\sin(\beta)\cos(\beta)$
⇨ sec(2α) = 1/2sin(β)cos(β)
⇨ 1/cos(2α) = 1/sin(2β)
⇨ cos(2α)= sin(2β)
⇨ 2α = 2β = π/4
⇨ (α + β) = π/4.

15. Consider the unit circle $x^2 + y^2 = 1$. The locus of a point P such that the tangents PA, PB at the points A, B respectively, of the circle are so that angle AOB = 60°, where O is the origin, is

(a) A circle of radius $2/\sqrt{3}$ with centre O.
(b) A circle of radius $\sqrt{3}$ with centre O.
(c) A circle of radius 2 with centre O.
(d) A pair of straight lines.

Solution: (A)



Angle AOB = 60°

⇨ Angle POB = 30°

Now, OB = 1 and OP = $\sqrt{(h^2 + k^2)}$

In triangle BOP, cos30° = OB/OP

⇨ $1/\sqrt{(h^2 + k^2)} = \sqrt{3}/2$

⇨ $h^2 + k^2 = 4/3$

⇨ Locus of point P is, $x^2 + y^2 = 4/3$

16. Let, x, y be integers. Consider the two statements (I) 10x + y is divisible by 7, and, (II) x + 5y is divisible by 7. Then
- **(a) (I) implies (II) but not conversely**
- **(b) (II) implies (I) but not conversely**
- **(c) The two statements are equivalent**
- **(d) Neither statement implies the other.**

Solution: (C)

Now, $10x + y \equiv 0 \pmod 7$

⇨ $3x - 6y \equiv 0 \pmod 7$

⇨ $x - 2y \equiv 0 \pmod 7$

⇨ $x + 5y \equiv 0 \pmod 7$

Now, $x + 5y \equiv 0 \pmod 7$

⇨ $x - 2y \equiv 0 \pmod 7$

⇨ $3x - 6y \equiv 0 \pmod 7$

⇨ $10x + y \equiv 0 \pmod 7$

17. The number of solutions of the equation 6m + 15n = 8 in integers m and n are
- **(a) Zero**
- **(b) One**
- **(c) More than one but finitely many**
- **(d) Infinitely many.**

Solution: (A)

The equation is, $6m + 15n = 8$

Dividing the equation by 3 we get.

$0 + 0 \equiv 2 \pmod 3$
Which is impossible.

No solution.

18. Let A, B be real numbers both greater than 0. The graph of the function $f(x) = Bx^5 + 2Ax + A\sin(2x)$ passes through the two points P = (-1, 2) and Q = (0, 1) for

(a) Finitely many values of A and infinitely many values of B
(b) Infinitely many values of A and infinitely many values of B
(c) No values of A and B
(d) None of the above.

Solution: (C)

The graph of the function passes through Q = (0, 1)

⇨ $f(0) = 1$
⇨ $B\times0^5 + 2A\times0 + A\sin(2\times0) = 1$
⇨ $0 = 1$

Which is impossible.

19. A triangle in the plane has area 1. Then its perimeter (= sum of the lengths of its three sides) p must satisfy

(a) $p < 1$
(b) $p < 2$
(c) $p > 2$
(d) $p = 2$.

Solution: (C)

Now, $\sqrt{s(s - a)(s - b)(s - c)} = 1$

Now, $\{s + (s - a) + (s - b) + (s - c)\}/4 > \{s(s - a)(s - b)(s - c)\}^{1/4}$ (AM > GM)

⇨ $\{4s - (a + b + c)\}/4 > 1$
⇨ $(4s - 2s)/4 > 1$
⇨ $s > 2$
⇨ $p/2 > 2$
⇨ $p > 4$

None of the option matches.

We take option (c) as it is closest.

20. A sequence is defined by $a_1 = 1$ and the inductive formula $a_{n+1} = \sqrt{(1 + a_n^u)}$ where u is a real number greater than 0. If this sequence converges to a finite limit then u must be

(a) > 0
(b) > 2
(c) < 2
(d) = 2.

Solution: (C)

Let u = 2.

Now, $a_2 = \sqrt{2}$, $a_3 = \sqrt{3}$, $a_4 = \sqrt{4}$,... i.e., in general $a_n = \sqrt{n}$ and it is not a converging sequence.

Now, u > 2, let u = 4

Now, $a_2 = \sqrt{2}$, $a_3 = \sqrt{5}$, $a_4 = \sqrt{26}$,.... which is not a converging sequence.

Options (b) and (d) cannot be true.

21. Let a, b, c be three nonzero real numbers. If $f(x) = ax^2 + bx + c$ has equal roots, then a, b, c are in

(a) Arithmetic progression
(b) Geometric progression
(c) Harmonic progression
(d) None of the above.

Solution: (D)

Roots are equal.

⇨ $b^2 = 4ac$

22. Let a, b, c be real numbers such that $3b > a^2$. Then the function g : R -> R given by $g(x) = x^3 + ax^2 + bx + c$ is

(a) One-one an onto
(b) Onto but not one-one
(c) One-one but not onto
(d) Neither one-one nor onto.

Solution: (A)

Let us take any two points x_1 and x_2
Let us see if $g(x_1) = g(x_2)$ or not.

So we have, $x_1^3 + ax_1^2 + bx_1 + c = x_2^3 + ax_2^2 + bx_1 + c$

⇨ $(x_1^3 - x_2^3)\ a(x_1^2 - x_2^2) + b(x_1 - x_2) = 0$

⇨ $(x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) + a(x_1 + x_2)(x_1 - x_2) + b(x_1 - x_2) = 0$

⇨ $(x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2 + ax_1 + ax_2 + b) = 0$

Now, we assume, $x_1^2 + x_1x_2 + x_2^2 + ax_1 + ax_2 + b = 0$

⇨ $x_1^2 + x_1(x_2 + a) + (x_2^2 + ax_2 + b) = 0$

Now, this is a quadratic on x_1^2.

Now, the discriminant of the equation is, $(x_2 + a)^2 - 4(x_2^2 + ax_2 + b) \geq 0$ (for real solution of x_1)

⇨ $x_2^2 + 2ax_2 + a^2 - 4x_2^2 - 4ax_2 - 4b \geq 0$

⇨ $0 \leq a^2 - 2ax_2 - 3x_2^2 - 4b < a^2 - 2ax_2 - 3x_2^2 - 4a^2/3$ (As $3b > a^2$)

⇨ $-a^2/3 - 2ax_2 - 3x_2^2 > 0$

⇨ $a^2 + 6ax_2 + 9x_2^2 < 0$

⇨ $(a + 3x_2)^2 < 0$

⇨ Our assumption was wrong.

⇨ If $g(x_1) = g(x_2)$ then $x_1 = x_2$.

⇨ g(x) is one-one.

Now, $g(x) = x^3 + ax^2 + bx + c$

⇨ $g'(x) = 3x^2 + 2ax + b$

Now, the discriminant of the equation is, $4a^2 - 12b = 4(a^2 - 3b) < 0$ (As $3b > a^2$)

⇨ $g'(x) = 0$ has no solution.

⇨ g(x) doesn't have any extreme point.

⇨ g(x) is stretching towards $-\infty$ to ∞.

⇨ g(x) is onto.

23. Express the polynomial $f(x) = (2 + x)^n$ as $f(x) = c_0 + c_1x + c_2x^2 + \ldots + c_nx^n$, where n is a positive integer. If Σc_j (summation running from j = 0 to j = n) = 81, then the largest coefficient c_j of f is

(a) 64
(b) 16
(c) 24
(d) 32.

Solution: (D)

Let n = 6, then the middle term = $2^3 \times {}^6C_3 = 8\times6\times5\times4/3\times2 = 160 > 81$

Let n = 5, then middle term = $2^2 \times {}^5C_2 = 4\times10 = 40$

There is another middle term and = 40.

First term i.e. $c_0 = 32$

The sum > 81.

Let n = 4,

Then $c_0 = 16$, $c_1 = 8\times{}^4C_1 = 32$, $c_2 = 4\times{}^4C_2 = 24$, $c_3 = 2\times{}^4C_1 = 8$, $c_4 = 1$

Now, $c_0 + c_1 + c_2 + c_3 + c_4 = 16 + 32 + 24 + 8 + 1 = 81$

⇨ n = 4 and largest $c_j = c_1 = 32$

24. Let l, m, n be any three positive such that $l^2 + m^2 = n^2$. Then,

(a) 3 always divides mn
(b) 3 always divides lm
(c) 3 always divides ln
(d) 3 does not divide lmn.

Solution: (B)

Now, $l^2 + m^2 = n^2$

Let 3 does not divide any of l, m and n

⇨ $l \equiv \pm1 \pmod 3$

⇨ $l^2 \equiv (\pm1)^2 = 1 \pmod 3$

Similarly, $m^2, n^2 \equiv 1 \pmod 3$

Now, dividing the equation by 3 we get,

$1 + 1 \equiv 1 \pmod 3$

⇨ $2 \equiv 1 \pmod 3$

Which is impossible.

Let 3 divides n.

Now, dividing the equation by 3 we get,

$1 + 1 \equiv 0 \pmod 3$

Which is impossible.

Let 3 divides any one of m or l

Let 3 divides m.

Now, dividing the equation by 3 we get,

$1 + 0 \equiv 1 \pmod 3$

Which is consistent equation.

⇨ 3 divides any one of l or m.

25. Let $a_1 = 10$, $a_2 = 20$ and define $a_{n+1} = a_{n-1} - 4/a_n$ for $n > 1$. The smallest k for which $a_k = 0$

(a) Does not exist.
(b) Is 200
(c) Is 50
(d) Is 52.

Solution: (D)