ISI BSTAT/BMATH 2013 SOLVED PAPER

1. (B) Let n = 1 then S = {i}

Let, n = 2 then S = {i – 1}

Let n = 3, then S = {i – 1 – i} = {-1}

Let n = 4, then S ={i – 1 – i + 1} = {0}

Let n = 5 then S = {0 + i} = {i}

Let n = 6 then S = {i – 1}

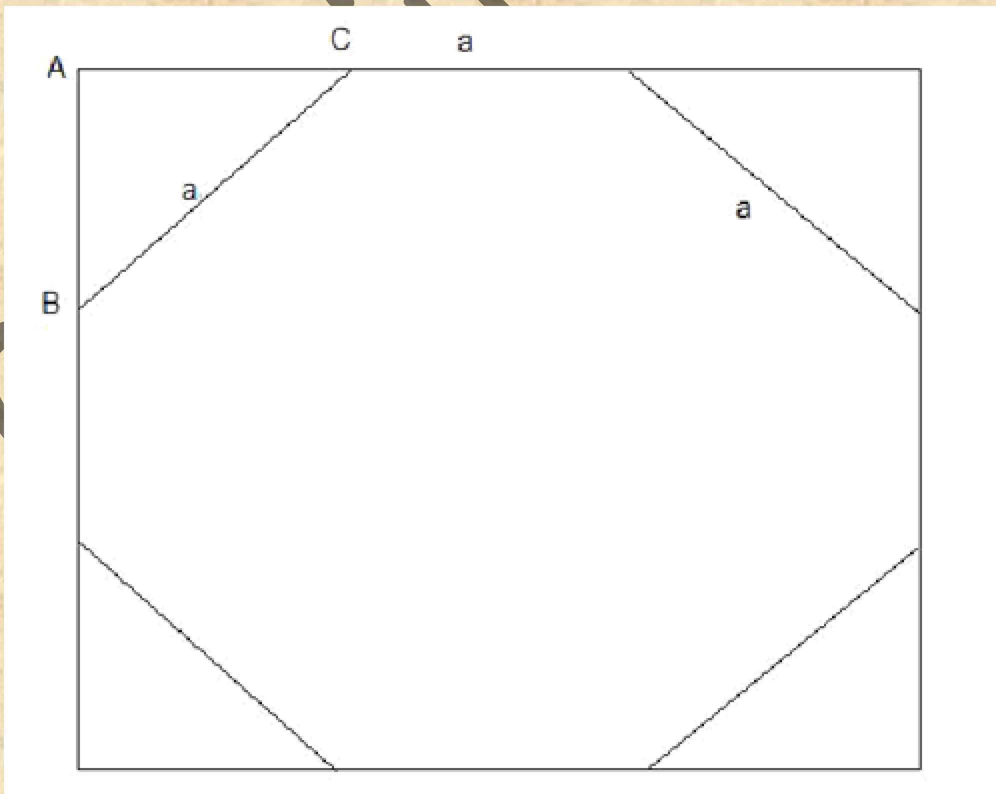
Let n = 7 then S = {i – 1 + i} = {-1}

Let n = 8 then S = {-1 + 1} = {0}

And it runs in a loop.

⇨ **2 real numbers viz. -1, 0**

2. (D)



Let BC = a

Now, from triangle ABC we get, $AC^2 + AB^2 = BC^2$

⇨ **AC = AB = a/√2**

Now, 2AC + a = 1 (As the length of the side of the square is 1)

⇨ **2a/ √2 + a = 1**

⇨ **a = 1/(√2 + 1)**

⇨ **AC = AB = 1/(2 + √2)**

Area of triangle ABC = (1/2){1/(2 + √2)}{(1/(2 + √2)}

= (1/2){1/(6 + 4√2) = (1/4){1/(3 + 2√2)}

Area of the removed portion = 4× (1/4){1/(3 + 2√2)} = 1/(3 + 2√2)
= (3 - 2√2) = $(\sqrt{2} - 1)^2$

3. (C)

The equation of the family of non-vertical lines through (1, 1) is,
y – 1 = m(x – 1)

y = mx + (-m + 1) c
= -m + 1
So, m + c = 1

Putting m = x and c = -y we get the locus as, x – y = 1

4. (A)

Now, cosα + cosβ = -cosγ

⇨ **$(\cos\alpha + \cos\beta)^2 = \cos^2\gamma$**

Similarly, $(\sin\alpha + \sin\beta)^2 = \sin^2\gamma$

Adding the above two equations we get,

$\cos^2\alpha + \sin^2\alpha + \cos^2\beta + \sin^2\beta + 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = \cos^2\gamma + \sin^2\gamma$

⇨ **1 + 1 + 2cos(α – β) = 1**

⇨ **cos(α – β) = -1/2**

5. (D)

Now, lim $(3^x + 7^x)^{1/x}$ as x -> ∞ = lim $7\{1 + (3/7)^x\}^{1/x}$ as x -> ∞ Now, $(3/7)^x$ -> 0 as x -> ∞ and 1/x -> 0 as x -> ∞

So, the limit is $7\times(1 + 0)^0 = 7$

6. (D)

The foci are (2, 2), (-2, -2).

Clearly the distance is 4√2.

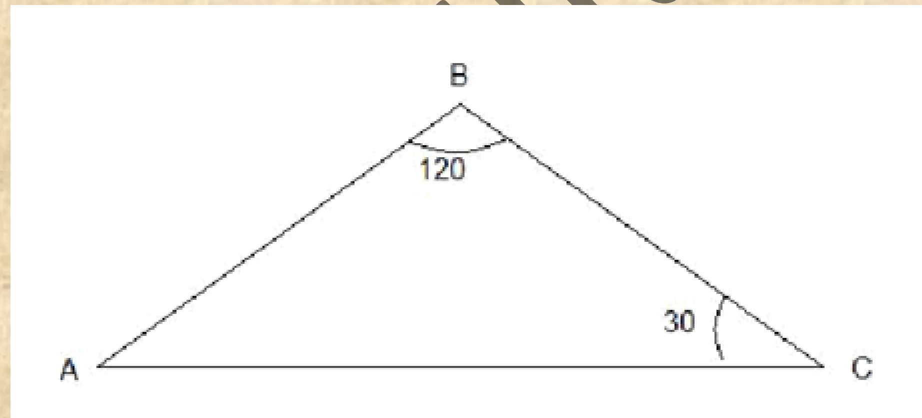
7. (D)

F'(t) = f(t) = 0 has a root in the open interval (0, 1) as f(0) < 0 < f(1).

Now, F"(t) = f'(t) > 0 (As f is increasing function)

⇨ **F(t) has a unique minimum in the open interval (0, 1).**

8. (D)



Now, AC/sin120° = AB/sin30°

⇨ **AC/AB = sin120°/sin30° = (√3/2)/(1/2) = √3**

⇨ **AC : AB = √3 : 1**

9. (A)

Now, $(x - y)^2 \geq 0$

⇨ $x^2 + y^2 \geq 2xy$

Similarly, $y^2 + z^2 \geq 2yz$ and $z^2 + x^2 \geq 2zx$

Adding the three inequalitieS

$x^2 + y^2 + z^2 \geq xy + yz + zx$

$(x^2 + y^2 + z^2)/(xy + yz + zx) \geq 1$

implying $\sin\theta \geq 1$

The equation holds only if $\sin\theta = 1$ i.e. $x^2 + y^2 + z^2 = xy + yz + zx$

The equality holds only if $x = y = z$.

10. **(D)_**

$\sin\theta = 4/5, \cos\theta = 3/5$

$\sec\alpha = 7/4, \cos\alpha = 4/7, \sin\alpha = -\sqrt{33}/7$

$$\sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha$$
$$= (4/5)(4/7) + (3/5)(-\sqrt{33}/7)$$
$$= (16 - 3\sqrt{33})/35$$

11. **(C)**

$z_2 = i^2 + i = i - 1$

$z_3 = (i - 1)^2 + i = -i$

$z_4 = (-i)^2 + i = i - 1$

$z_5 = (i - 1)^2 + i = -i$

And it runs in a loop.

If n is odd then $z_n = -i$

- ⇨ $Z2013 = -i$
- ⇨ $|z2013 - z1| = |-i - i| = 2$

12. (D) Now, $2^4 \equiv 6 \pmod{10}$

⇨ $2^{100} \equiv 6^{25} \equiv 6 \pmod{100}$

Last digit of 5^{100} is 5

Last digit of 8^{100} is 6 as $8^{100} = 2^{300}$ and 300 is divisible by 4.

⇨ Last digit of the given expression = $6 + 5 + 6 \equiv 7 \pmod{10}$

13. (C)

Now, $|x^2 - 4| \leq 5$

⇨ $-5 \leq x^2 - 4 \leq 5$

⇨ $-1 \leq x^2 \leq 9$

⇨ $|x| \leq 3$

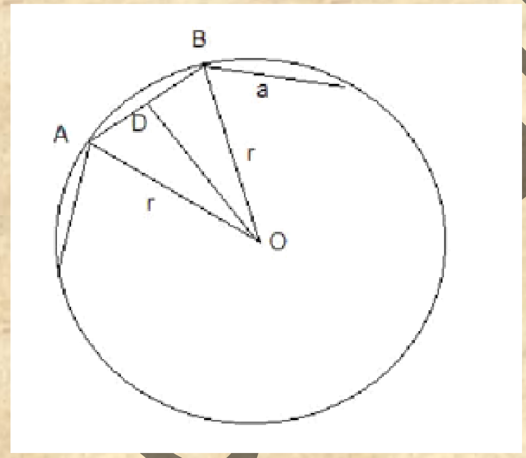
⇨ $-3 \leq x \leq 3$

⇨ $-4 \leq x - 1 \leq 2$

⇨ Maximum value of $|x - 1|$ is $|-4| = 4$.

14. (A)

15. (C)



Now equation of the circle is, $x^2 + y^2 - 6x + 5 = 0$

⇨ $(x - 3)^2 + y^2 = 2^2$

⇨ $r = 2$

Now, Angle A = $(12 - 2)\times180°/12 = 150°$

⇨ Angle OAB $=150°/2 = 75°$

⇨ Angle BOD = 180° - 2×75° = 30°

From triangle OAB, we get, r/sin75° = a/sin30°

⇨ a = rsin30°/sin75°

⇨ AD = a/2 = rsin30°/2sin75°

$OD = \sqrt{\{r^2 - (a/2)^2\}} = r\sqrt{\{1 - \sin^2 30°/4\sin^2 75°\}}$

$= r\sqrt{\{1 - (1/4)/(2 + \sqrt{3})\}} = r\sqrt{\{(4 - 2 + \sqrt{3})\}}$

$= r\sqrt{(2 + \sqrt{3})/2} = r\sqrt{\{(4 + 2\sqrt{3})/2\}/2} = r(\sqrt{3} + 1)/2\sqrt{2}$

Area of triangle OAB = $(1/2)(r\sin30°/\sin75°) \times r(\sqrt{3} + 1)/2\sqrt{2}$

$= r^2/4$

⇨ Area of the polygon = $12 \times (r^2/4) = 3r^2 = 3 \times 2^2 = 12$ square units.

16. (D)

x > 0 and $\log_2 x - 1 > 0$ i.e. x > 2

The function is defined for x > 2

Now, f(x) < 0

⇨ $\sqrt{(\log 2x - 1)} + (1/2)\log 1/2x^3 + 2 < 0$

⇨ $\sqrt{(\log 2x - 1)} < (3/2)\log 2x - 2$

⇨ $\log 2x - 1 < \{(3/2)\log 2x - 2\}^2$

⇨ $\log 2x - 1 < (9/4)(\log 2x)^2 - 6\log 2x + 4$

⇨ $9(\log 2x)^2 - 28\log 2x + 20 > 0$

⇨ $(3\log 2x - 14/3)^2 > 196/9 - 20 = 16/9$

⇨ $3\log 2x - 14/3 > 4/3$

⇨ $3\log 2x > 14/3 + 4/3$

⇨ $3\log 2x > 6$

⇨ $\log 2x > 2$

⇨ $x > 4$

17. (C)

If 7b/8 is an integer then 7b/8 – a = 1 (strictly less than)

If 7b/8 is not an integer then 7b/8 – a = fraction < 1.

⇨ (1) is correct.

Now, if (7b/8) is an integer then 7b/8 – a = 1 > 1/8

Now, if we divide 7b by 8 then remainder can be 1, 2, …, 7.

Now, the least remainder is 1. So if 7b ≡ 1 (mod 8)

Then 7b/8 = a + 1/8

⇨ **7b/8 – a = 1/8**

In other cases 7b/8 – a > 1/8

⇨ **(2) is also correct.**

18. (B)

Clearly the limit -> ∞

19. (B)

Now, $f_{m,n}(x)$ = mx + n

Now, g(f(x)) = x

⇨ **g(mx + n) = x**

Putting, x = (x – n)/m we get,

g(x) = (x – n)/m

Also, f(g(x)) = f((x – n)/m) = m(x – n)/m + n = x

So, we see that if there is an element g such that m divides x – n then f is invertible.

Now, depending on values of m, n there can be infinitely many invertible and also infinitely many non-invertible elements in F.

20. (C)

The teams are {P_1, P_2}; {P_1, P_3}; {P_1, P_4}; {P_1, P_5}; {P_1, P_6} {P_2, P_3}; {P_2, P_4}; {P_2, P_5}; {P_2, P_6} {P_3, P_4}; {P_3, P_5}; {P_3, P_6} {P_4, P_5}; {P_4, P_6} {P_5, P_6}

Now, the first row teams each can play 6 matches. Thus making 6×5 = 30 matches

The second row teams can play 3 matches each. Thus making 4×3 = 12 matches. (Do not go to upper rows as the matches has already been considered)

The third row team can play 1 math each. Thus making 1×3 = 3

matches. There are no more combination available for 4th and 5th rows.

Thus, total number of matches = 30 + 12 + 3 = 45.

21. (A)

Now, $9\cos^2\theta + 16\sec^2\theta = (3\cos\theta - 4\sec\theta)^2 + 24$

Now, $f(\theta)$ will be minimum when $3\cos\theta - 4\sec\theta$ attains minimum value.

⇨ **The minimum value of $9\cos^2\theta + 16\sec^2\theta = 9\times1 + 16/1 = 25$.**

22. (B)

$100! - 101! + \ldots - 109! + 110! = 100!(1 - 101 + 101\times102 - \ldots - 101\times102\times..109 + 101\times102\times\ldots110)$

Now, $-101\times102\times..105 + 101\times102\times\ldots106 - \ldots + 101\times102\times\ldots110$ has 2 0's at the end.

Now, $1 - 101 + 101\times102 - 101\times102\times103 + 101\times102\times103\times104$ has last digit $= 1 - 1 + 2 - 6 + 4 = 0$

Now, 100! Has $[100/5] + [100/5^2] = 20 + 4 = 24$ 0's

So, there are 24 + 1 = 25 0's.

23. (C)

Clearly, $10^a = 1\times10\times10^2\times\ldots\times10^{10} = 10^{(1+2+\ldots+10)} = 10^{10*11/2} = 10^{55}$

⇨ **a = 55.**

Let, $f(x) = (1 + x)(10 + x)(10^2 + x)\ldots(10^{10} + x)$

⇨ **$f'(x) = (10 + x)(10^2 + x)\ldots(10^{10} + x) + (1 + x)(10^2 + x)\ldots(10^{10} + x) + \ldots + (1 + x)(10 + x)\ldots(10^9 + x)$**

Now, $f'(0) = 10^b$

⇨ **$10*10^2*\ldots10^{10} + 1*10^2*\ldots10^{10} + \ldots + 1*10*10^2*\ldots*10^9 = 10^b$**

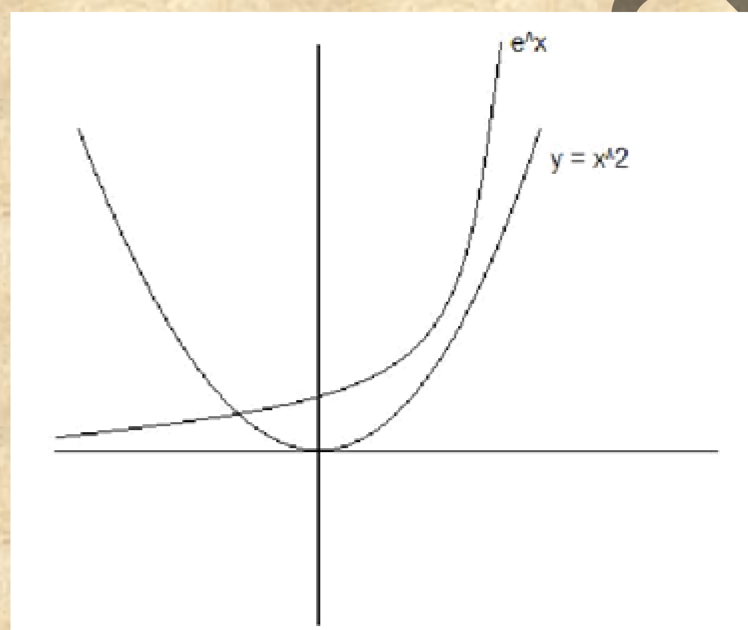
⇨ **$10^b = (1*10*10^2*\ldots10^{10})(1 + 10^{-1} + 10^{-2} + \ldots + 10^{-10})$**

- ⇨ $10^b = 10^{(1 + 2 + .. +10)}*1*(1 - 10^{-11})/(1 - 10^{-1})$
- ⇨ $10^b = 10^{10*11/2}*(1 - 10^{-11})/(1 - 10^{-1})$
- ⇨ $10^b = 10^{55}*(1 - 10^{-11})/(1 - 10^{-1})$
- ⇨ $10^b = 10^a*(1 - 10^{-11})/(1 - 10^{-1})$
- ⇨ $10^{b - a} = (1 - 10^{-11})/(1 - 10^{-1}) > 1$
- ⇨ $10^{b - a} > 10^0$
- ⇨ $b - a > 0$
- ⇨ $b > a$

As a and b are both integers, [b] > [a]

24. (C)

25. (B)



26. (A)

Now, $x^4 + x^2 + 1 = 0$

- ⇨ $x^2 = \{-1 \pm \sqrt{(1 - 4)}\}/2 = (-1 \pm i\sqrt{3})/2$
- ⇨ $x^2 = (-2 \pm i2\sqrt{3})/4$
- ⇨ $x^2 = (-3 \pm i2\sqrt{3} + 1)/4$
- ⇨ $x^2 = (i\sqrt{3} \pm 1)^2/4$

- ⇨ $x = \pm(i\sqrt{3} \pm 1)/2$
- ⇨ $x = -w, w, w^2, -w^2$ (where w is complex cube root of unity)
- ⇨ $\alpha_1^4 + \alpha_2^4 + \alpha_3^4 + \alpha_4^4 = (-w)^4 + w^4 + (w^2)^4 + (-w^2)^3$

$= w + w + w^2 + w^2 = -1 - 1 = -2$

27. (B)

Clearly, the answer is 5:26 p.m. as the hour hand has not crossed the mid-point of 5 and 6 so at 5:26 the angle will be smaller than angle at 5:29 p.m.

28. (B)

Now, $f(x) = a_{10}(e^{10x}/10) + a_9(e^{9x}/9 + \dots + a_1e^x + a_0x$

$f'(x) = a_{10}e^{10x} + a_9e^{9x} + \dots + a_1e^x + a_0$

Now, all the roots of P(x) < 1.

For all real numbers greater than 1 P(x) is either negative or positive.

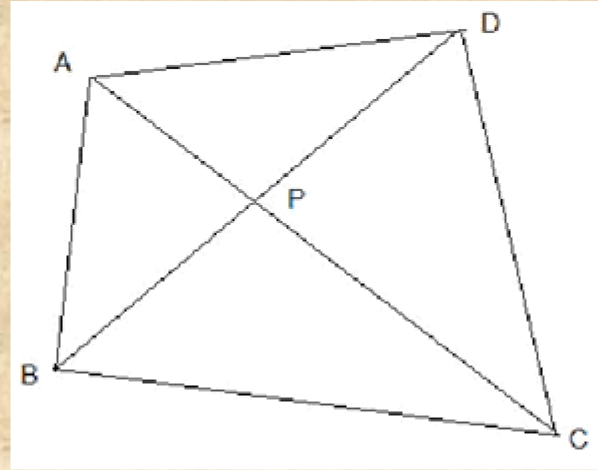
Because if P(a) is positive and P(b) is negative where a, b > 1 then there is a root between P(a) and P(b) which contradicts the statement that all roots of P(x) < 1.

Now, $f''(x) = P(e^x)$ **which is either greater than 0 for all values of x or less than 0 for all values of x.**

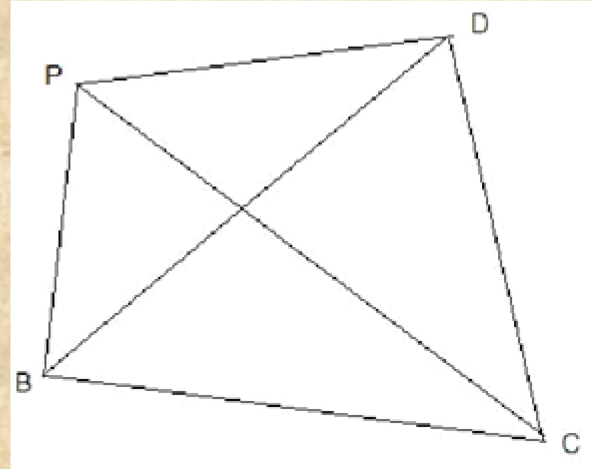
Now, $e^x < 1$ **when x < 0 but x > 0.**

- ⇨ e^x **is always greater than 1 for x > 0**
- ⇨ **f(x) is either increasing or decreasing.**

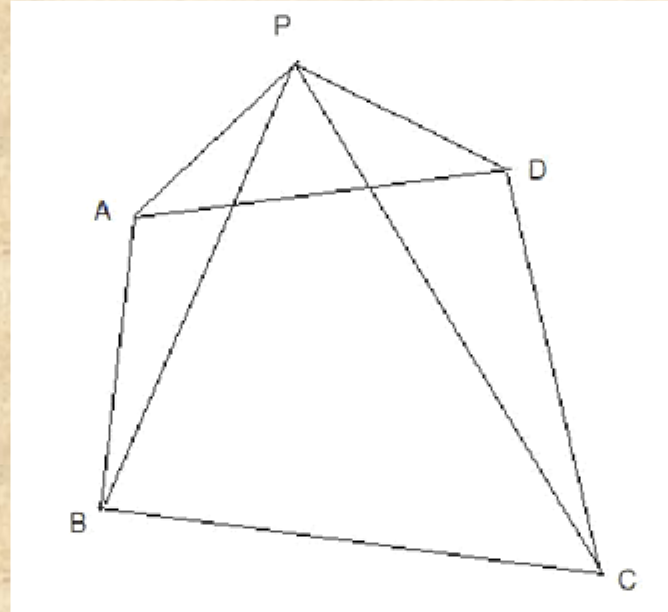
29. (C)



Clearly if the point P is on the intersection of the diagonals then the area of the six triangles = area of the quadrilateral.



If the point P is on vertex A then triangle PCD and PBC cover the area of the quadrilateral. Now consider one more triangle PBD and the sum of the area is more.



If P is outside the quadrilateral then triangles PDC, PBC, PAB cover the quadrilateral and also extra area.

Now, if the point P was inside the quadrilateral then also the area of the 4 triangles PAB, PBC, PCD, PDA cover the area of the quadrilateral but area of triangles PAC and PBD are not zero.

30. (B)

Now, $x^3 + 3x^2y + 3xy^2 + y^3 - x^2 + y^2 = 0$

⇨ $(x + y)^3 - (x^2 - y^2) = 0$

⇨ $(x + y)^3 - (x + y)(x - y) = 0$

⇨ $(x + y)(x^2 + 2xy + y^2 - x + y) = 0$