STATISTICAL PROCESS & QUALITY CONTROL TECHNIQUES

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STATISTICAL QUALITY CONTROL

Introduction: — The most important word in the term 'Statistical Quality Control' is quality.

[A] Quality and Quality Control: — The quality of a product is the most important property that one desires while purchasing it. A product is of good quality if it meets the required specifications, otherwise not. By quality, we mean an attribute of the product that determines its suitability or fitness for use.

Quality control is a powerful productivity technique for effective diagnosis of lack of quality in any of the materials, processes, machines, etc. Quality control covers all the factors and processes of production which may be broadly classified as follows:

(i) Quality of materials. Material of good quality will result in smooth processing thereby reducing the waste and increasing the output. It will also give better finish to end products.

(ii) Quality of manpower. Trained and qualified personnel will give increased efficiency due to the better quality production through the application of skill and also reduce production cost and waste.

(iii) Quality of machines. Better quality equipment will result in efficient work due to lack of scarcity of breakdowns and thus reduce the cost defectives.

(iv) Quality of management. A good management is imperative for increase in efficiency, harmony in relations, growth of business and markets.

Chance and Assignable Causes of Variation

Variation in the quality of manufactured product in the repetitive process in industry is inherent and inevitable. These variations are broadly classified as being due to two causes viz.,

(i) Chance causes, and
(ii) assignable causes.

(i) Chance causes :~ Some "stable pattern of variation" or "a constant cause system" is inherent in a manufacturing process. This pattern results from some minor causes or this variation to which no reason can be assigned, and is of random nature. Therefore, these causes of variation are known as chance causes. The variation due to these causes is beyond the control of human hand and can not be prevented or eliminated under any circumstances. One has got to allow for variation within this suitable pattern, usually termed as allowable variation. This type of variation is tolerable and does not affect the quality and the utility of the process. The range of such variation is known as natural tolerance of the process.

(ii) Assignable Causes :~ Sometimes, the products show marked deviation from the given specifications of a product. This affects the utility of the product and causes worry to the manufacturer. Such a major variation from the specifications may be due to various reasons, such as, defective raw materials, faulty equipment, negligence of the operators, wrong or improper handling of the machines, etc. these causes are non-random and known as so called 'assignable causes'. These causes can be identified and eliminated and are to be discovered in a defective production process. This type of variation due to assignable causes is termed as preventable variation.

What do you mean by SQC?

By statistical quality control we mean the various statistical methods used for the maintenance of quality in a continuous flow of manufactured products. The main purpose of SQC is to derive statistical methods for seperating allowable variation from preventable variation, so that we may take appropriate steps as quickly as possible whenever assignable causes are operating in the process. The elimination of assignable causes of erratic fluctuations is described as bringing a process under control. A production process is said to be in a state of statistical control, if it is governed by chance causes alone, in the absense of assignable causes of variation.

" SQC is simply a statistical method for determining the extent of which quality goals are being met without necessarily checking every item produced and for indicating whether or not the variations which occur are exceeding normal expectations. SQC also enables us to decide whether to reject or accept a particular product." — Grant.

Uses of S.Q.C. :– We briefly outline some of the advantages that might result when a process is brought in good statistical control.

1. The act of getting a process in statistical control involves the identification and elimination of assignable causes of variation and possibly the inclusion of good ones viz., new material on methods.
2. It tells us when to leave a process alone and when to take action to correct troubles, thus preventing frequent and unwarranted adjustments.
3. If a process in control is not good enough, we shall have to make more or less a radical change in the process – just meddling with it won't help.
4. It provides better quality assurance at lower inspection cost.
5. The very presence of a quality control scheme in a plant improves and alerts the personnel. Such a scheme is likely to breed 'quality consciousness' throughout the organisation which is of immense long run value.
6. S.Q.C. reduce waste of time and material to the absolute minimum by giving an early warning about the occurrence of defects.

Remark:–

1. An S.Q.C. department is, thus, an essential part of a modern plant, and its important functions are as follows;

(i) Evaluation of quality standards of incoming materials, products in process and of finished goods.

(ii) Judging the conformity of the process to established standards and taking suitable action when deviations are noted.

(iii) Evaluation of optimum quality obtainable under given conditions.

(iv) Improvement of quality and productivity process control and experimentation.

2. Advantages of Quality control in industry :

Planned collection of data, analysis and interpretation
↓
Improvement in product quality and design
Reduction is scrap
Saving in excess use of materials
Reduction in inspection
Quality consciousness
Greater consumer satisfaction
↓
Enhanced Productivity

The meaning of Control: —— Variability is of two types

(i) systematic, which is attributable to assignable causes

(ii) random, which is due to a number of small independent causes within a system of causes, i.e. due to chance causes.

When we have eliminated all assignable causes of variation which is economical to eliminate, there still remains a type of variability which may behave statistically in a way that we call random and this due to chance causes.

Thus, if all non-random types of variation have been eliminated, then we have a manufacturing process operating in a random manner and consequently the probability distn. of the random variation can be obtained. A process that is operating with only chance causes of variation is said to be in <u>statistical control</u>.

A process that is operating in the presence of assignable causes is said to <u>out of Control</u>.

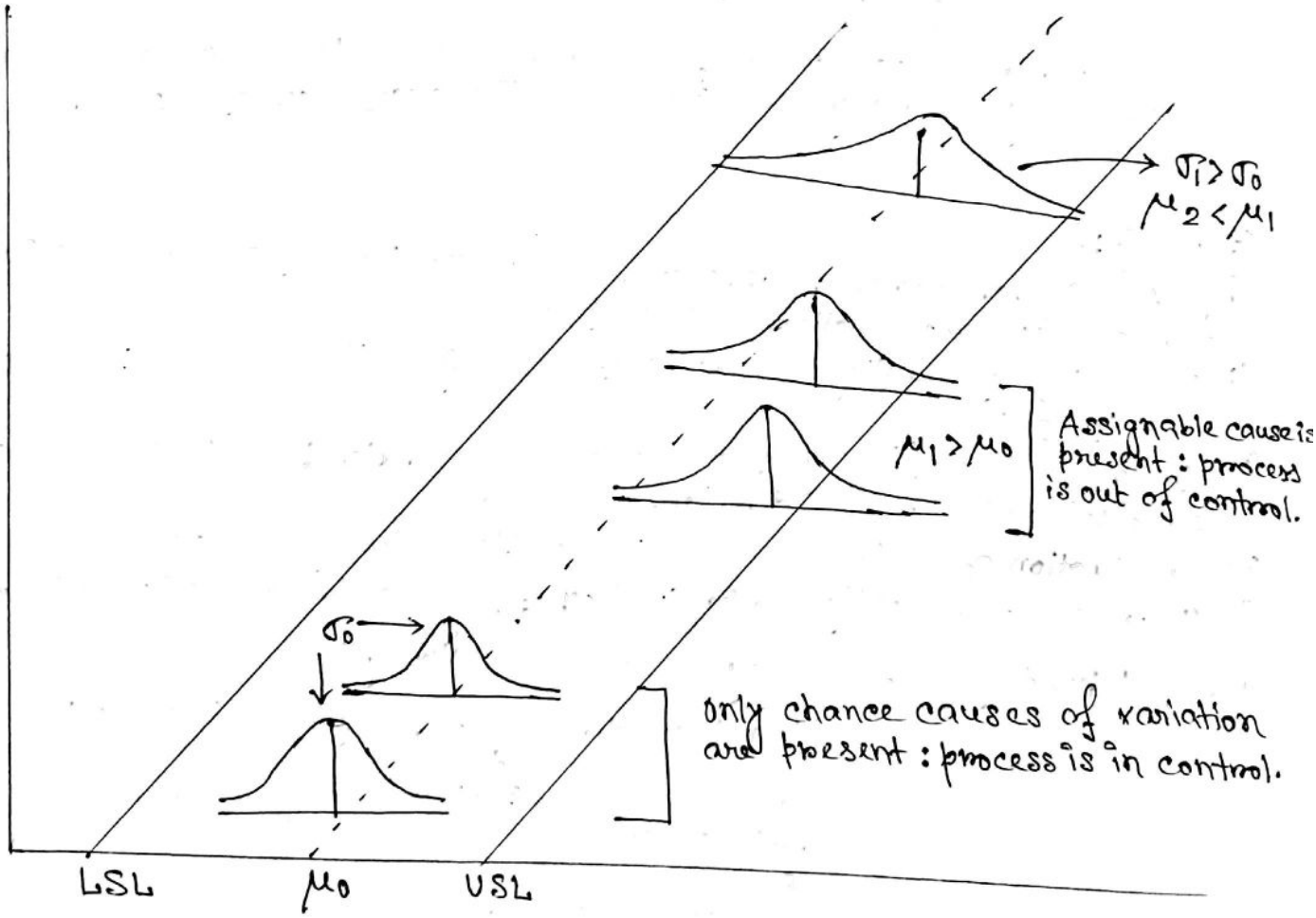


Fig: Chance and assignable causes of variation

Process control and Product control:

It is appearent that a manufacturing is faced with two quality control problems:

(i) His manufacturing process should be so controlled that the proportion of defective units is not excessive.

(ii) He should not ship out lots that contain an excessive proportion of defective pieces. We should refer to these two aspects of quality control as (a) Process Control (b) Product Control or lot control.

It is important to realise that the process may be in satisfactory control, so that the number of defective items will not be excessive for the entire output over a long period of time, individual lots, occassionally may not be satisfactory and the objectives of process control and product control are distinct. The primary object of process control is to keep the process in control. The main statistical tool is the control chart. The primary object of product control is to decide whether to accept or reject a lot on the basis of evidence afforded by one or more samples drawn at random from the lot in question and it is achieved through sampling inspection.

If the process is kept in control, product control is made more economical. If the process is in control one can make a valid estimate of the quality being manufactured. Knowledge of the process quality, in turn, may enable one to select the most economical sampling inspection plan.

(a) Process Control :~ A process that is operating with only chance causes is said to be 'in control' and a process that is operating in the presence of assignable causes is said to be out of control. A major objective of process control is to quickly detect the occurrence of assignable causes of process shifts so that the investigation of the process and the corrective action may be undertaken before many nonconforming units are manufactured. The question to be answered by the "process control" is:

"Do the samples show statistical control?" ⇔ "Do the samples indicate a stable pattern of variation?" ⇔ "Is there one popln. from which the samples appear to come?"

In quality control in manufacturing, the answer "No, this is not a constant-cause system", leads to a hunt for an assignable cause of variation, and an attempt to remove it, if possible. The answer, "Yes, this is a constant-cause system", leads to leaving the process alone, making no effect to hunt for causes of variation.

Control charts:

Shewhart's control chart provides a powerful tool of discovering and correcting the assignable causes of variation outside the "stable pattern" of chance causes, thus enabling us to stabilize and control our processes at desired performance; and thus being the process under statistical control. A typical control chart is shown in the figure, which is a graphical display of a quality characteristic that has been measured from a sample versus the sample number.

A typical control chart consists the following three horizontal lines:

(1) A central line (CL), indicating the desired standard or the level of the process.

(2) Upper control limit (UCL), indicating the upper limit of tolerance.

(3) Lower control limit (LCL), indicating the lower limit of tolerance.

In the control chart, UCL and LCL are usually plotted as dotted lines and CL is plotted as a bold line.

We may give a general model for a control chart.

Let T be a (sample) statistic that measures some quality characteristic of interest.

Suppose that $E(T) = \mu_T$, and, $Var(T) = \sigma_T^2$, when the process is in control.

Then CL, UCL, LCL become

$$UCL = \mu_T + L\sigma_T$$
$$CL = \mu_T$$
$$LCL = \mu_T - L\sigma_T ;$$

where, L is the 'distance' of the control limits from the central line, expressed in standard deviation units.

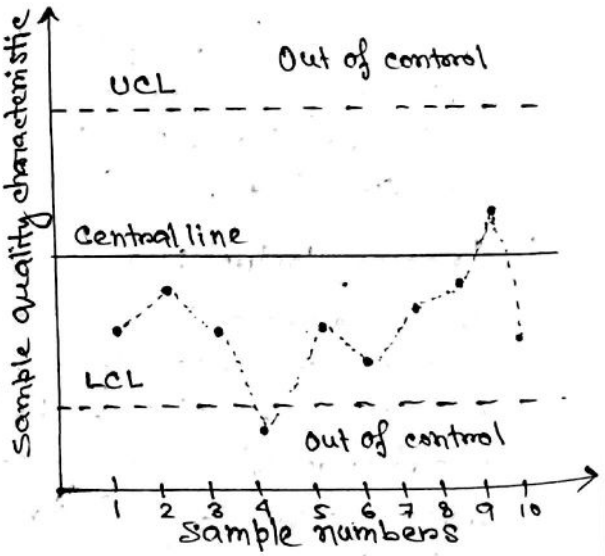


Fig:- Outline of a control chart

This general theory of control charts was first proposed by Dr. Walter Shewhart, and control charts developed according to the principles are often called Shewhart control charts.

The appropriateness of 3-σ limits:

3-σ limits:
$$\left.\begin{aligned} UCL &= \mu_T + 3\sigma_T \\ CL &= \mu_T \\ LCL &= \mu_T - 3\sigma_T \end{aligned}\right\}$$
, were proposed by Dr. Shewhart for his control charts for various considerations, the main being probabilistic considerations.

Here T is a statistic that measures some quality characteristic of the popln. If the process is in control, then, let $E(T) = \mu_T$ and $V(T) = \sigma_T^2$ and the fluctuations in the value of T from the sample to sample should be due to random variation alone.

Note that, by Chebyshev's inequality, $P[|T-\mu_T| < 3\sigma_T] > 1 - \frac{1}{9}$

$\Leftrightarrow P[\mu_T - 3\sigma_T < T < \mu_T + 3\sigma_T] > \frac{8}{9} \simeq 0.9$, in the case where the process is in control, whatever the distn. of T may be.

In particular, in case T is normally distributed and the process is in control, $P[\mu_T - 3\sigma_T < T < \mu_T + 3\sigma_T] = 2\Phi(3) - 1 = 0.9973$

$\Rightarrow P[|T-\mu_T| > 3\sigma_T] = 0.0027$; that is, the probability that a random value of T goes outside the 3-σ limits is 0.0027, which is very small.

The rule for establishing the control limits depends on the control over the two types of errors — (i) the error of hunting for trouble or assignable causes when it is absent (ii) the error of not hunting for trouble or assignable causes when it is really present.

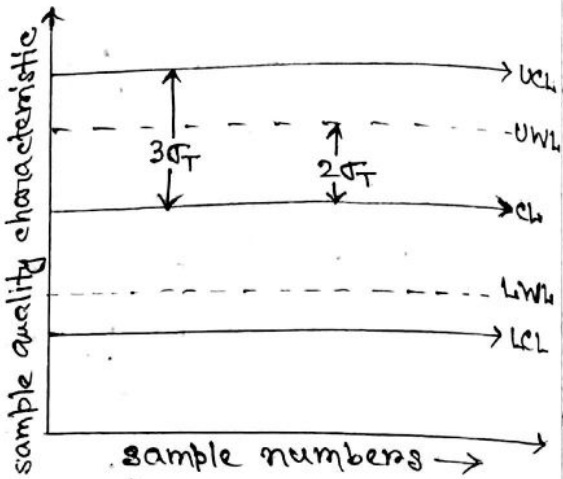
It has been pointed out that as long as the samples are really random samples from one population (or, from a process which is in control), the observed value of T will nearly always fall within 3-σ limits. Also, the probability of type I error, i.e. the probability of indication of out of control when the process is in control, i.e. $P[|T-\mu_T| > 3\sigma_T \mid \text{the process is in control}] = 0.0027$, if T is normally distributed. The 3-σ limits seldom make the error of indicating an assignable cause of variation when there is no assignable causes present. Therefore, if the observed T for the i^{th} sample lies between $\mu_T - 3\sigma_T$ and $\mu_T + 3\sigma_T$, it is taken to be a fairly good indication of non-existance of assignable causes of variation at the time when the i^{th} sample was taken and if the observed T for the i^{th} samples lies outside the 3-σ limits, it is considered to be a danger signal indicating that some assignable cause has present and it must be identified and eliminated.

Instead of using 3-σ limits, we may use other limits, as for e.g., if $L = 3.09$ then:

$$UCL = \mu_T + 3.09\sigma_T$$
$$CL = \mu_T$$
$$LCL = \mu_T - 3.09\sigma_T$$

with probability of type I error 0.002.

Warning limits :~ The outer limits - say, 3-σ limits — are usual action limits. The inner limits, usually at two-sigma, are called warning limits. If one or more points fall between the warning limits and the control limits, or very close to the warning limit, we should be suspicious that the process may not be operating properly, one possible action to take when this occur is to increase the sample size so that more information about the process can be obtained quickly. Process schemes that the change of the sample size depending on the position of the current sample value is called variable sample size.



Rational Subgroups :~ [CU]

To explain this term, suppose that we are using some control chart to detect changes in process quality. A fundamental idea in the use of control charts is the collection of sample data according to what Shewhart called the rational subgroups. The rational subgroup should concept means that subgroups or samples be selected so that if assignable causes are present, the chance for differences between subgroups will be maximized, while the chance for differences due to these assignable causes within a subgroup will be minimized. The use of such subgroups would tend to reveal assignable causes of variation.

When control charts are applied to production processes, the time order of production is logical basis for rational subgrouping. Each sample consists of units that were produced at the same time (or, as closely together as possible). It minimizes the chance of variability due to assignable causes within a sample, and it maximizes the chance of variability between the samples if assignable causes are present. Time order is frequently a good basis for forming subgroups because it allows us to detect assignable causes that occur over time.

There are other bases for forming rational subgroups. For example, a group of machines in a factory may have different variation, and it may be necessary to have different subgroups for different machines, or for different operators or different shifts.

The rational subgroup concept is very important. The proper selection of subgroups requires careful consideration of the process, with the objective of obtaining as much useful information as possible from the control chart analysis.

Control charts for Variables :~ When dealing with a quality characteristic that is a variable, it is usually necessary to monitor both the mean value of the quality characteristic and its variability. Control of the process mean quality level is done with the control chart for mean or $\bar{x}$ chart. Process variability can be monitored with either a control chart for the standard deviation, called the S chart, or a control chart for the range, called a R chart. The R chart is more widely used. Usually, seperate $\bar{x}$ and R charts are maintained for each quality characteristic of interest.

The four types of situation that may be encountered here are: (i) the process is in control, (ii) the mean is out of control but not the variability, (iii) the variability is out of control but not the mean, (iv) both mean and variability are out of control.

We have assumed that the distribution of the quality characteristic is normal. However, the above assumption is still approximately correct if the underlying distribution is non-normal, because of the Central-limit theorem.

[A]. Control charts for $\bar{x}$ and R :~ Suppose that a quality characteristic is normally distributed with mean μ and standard deviation σ, where both μ and σ are usually unknown. If $x_1, x_2, \ldots, x_n$ is a sample of size n, then the sample mean is

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} \quad \text{and} \quad \bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right).$$

Control charts for mean or $\bar{x}$ chart :—— [C.U.]

Case I: Standards given

Suppose that the values for μ and σ are specified as μ_0 and σ_0. Then the control chart for $\bar{x}$ is given by

$$LCL = \mu_{\bar{x}} - 3\sigma_{\bar{x}} = \mu_0 - 3\frac{\sigma_0}{\sqrt{n}} = \mu_0 - A\sigma_0$$

$$CL = \mu_{\bar{x}} = \mu_0 = \mu_0$$

$$UCL = \mu_{\bar{x}} + 3\sigma_{\bar{x}} = \mu_0 + 3\frac{\sigma_0}{\sqrt{n}} = \mu_0 + A\sigma_0, \text{ where, } A = \frac{3}{\sqrt{n}}.$$

Case II: Standards not given

In practice, we usually will not know μ and σ; therefore, they must be estimated from preliminary samples taken when the process is thought to be in control. Suppose that 'm' samples are available, each containing n observations on the quality characteristic. Let $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m$ be the means of the samples. Then an unbiased estimator of μ is the grand mean

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \cdots + \bar{x}_m}{m}.$$

Let $R_1, R_2, \ldots, R_m$ be the ranges of the m samples. The average range is $\bar{R} = \frac{R_1 + R_2 + \cdots + R_m}{m}$.

[The RV $W = R/\sigma$ is called the relative range and $E(W) = d_2$, a function of the sample size n. Consequently, an estimator of σ is $\frac{R}{d_2}$.]

Then, $\hat{\sigma} = \frac{\bar{R}}{d_2}$ is an unbiased estimator of σ.

If we use $\bar{\bar{x}}$ as an estimator of μ and $\frac{\bar{R}}{d_2}$ as an estimator of σ, then the control chart for $\bar{x}$ is given by

$$LCL = \bar{\bar{x}} - \frac{3}{d_2\sqrt{n}}\bar{R} = \bar{\bar{x}} - A_2\bar{R}$$
$$CL = \bar{\bar{x}} = \bar{\bar{x}}$$
$$UCL = \bar{\bar{x}} + \frac{3}{d_2\sqrt{n}}\bar{R} = \bar{\bar{x}} + A_2\bar{R}$$

, where $A_2 = \frac{3}{d_2\sqrt{n}}$ is tabulated for various sample sizes. [C.U.]

Control chart for range or R-chart : — Assuming that the quality characteristic is normally distributed, then the relative range $W = \frac{R}{\sigma}$ has mean $E(W) = d_2$ and $Var(W) = d_3$.

Then $\mu_R = E(R) = d_2\sigma$ and $\sigma_R = d_3\sigma$.

Case-I : Standard given

To construct the R-chart with a standard value σ_0 of σ. Then the control chart for the range is given by

$$LCL = \mu_R - 3\sigma_R = d_2\sigma_0 - 3d_3\sigma_0 = D_1\sigma_0$$
$$CL = \mu_R = d_2\sigma_0$$
$$UCL = \mu_R + 3\sigma_R = d_2\sigma_0 + 3d_3\sigma_0 = D_2\sigma_0$$

, where $D_1 = d_2 - 3d_3$ and $D_2 = d_2 + 3d_3$ are tabulated for different values of 'n'.

Case-II : Standard not given

Here the process s.d. σ is unknown. To determine the control limits we need an estimator of μ_R as well as σ_R.

Since $\mu_R = d_2\sigma$ and $\sigma_R = d_3\sigma$, hence $\hat{\sigma} = \frac{\bar{R}}{d_2}$ is an unbiased estimator of σ and consequently $\hat{\sigma}_R = d_3\frac{\bar{R}}{d_2}$ is an unbiased estimator of σ_R.

Hence, the control chart for the range is given by

$$LCL = \hat{\mu}_R - 3\hat{\sigma}_R = \bar{R} - \frac{3d_3}{d_2}\bar{R} = D_3\bar{R}$$
$$CL = \hat{\mu}_R = \bar{R}$$
$$UCL = \hat{\mu}_R + 3\hat{\sigma}_R = \bar{R} + \frac{3d_3}{d_2}\bar{R} = D_4\bar{R}$$

, where, $D_3 = \left(1 - \frac{3d_3}{d_2}\right)$, $D_4 = \left(1 + \frac{3d_3}{d_2}\right)$ are tabulated for different values of 'n'.

B. Control charts for $\bar{x}$ and S : ~*

The R chart is relatively insensitive to shifts in the process S.d. for small samples. Larger samples would seem to be more effective but we also know that the range method for estimating the standard deviation drops dramatically in efficiency as n increases. Consequently, for large n, say, $n > 10$, it is probably best to use control charts based on S instead of R.

Construction :

If the quality characteristic x is normally distributed with mean μ and standard deviation σ. If $x_1, x_2, \ldots, x_n$ be a sample of size n, then $\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$, where $S^2 = \frac{1}{(n-1)} \sum_i (x_i - \bar{x})^2$ is the sample variance.

We also have, $E(S) = c_4\sigma$ and $Var(S) = \sigma^2(1 - c_4^2)$

$\Leftrightarrow \mu_S = c_4\sigma$ and $\sigma_S = \sigma\sqrt{1 - c_4^2}$, where c_4 is a constant that depends on 'n'.

Control chart for $\bar{x}$:

Case-I: Standards given

Suppose that standard values of μ and σ are given, say, μ_0 and σ_0, then

$$LCL = \mu_0 - 3\frac{\sigma_0}{\sqrt{n}} = \mu_0 - A\sigma_0$$
$$CL = \mu_0 = \mu_0$$
$$UCL = \mu_0 + 3\frac{\sigma_0}{\sqrt{n}} = \mu_0 + A\sigma_0, \text{ where } A = \frac{3}{\sqrt{n}}.$$

Case-II Standards not given

If no standards are given for μ and σ; then it must be estimated by analyzing the past data. Suppose that m preliminary samples are available, each of size n, and let $\bar{x}_i$, S_i be the mean and the s.d. of the i^{th} sample.

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* When subgroup size n is moderately large (say $n > 10$ or 12). Range may not be a good measure of variation. It is desirable to estimate the variation using standard deviation.

Define, $\bar{\bar{x}} = \frac{1}{m}\sum_{i=1}^{m} \bar{x}_i$, $\bar{s} = \frac{1}{m}\sum_{i=1}^{m} s_i$.

Note that $E(\bar{\bar{x}}) = \mu$ and $E\left(\frac{\bar{s}}{c_4}\right) = \sigma$. Hence $\hat{\mu} = \bar{\bar{x}}$ and $\hat{\sigma} = \frac{\bar{s}}{c_4}$ are unbiased estimators of μ and σ.

Hence the control chart for $\bar{x}$ (based on $\bar{s}$) is given by :

$$LCL = \hat{\mu} - 3.\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} - \frac{3}{c_4\sqrt{n}}\bar{s} = \bar{\bar{x}} - A_3\bar{s}$$
$$CL = \hat{\mu} = \bar{\bar{x}}$$
$$UCL = \hat{\mu} + 3.\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} + \frac{3}{c_4\sqrt{n}}\bar{s} = \bar{\bar{x}} + A_3\bar{s}, \text{ where } A = \frac{3}{c_4\sqrt{n}}.$$

Control chart for S :

Case-I : **Standard given**

Suppose that a standard value σ is given, say , σ_0 .

The control chart for S is :

$$LCL = \mu_S - 3\sigma_S = c_4\sigma_0 - 3\sqrt{1-c_4^2}\,\sigma_0 = B_5\sigma_0$$
$$CL = \mu_S = c_4\sigma_0 = c_4\sigma_0$$
$$UCL = \mu_S + 3\sigma_S = c_4\sigma_0 + 3\sqrt{1-c_4^2}\,\sigma_0 = B_6\sigma_0$$

Case-II : **Standard not given**

Here σ is unknown and $\hat{\sigma} = \frac{\bar{s}}{c_4}$ is an unbiased estimator of σ.

Therefore, the control chart for S is given by :

$$LCL = \hat{\mu}_S - 3\hat{\sigma}_S = c_4\hat{\sigma} - 3\sqrt{1-c_4^2}\,.\,\hat{\sigma} = \bar{s} - 3.\frac{\sqrt{1-c_4^2}}{c_4}.\bar{s} = B_3\bar{s}$$
$$CL = \hat{\mu}_S = c_4\hat{\sigma} = \bar{s} = \bar{s}$$
$$UCL = \hat{\mu}_S + 3\hat{\sigma}_S = c_4\hat{\sigma} + 3\sqrt{1-c_4^2}\,\hat{\sigma} = \bar{s} + \frac{3\sqrt{1-c_4^2}}{c_4}\bar{s} = B_4\bar{s}$$

where, $B_3 = 1 - \frac{3}{c_4}\sqrt{1-c_4^2}$, $B_4 = 1 + \frac{3}{c_4}\sqrt{1-c_4^2}$

$\bar{X}$ - R Chart : Methodology :-

- Decide on Total Number of samples N. ($N > 19$)
- Decide on Subgroup size n. ($n > 3$)
- Decide on Frequency of sampling (eg: once in a hour, once in 2 hours, etc.)

Interpretation of $\bar{x}$ and R charts | Analysis of patterns of control charts :— In interpreting patterns on $\bar{x}$ chart, we must first determine whether or not the R chart is in control. Some assignable causes show up on both the $\bar{x}$ and R charts. If both the $\bar{x}$ and R charts exhibit a non-random pattern, the best strategy is to eliminate the R chart assignable causes first. In many cases, this automatically will eliminate the non-random pattern on the $\bar{x}$-chart. Never attempt to interpret to $\bar{x}$-chart when the R chart indicates an out-of-control condition. Situations exist where R-chart is in state of control but $\bar{x}$-chart is not.

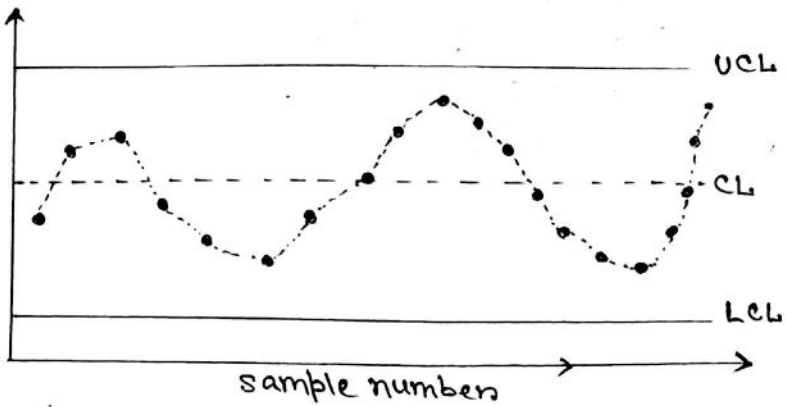
A control chart may indicate an out-of-control condition either when one or more points fall beyond the control limits or when the plotted points exhibit some non-random pattern of behaviour. If the points are truly random, we should expect a more even distn. of them above and below the central line. In general, we define a run as a sequence of observations of same type. In addition, to runs up & runs down, we could define the types of the observations as those above and below the central line, respectively.

A run of length 8 or more points has a very low probability of occurrence in a random sample of points. Consequently, any type of run of length 8 or more is often taken as a signal of an out-of-control state.

Although runs are an important measure of non-random behaviour of a control chart, other types of patterns may also indicate an out-of-control condition:

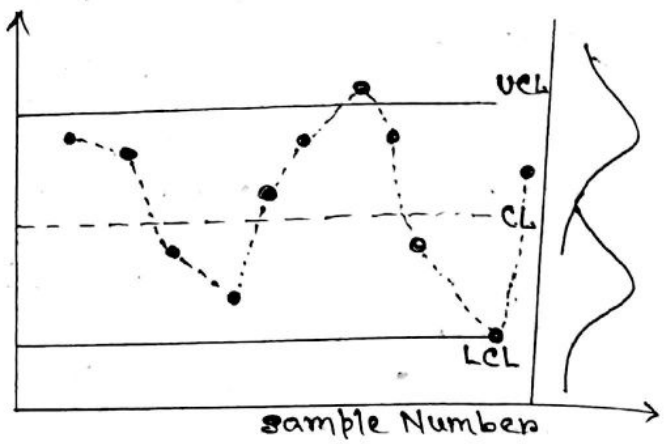
(i) Cyclic Patterns occasionally appear on the control chart. Such a pattern may indicate a problem with the process such as operator fatigue, raw material deliveries, heat or stress build up, etc. Although the process is not really out of control, the yield may be improved by elimination or reduction of the source of variability.

R charts will sometimes reveal cycles because of maintainance schedules, operator fatigue, or tool wear.



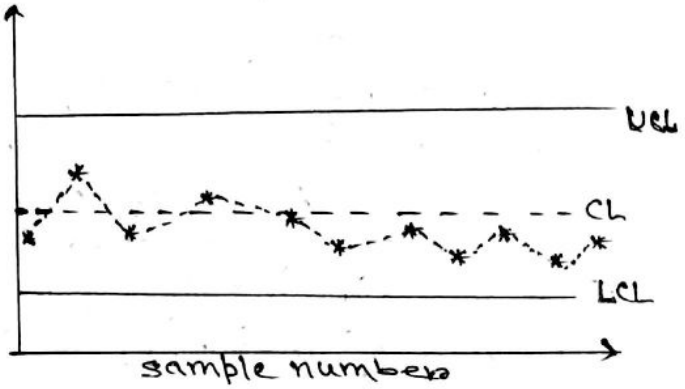
Cyclic pattern

(ii) A Mixture pattern is indicated when the plotted points tend to fall near or slightly outside the control limits, with relatively few points near the central line. A mixture pattern is generated by two or more overlapping distn.s, generating the process output.



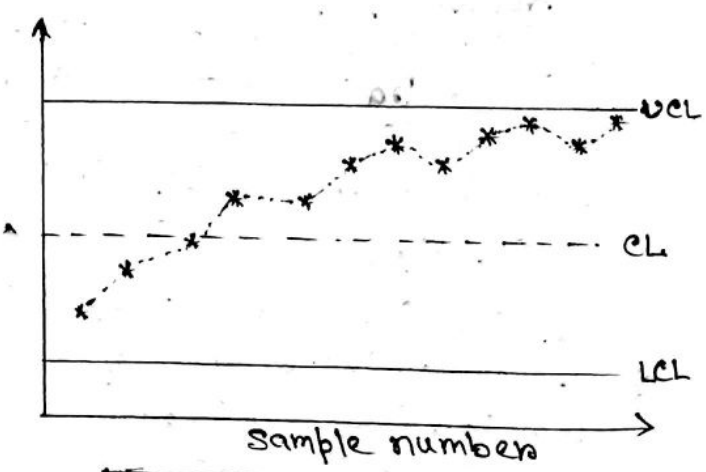
A mixture pattern

(iii) A shift in process level is illustrated in the following figure:
These shifts may result from the introduction of new workers, methods, raw materials, or machines.



A shift in process

(iv) A trend or continuous movement in one direction, is shown on the control chart:
Trends are usually due to a gradual wearing out or deterioration of a tool or some other critical process component.



A trend in process level

The effect of non-normality on $\bar{x}$ and R (or S) charts :~

A fundamental assumption in the development of $\bar{x}$ and R (or S) charts is that the underlying distn. of the quality characteristic is normal. In many situations we may have reason to doubt the validity of this assumption. Now if we know the form of the underlying distn., it is possible to derive the sampling distn. of $\bar{x}$ and R (or S), and to obtain exact probability limits for the control charts. This approach could be difficult in some cases, and most analysts could probably prefer to use the standard approach based on the normality assumption if they felt that the effect of departure from this assumption was not serious. However, we may know nothing about the form of the underlying distribution, and then our only choice may be to use the normal theory result. In either case, we would be interested in knowing the effect of departures from normality on the control chart for $\bar{x}$ and R (or S).

Operating characteristic and Average run length of Control-Chart

For a control chart, define $\beta(\theta) = P$ [a sample is taken from 'in-control process' as decided by the control chart, when the true process parameter is θ]. The $\beta(\theta)$, considered as a function of true process parameter θ, is called the OC function of the control chart.

The ability of the $\bar{x}$ and R charts to detect shift in process quality is described by their OC functions or curves.

Consider the OC curve for an $\bar{x}$-chart with s.d. σ (known). If the mean shifts from the in-control value – say, μ_0 – to another value $\mu_1 = \mu_0 + k\sigma$, the probability of not detecting this shift on the first subsequent sample (or β-risk) is

$$\beta = P\left[LCL \le \bar{x} \le UCL \;\middle|\; \mu = \mu_1 = \mu_0 + k\sigma \right]$$

since $\bar{x} \sim N\left(\mu_1, \frac{\sigma^2}{n}\right)$, and the upper and lower control limits are $LCL = \mu_0 - \frac{3\sigma}{\sqrt{n}}$, $UCL = \mu_0 + 3\sigma/\sqrt{n}$, we have

$$\begin{aligned}
\beta &= P\left[\mu_0 - 3\sigma/\sqrt{n} < \bar{x} < \mu_0 + 3\sigma/\sqrt{n} \;\middle|\; \mu = \mu_0 + k\sigma \right] \\
&= P\left[(-3 - k\sqrt{n}) < \frac{\sqrt{n}\{\bar{x} - (\mu_0 + k\sigma)\}}{\sigma} < (3 - k\sqrt{n}) \right] \\
&= \Phi(3 - k\sqrt{n}) - \Phi(-3 - k\sqrt{n})
\end{aligned}$$

The probability that such a shift will be detected on the first subsequent sample is $(1-\beta)$. To construct OC curve for the $\bar{x}$-chart, plot β-risk against the magnitude of shift, for a given sample size(n).

Average Run Length (ARL) :~ The OC curve does not give an entirely fair comparison between two control charts.

Note that the probability that the shift will be detected on the first sample is $(1-\beta)$. The probability that the shift will be detected on the nth subsequent sample is $\beta^{n-1}(1-\beta)$.

The expected number of samples taken to detect the shift is simply the average run length (ARL) or

$$ARL = \sum_{n=1}^{\infty} n\beta^{n-1}(1-\beta) = \frac{1}{1-\beta} \qquad \text{---------- (*)}$$

It is also convenient to express the performance of the control chart in terms of its average time to signal (ATS). If samples are taken at fixed intervals of time that are 'h' hours apart, then

$$ATS = (ARL) \times h \qquad \text{---------- (**)}$$

The equations (*) and (**) can be used to evaluate the performance of the control charts.

It may also be useful to express the ARL in terms of the expected number of individual units sampled – say I – rather than the number of samples taken to detect a shift. If the sample size is n, the relationship between I and ARL is

$$I = nARL$$

Examples:-

(1) i) Show that p_n, the probability of the mean of a random sample of size n exceeding $UCL = \mu_0 + 3\sigma/\sqrt{n}$, when the population mean has shifted to $\mu_0 + k\sigma$ is $\Phi(k\sqrt{n} - 3)$.

ii) If the rth sample mean is the first to exceed UCL, show that $E(r) = 1/p_n$.

Solution:-

i) $p_n = P[\bar{x} > UCL] = P\left[\bar{x} > \mu_0 + 3\frac{\sigma}{\sqrt{n}}\right]$

$= P\left[\frac{\bar{x} - (\mu_0 + k\sigma)}{\sigma/\sqrt{n}} > 3 - k\sqrt{n}\right]$, since $\bar{x} \sim N\left(\mu_0 + k\sigma, \frac{\sigma^2}{n}\right)$

$= \Phi(k\sqrt{n} - 3)$,

ii) If the rth sample mean is the first to exceed the UCL, the preceeding $(r-1)$ sample means must be $\le$ UCL. Thus if Y is the random variable such that $Y = r\ (1, 2, \ldots)$ implies that the rth sample mean is the first to exceed UCL then $Y \sim Geo(p_n)$, i.e. $P[Y = r] = (1 - p_n)^{r-1} p_n$

Then, $E(Y) = \frac{1}{p_n}$.

(2) Show that the probability that at least one of the two points $\bar{x}$ and R goes outside the control limits is:

$$1 - \left[\Phi(\sqrt{n}T + 3\rho) - \Phi(\sqrt{n}T - 3\rho)\right]\left[P\left(\frac{R}{\sigma} \le D_2\rho\right) - P\left(\frac{R}{\sigma} \le D_1\rho\right)\right]$$

where $\rho = \sigma'/\sigma$, $T = \frac{\mu' - \mu}{\sigma}$, assuming that the control charts are based on μ' and σ' as standards, where the actual values of these parameters are μ and σ respectively.

Solution:- The probability that at least one of the two points $\bar{x}$ and R goes outside the control limits

$= 1 - P[$none of the points $\bar{x}$ and R goes outside the control limits$]$

$= 1 - P[LCL_{\bar{x}} \le \bar{x} \le UCL_{\bar{x}},\ LCL_R \le R \le UCL_R]$

$= 1 - P[LCL_{\bar{x}} \le \bar{x} \le UCL_{\bar{x}}]\, P[LCL_R \le R \le UCL_R]$

$= 1 - P\left[\frac{\mu' - \frac{3\sigma'}{\sqrt{n}} - \mu}{\sigma/\sqrt{n}} \le \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \le \frac{\mu' + 3\sigma'/\sqrt{n} - \mu}{\sigma/\sqrt{n}}\right] \times P[D_1\sigma' \le R \le D_2\sigma']$

[Since in normal population $\bar{x}$ and R are independently distributed]

$$= 1 - \left\{ \Phi\left(\sqrt{n}.T + 3\rho\right) - \Phi\left(\sqrt{n}.T - 3\rho\right) \right\} \times \left\{ P\left(\frac{R}{\sigma} \le D_2\rho\right) - P\left(\frac{R}{\sigma} \le D_1\rho\right) \right\}$$

(3) Let p_n is the probability of the mean of a sample of size n falling outside the control limits. Show that

(a) the probability that at most x samples are to be taken for r points to go out of control is

$$1 - \sum_{s=0}^{r-1} \binom{x}{s} p_n^s (1-p_n)^{x-s}$$

(b) the probability that exactly x samples are to be taken for r points to go out of control is

$$\left(\frac{p_n}{1-p_n}\right)^r \cdot \binom{x-1}{r-1} (1-p_n)^x, \quad x \ge r.$$

Solution:-

(a) Let Y be the RV which represents the number of points (sample means) falling outside the control limits in x samples. Then $Y \sim \text{Bin}(x, p_n)$.

Hence the probability 'p' that in x samples the number of points going out of the control limits is greater than or equal to r is the required probability.

$$\therefore p = P[Y \ge r] = 1 - P[Y < r] = 1 - \sum_{s=0}^{r-1} \binom{x}{s} p_n^s (1-p_n)^{x-s}$$

(b) the event 'E' that exactly x samples are required for r points to go out of control limits, happens if

(i) the rth point goes out of control limits at the xth sample and

(b) X : the no. of samples required for r points to go out of control limits.

$X \sim$ Negative Binomial (r, p_n).

$\therefore$ Required probability $= \binom{x-1}{r-1} p_n^r (1-p_n)^{x-r}$, if $x \ge r$.

$$= \binom{x-1}{r-1} \left(\frac{p_n}{1-p_n}\right)^r (1-p_n)^x, \quad x \ge r$$

CONTROL CHART FOR ATTRIBUTES :

A defective or non conforming item is a unit of product that does not satisfy one or more of the specifications for that product. Each specific point at which a specification is not satisfied results in a defect or non conformity.

Usually a unit is considered defective when it is qualitatively unsatisfactory. It may be usable, but have a major defect or too many minor defects.

A. Procedures with constant sample size : [C.U]

The control chart for fraction defective or non-conforming :

The fraction defective is defined as the ratio of the number of defective items in a population to the total number of items in that popln..

The statistical principles underlying the control chart for fraction non-conforming or defectives are based on the binomial distribution. Suppose the production process is operating in a stable manner, such that the probability that any unit will not conform to specifications is p, and successive units produced are independent. Then each unit produced is a realization of a Bernoulli random variable with parameter 'p'. If a random sample of n units of product is selected, and if D is the number of units of product that are non conforming, then D has a binomial distn. with parameters n and p, i.e.

$$P[D=d] = \binom{n}{d} p^d (1-p)^{n-d}, \quad d = 0, 1, 2, \ldots, n.$$

Note that $E(D) = np$, $V(D) = np(1-p)$.

The sample fraction conforming or defective is defined as the ratio of the number of non-conforming units in the sample D to the sample size n; that is, $\hat{p} = \frac{D}{n}$.

Again, $\mu_{\hat{p}} = E(\hat{p}) = p$ and $\sigma_{\hat{p}}^2 = V(\hat{p}) = V\left(\frac{D}{n}\right) = \frac{1}{n^2} V(D) = \frac{p(1-p)}{n}$.

Because the chart monitors the process function non-conforming 'p', it is also called the p-chart.

Development of the control chart : — If T is a statistic that measures a quality characteristic, and if the mean of T is μ_T and the variance of T is σ_T^2, then the general structure of Shewhart control chart is as follows:

$$\left.\begin{aligned} UCL &= \mu_T + 3\sigma_T \\ CL &= \mu_T \\ LCL &= \mu_T - 3\sigma_T \end{aligned}\right\} \rightarrow 3\text{-sigma limits}.$$

(1) **Standard value is given** :

Suppose that the true fraction defective p in the production process is known or is a standard value specified by management. Then the central line and control limits of the fraction defective control chart are:

$$UCL = \mu_{\hat{p}} + 3\sigma_{\hat{p}} = p + 3\sqrt{\frac{p(1-p)}{n}}$$
$$CL = \mu_{\hat{p}} = p$$
$$LCL = \mu_{\hat{p}} - 3\sigma_{\hat{p}} = p - 3\sqrt{\frac{p(1-p)}{n}}.$$

(2) **No standard given** :

When the process fraction defective p is not known, then it must be estimated from observed data. The usual procedure is to select m preliminary samples, each of size n. Then if there are D_i defective units in the i^{th} sample, we compute the fraction defective in the i^{th} sample as

$$\hat{p}_i = \frac{D_i}{n}, \quad i = 1, 2, \ldots, m.$$

and the average of these individual sample fractions defective is $\bar{p} = \frac{1}{m}\sum_{i=1}^{m}\hat{p}_i = \frac{1}{mn}\sum_{i=1}^{m} D_i$. The statistic $\bar{p}$ estimates the unknown fraction non-conforming (defective) p.

The centralline and control limits of the control chart for fraction non-conforming (defective) are:

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$
$$CL = \bar{p}$$
$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

The control chart for the number of defectives or the np control chart :~ It is also possible to base a control chart on the number of defectives rather than the fraction defective. This is often called **np-control chart**.

(1) **Standard given** : p is given.

$$UCL = np + 3\sqrt{np(1-p)}$$
$$CL = np$$
$$LCL = np - 3\sqrt{np(1-p)}$$

(2) **Standard not given** :

If the process fraction defective p is not known then it must be estimated from observed data.

Then $\bar{p} = \frac{1}{mn}\sum_{i=1}^{m} D_i = \frac{1}{m}\sum_{i=1}^{m} \hat{p}_i$ can be used to estimate p. Then

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$CL = n\bar{p}$$

$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

• Remark: ~

(1) Note that p or np can never be negative. Hence, if LCL, in p chart or np-chart, comes out negative, then it is to be taken as zero.

(2) When a control chart for defectives in stead of means and range is used, much of the information is thrown away, because we utilize only the information that the measurement is or is not within a specified range of values, rather than its actual value. The sample must thus be larger to provide a test of the same power. Using a large sample may, however, be more economical. It is generally cheaper to use some sort of a gage that tells whether the object conforms to standard, and then count the number of defectives, that it is to weigh or measure the object, record the observations, and compute their mean and range.

(3) Care must be exercised in interpreting points that plot below the lower control limit. These points often do not represent a real improvement in process quality. Frequently, they are caused by errors in the inspection process from inadequately trained inspectors or from improper inspection equipment.

B. Variable sample size : In some applications of the control chart for fraction defective, the sample is a 100% inspection of process output over some period of time. Since different numbers of units could be produced in each period, the control chart would then have a variable sample size. There are three approaches to constructing and operating a control chart with a variable sample size.

p-chart :

(a) Variable-width Control limits :~

The first approach is to determine control limits for each individual sample that are based on the specific sample size. That is if the i^{th} sample is of size n_i, then the upper and lower control limits for p-chart are $\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$. Note that the width of the control limit is inversely proportional to the square root of the sample size.

(b) Control limits based on an Average sample size :~

The second approach is to base the control chart on an average sample size, resulting in an approximate set of control limits. This assumes that future sample sizes will not differ greatly from those previously observed. If this approach is used, the control limits will be constant.

Therefore, the approximate control limits for p-chart are:

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}}, \quad LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{\bar{n}}},$$

where, $\bar{n}$ is average sample size and $\bar{p}$ is the average fraction defective based on all the samples.

(c) The standard control chart :~

The third approach to dealing with variable sample size is to use a "standardized" control chart, where the points are plotted in standard deviation units. Such a control chart has the central line at zero, the $UCL = 3$, $LCL = -3$.

The variable plotted on the chart is

$$Z_i = \frac{\hat{p}_i - p}{\sqrt{\frac{p(1-p)}{n_i}}} \quad \text{or} \quad \frac{\bar{p}_i - \bar{p}}{\sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}}$$

where p (given) or $\bar{p}$ is the estimate of the process fraction defective in the in-control state.

np-chart :

(a) variable-width control limits :~

$$UCL = n_i\bar{p} + 3\sqrt{n_i\bar{p}(1-\bar{p})}$$
$$CL = n_i\bar{p}$$
$$LCL = n_i\bar{p} - 3\sqrt{n_i\bar{p}(1-\bar{p})}$$

(b) Control limits based on average sample size. :~

$$UCL = \bar{n}\bar{p} + 3\sqrt{\bar{n}\bar{p}(1-\bar{p})}$$
$$CL = \bar{n}\bar{p}$$
$$LCL = \bar{n}\bar{p} - 3\sqrt{\bar{n}\bar{p}(1-\bar{p})}$$

(c) The standardised control chart :~

$$UCL = 3, \quad CL = 0, \quad LCL = -3$$

The variable plotted on the chart is

$$Z_i = \frac{d - n_i\bar{p}}{\sqrt{n_i\bar{p}(1-\bar{p})}} \text{ or } \frac{d - n_i p}{\sqrt{n_i p(1-p)}}.$$

Choice between chart for p and chart for np :~ Whenever subgroup (sample) size is variable, the control chart must show the fraction defective rather than the numbers of defectives. If actual numbers of defectives were plotted the central line on the np-chart (as well as limits) would need to be changed with every change in sample size. When the sample size is constant, both the charts are equivalent.

Control charts for defects or non-conformities :— It is possible to develop control charts for either the total number of defects in a unit or the average number of defects per unit.

The poisson distribution is used with two types of data in quality control work (i) for defectives when n is large and p is small,
(ii) for defects per unit of output.

Essentially, this requires that the number of opportunities for defects be indefinitely large and that the probability of occurrence of a non-conformity at any location be small and constant.

A. Control charts for Constant sample size :—

In most cases, the inspection unit will be a single unit of product, although this is not necessarily always so. The inspection unit is simply an entity for which it is convenient to keep records. It could be a group of 5 or 10 units.

Suppose that defects or non-conformities occur in this inspection unit according to the Poisson distribution; i.e.

$$P[X = x] = e^{-c} \cdot \frac{c^x}{x!}, \quad x = 0, 1, 2, \ldots,$$

where X is the no. of defects and $c > 0$ is the parameter.

Note that $E(X) = c = V(X)$.

Therefore, a control chart for defects with $3-\sigma$ limits could be defined as follows:

$$UCL = E(X) + 3\sqrt{V(X)} = c + 3\sqrt{c}$$
$$CL = E(X) = c$$
$$LCL = E(X) - 3\sqrt{V(X)} = c - 3\sqrt{c}$$

The c-chart or control chart for defects: —

Standard given: Assuming that the standard value of c is available.

$$UCL = c + 3\sqrt{c}$$
$$CL = c$$
$$LCL = c - 3\sqrt{c}$$

Should these calculation yield a negative value for the LCL, set $LCL = 0$ as LCL can't be negative.

Standard not given: If no standard is given, then c may be estimated as the observed average number of defects in a preliminary sample of inspection units — say, $\bar{c} = \frac{1}{m}\sum_{i=1}^{m} c_i$; where c_i is the no. of defects in the i^{th} inspection unit.

In this case, the control chart is given by

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$
$$CL = \bar{c}$$
$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

The u-chart or the control chart for the average number of defects per unit: There is no reason why the sample size must be restricted to one inspection unit. In fact, we could often prefer to use several inspection units in the sample, thereby increasing the area of opportunity for the occurrence of defects. The sample size should be chosen according to statistical consideration such as – cost, probability of detecting a process shift. -s,

If we find x total defects in the sample of n inspection units, then the average number of defects per inspection unit is

$$u = x/n .$$

Here, $X \sim P(c)$. $E(U) = \frac{c}{n}$, $V(U) = \frac{c}{n^2}$.

$$\Rightarrow \mu_U = \frac{c}{n}, \quad \sigma_U = \frac{\sqrt{c}}{n}.$$

(i) If we have taken m samples of size 'n', we make estimates of the parameters $\hat{\mu}_U = \bar{u} \Leftrightarrow \hat{\mu}_U = \frac{\sum_{i=1}^{m} c_i}{\sum_{i=1}^{m} n} = \frac{1}{n}\bar{c}$; where $\bar{c} = \frac{1}{m}\sum_{i=1}^{m} c_i$

and $\hat{\sigma}_U = \frac{\sqrt{\bar{c}}}{n} = \frac{\sqrt{\bar{u}n}}{n} = \sqrt{\frac{\bar{u}}{n}}$.

(ii) If c is given $\Leftrightarrow$ if $u' = \frac{c'}{n}$ is given, then $\mu_U = u'$, $\sigma_U = \sqrt{\frac{u}{n}}$.

Standard given: If c is given, say $u' = \frac{c'}{n}$, then the control limits are:

$$UCL = \mu_U + 3\sigma_U = \mu' + 3\sqrt{\frac{u'}{n}}$$
$$CL = \mu_U = \mu'$$
$$LCL = \mu_U - 3\sigma_U = \mu' - 3\sqrt{\frac{u'}{n}}, \quad \text{from (ii)}$$

Standard not given: If no standard is given, then from (i), we get

$$UCL = \hat{\mu}_U + 3\hat{\sigma}_U = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$
$$CL = \hat{\mu}_U = \bar{u}$$
$$LCL = \hat{\mu}_U - 3\hat{\sigma}_U = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}.$$

B. Control charts for variable sample size :

When a 100% inspection of the product is observed, the number of inspection units in a sample will usually not be constant. For example the inspection of rolls of cloth or paper often leads to a situation in which the size of the sample varies, because not all rolls are exactly the same length or width. If a control chart for defects (c chart) is used in this situation, both the central line and control limits will vary with the sample size — such a control chart for non-conformities per unit (u-chart). This control chart will have a constant central line; however, the control limits will vary inversely with the square root of the sample size n.

(i) **U-chart :**

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$$

$$CL = \bar{u}$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}} \quad ; \text{ where, } \bar{u} = \sum_{i=1}^{m} c_i \Big/ \sum_{i=1}^{m} n_i$$

(ii) **Use of control limits based on the average sample size :**

$$\bar{n} = \frac{1}{m}\sum_{i=1}^{m} n_i$$

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

$$CL = \bar{u}$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$$

(iii) **Use of a standardized control chart (this is the preferred option) :**

Here, we plot the standardized statistic :

$$Z_i = \frac{u_i - \bar{u}}{\sqrt{\frac{\bar{u}}{n_i}}}$$ on a control chart with

$LCL = -3$, $UCL = 3$ and $CL = 0$.

Process Capability and Modified Control charts :

Statistical techniques can be useful throughout the product cycle, including development activities prior to manufacturing, in quantifying process variability, in analysing this variability relative to product requirements or specifications. This general activity is called process capability analysis.

Natural Tolerance Limits : [C.U]

Process capability refers to the uniformity of the process. It is customary to take the six-sigma spread in the distn. of the product quality characteristic as a measure of the process capability. If μ and σ are the process average and process standard deviation respectively, then the limits $\mu \pm 3\sigma$ (three sigma above and below the mean) are called the 'Natural Tolerance Limits'. The upper and lower "natural tolerance limits" of the process fall at $\mu+3\sigma$ and $\mu-3\sigma$, respectively, that is,

$$UNTL = \mu + 3\sigma$$
$$LNTL = \mu - 3\sigma$$

The width '6σ' which is the inherent variability of the process is given a special name Natural tolerance.
For normal distribution, only 0.27% of the process output will fall outside natural tolerance limits. If the distribution of process output is non-normal then the percentage of output falling outside $\mu \pm 3\sigma$ may differ considerably from 0.27%.

If μ and σ are unknown the $\hat{\mu} \pm 3\hat{\sigma}$ are the estimates of the natural tolerance limits, where $\hat{\mu} = \bar{\bar{X}}$, $\hat{\sigma} = \bar{R}/d_2$ or $\frac{\bar{s}}{c_4}$.

Specification Limits :

It might happen that even though the process is in statistical control as exhibited by control limits / chart, the consumer may not be satisfied with the products. The specification limits, are determined (externally) by the management, the manufacturing engineers, the customer such that a product having quality outside the specification limits is considered as unsatisfactory, one should have knowledge of 'inherent variability' of the process while setting specifications, but remember that there is no mathematical or statistical relationship between the control limits and specification limits.

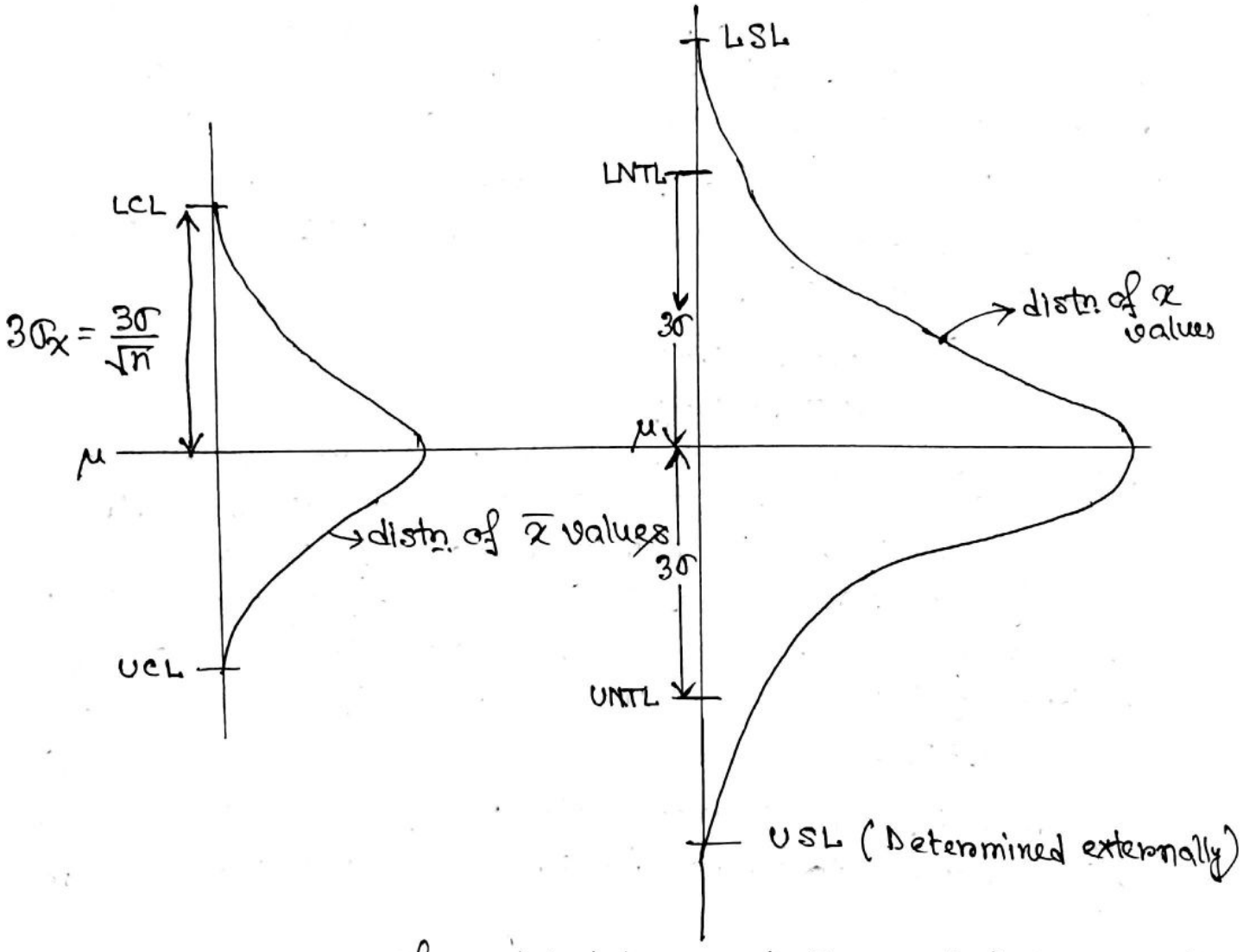


Fig:- Relationship of natural tolerance limits, control limits and specification limits

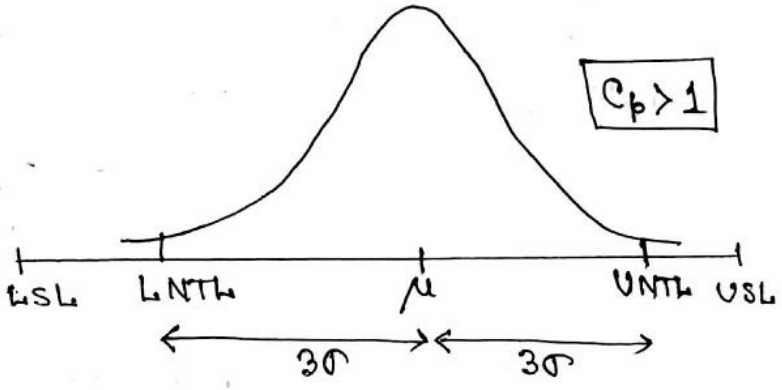
Process Capability Ratio :— Another way to express process capability is in terms of the process capability ratio (PCR) C_p, which for a quality characteristic with both USL and LSL is

$$C_p = \frac{USL - LSL}{6\sigma}.$$

Case-(I): $C_p > 1 \Leftrightarrow USL - LSL > 6\sigma$.

This implies that natural tolerance limits in the process are well inside the USL and LSL. In such a case almost all the products will conform to specifications as long as the process is in statistical control. The larger the C_p, the greater is the likelihood of getting good product without assistance from any control chart. This will imply that the process is too good for the product, less costly processing on material could be allowed on it may also be worthwhile to 'squeeze' the specification limits, to produce a product superior to the one originally intended.

Here, the process mean can sometimes be allowed to vary over an interval without appreciably affecting the overall performance of the process.

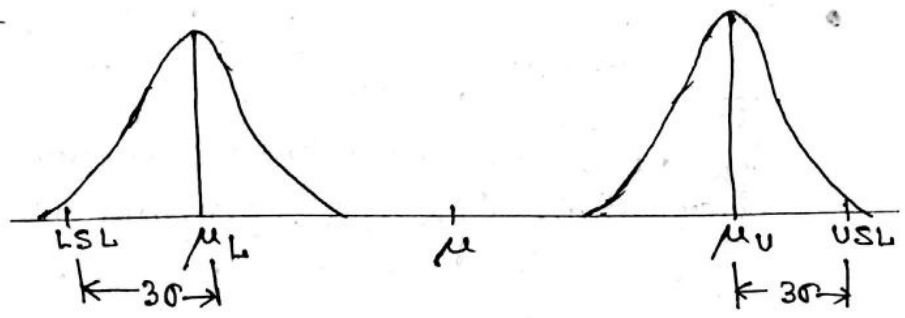


Modified Control Chart :— When this situation occurs, we can use a modified control charts or reject charts

In effect, μ is allowed to vary over an interval, say, $\mu_L \le \mu \le \mu_U$ - where, μ_L and μ_U are chosen as the smallest and largest permissible values of μ, respectively, until they reach at a danger point. To specify the control limits (reject limits) for a modified $\bar{x}$-chart, we will assume that the process output is normally distributed.

From the figure, we have

$$\left.\begin{aligned} \mu_L &= LSL + 3\sigma \\ \mu_U &= USL - 3\sigma \end{aligned}\right\}$$



Hence the control limits for the modified chart or the reject limits are:

$$URL_{\bar{x}} = \mu_U + 3\sigma/\sqrt{n} = USL - 3\sigma + \frac{3\sigma}{\sqrt{n}} = USL - 3\sigma\left(1 - \frac{1}{\sqrt{n}}\right)$$

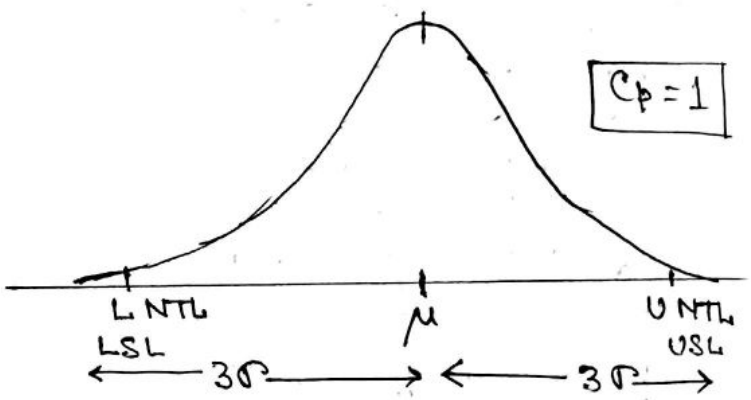
$$LRL_{\bar{x}} = \mu_L - 3\sigma/\sqrt{n} = LSL + 3\sigma - \frac{3\sigma}{\sqrt{n}} = LSL + 3\sigma\left(1 - \frac{1}{\sqrt{n}}\right)$$

These reject limits, when used in place of control limits, are called modified control limits.

To design a modified control chart, one must have a good estimate of σ available. If the process variability shifts, then the modified control limits are not appropriate. Consequently, an R or an S chart should always be used in conjunction with the modified control chart.

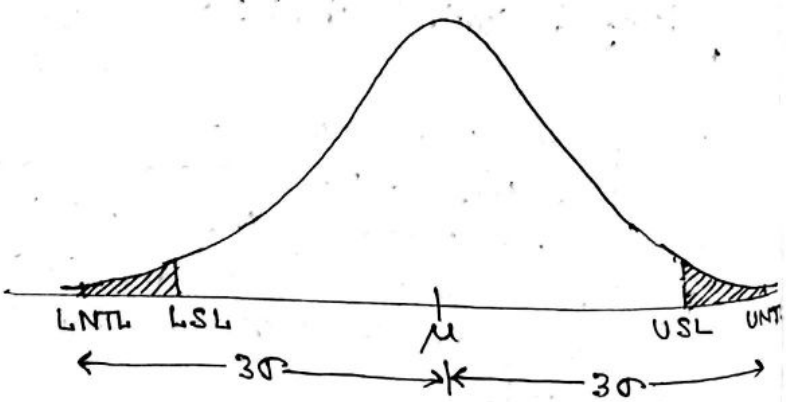
Case-II : $C_p = 1 \Leftrightarrow USL - LSL = 6\sigma$

For a normal distn., this could imply about 0.27% non-conforming units.



Case III: $C_p < 1 \Leftrightarrow USL - LSL < 6\sigma$

In this case, the process is very yield-sensitive, and a large number of non-conforming units will be produced.



- Process Performance Index :- (C_{pk}) $C_{pk} = \text{Min}[C_{pl}, C_{pu}]$, where $C_{pl} = \frac{\mu - LSL}{3\sigma}$, $C_{pu} = \frac{USL - \mu}{3\sigma}$.

C_{pk} checks whether the process is centered at the middle of the specification. $C_{pk} < 1$, performance is not OK.

$C_{pk} = 1 \Leftrightarrow C_{pu} = C_{pl} = C_{pk} = C_p$, otherwise $C_{pk} < C_p$, then performance is not optimum.

(b) Product control :— The object of product control is to decide whether to accept or reject a lot on the basis of evidence afforded by one or more samples drawn at random from the lot in question.

Lot acceptance sampling plans refer to the use of sampling inspection by a purchaser to decide whether to accept or reject a lot of given product. Acceptance sampling plans are often designed so as to accomplish at least two of the following objectives :

(1) the probability of rejecting a good lot is some specified value (Producer's risk).
(2) the probability of accepting a bad lot is some specified value (Consumer's risk).
(3) the average quality of goods shipped out shall not be coarse than some specified standard.
(4) the amount of inspection (consistent with the conditions imposed) shall be minimized.

Advantages & disadvantages of sampling :—

When acceptance sampling is constructed with 100% inspection, it has the following advantages :—

(1) It is usually less expensive because there is less inspection.
(2) There is less handling of the product, hence reduced damage.
(3) It is applicable to destructive testing.
(4) Fewer personnel are involved in inspection activities.
(5) It often greatly reduces the amount of inspection error.

Acceptance sampling also has several disadvantages :—

(1) There are risks of accepting 'bad' lots and rejecting 'good' lots.
(2) Less information is usually generated about the product or about the process.
(3) Acceptance sampling requires planning and documentation of the acceptance — sampling procedure where as 100% inspection does not.

Lot Acceptance Sampling for Attributes :

Acceptance sampling is concerned with inspection and decision making regarding products, one of the oldest aspects of quality assurance. A typical example or application of acceptance sampling is as follows :
A company receives a shipment of product from a vendor. A sample is taken from the lot, and some quality characteristic of the units in the sample is inspected. On the basis of the information in this sample, a decision is made regarding lot disposition. Usually, this decision is either to accept or to reject the lot. Accepted lots are put for sale (production); rejected lots may be returned to the vendor or may be subjected to some other lot-desposition action.

A sampling plan may be either the acceptance-rejection or the acceptance-rectification type.

Acceptance-rejection Inspection Plan :—

In this plan, the lot is accepted or rejected on the basis of the sample(s) drawn from the lot and rejected lot is returned to the vendor. The accepted lot, after replacing the defective items in the drawn sample, is put for sale/production.

Acceptance-Rectifying Inspection Plan :—

In this plan, the lot is accepted or rejected on the basis of the sample(s) drawn from the lot and the rejected lot is subjected to corrective action. This generally takes the form of 100% inspection or screening of rejected lots, with all discovered defective items either removed for subsequent recoork or replaced from a set of known good items. Such sampling programs are called rectifying inspection programs, because the inspection activity affects the final quality of the outgoing product. The rejected lots will be screened, and their final fraction defective will be zero.

Notions:-

Producer: Any person, company or department that sells or prepares goods to be received by another person or company or another department of the same business.

Consumer: The recipient of product. It may be a buyer, or another department of the producer.

$\bar{p}$: Process average or fraction defective turned out by a process over a long period of time.

p : The fraction defective in a lot.

Acceptable quality level (AQL), p_1 : A relatively small fraction defective. The AQL represents the poorest level of quality for the vendor's process that the consumer could consider to be acceptable as a process average. More specially a lot with this fraction defective (p_1) is a lot of sufficiently good quality that we do not coish to reject if more often than a specified small proportion (usually 1%, 5%) of the time. Usually,

$$P[\text{Rejecting a lot of quality } p_1] = 0.05$$
$$\Rightarrow P[\text{accepting a lot of quality } p_1] = 0.95$$

'p_1' is known as the Acceptance Quality Level and a lot of this quality is considered as satisfactory by the consumer.

Lot Tolerance Proportion or Percent Defective (LTPD), p_t : A relatively large fraction defective. The LTPD is the lot quality which is considered to be bad by the consumer. The consumer is not willing to accept lots having proportion defective p_t or greater. $100\,p_t$ is called Lot Tolerance Percentage Defective. In other words, this is the quality level which the consumer regards as rejectable and is usually abbreviated as R.Q.L. (Rejecting Quality Level). A lot of quality p_t stands to be accepted some arbitrary and small fraction of time (usually 5%, 10%).

Operating-Characteristic (OC) Function: For an acceptance-sampling plan, define $L(p) = P[\text{accepting a lot when the fraction defective of the lot is } p]$. The $L(p)$, considered as a function of the fraction defective of the lot (p); is called the OC function of the sampling plan. The curve obtained by plotting $L(p)$ against p is called the OC curve and it is an important measure of the acceptance-sampling

Process Average Fraction Defective ($\bar{p}$): $\bar{p}$ represents the quality turned out by the manufacturing process over a long period of time. The process average of any manufactured product is obtained by finding the percentage of defectives in the product over a fairly long time.

Producer's risk: Any acceptance sampling plan for acceptance-rejection has certain risk on the part of the producer — the producer has to face the situation that some good lots will be rejected. The probability of rejecting a lot, with a fraction defective p_1 (AQL), under the acceptance–rejection sampling plan, is called the producer's risk.

Clearly, in terms of oc function we have $\{1 - L(p_1)\}$ as the producer's risk and it is denoted by 'α' or p_p.

Consumer's risk: The consumer has also to face the situation sometimes that a bad lot will be accepted, on the basis of an acceptance–rejection sampling plan. The probability of accepting a lot with fraction defective p_t (LTPD), under the acceptance–rejection sampling plan, is called the Consumer's risk.

Clearly, in terms of oc function, we have $L(p_t)$ as the consumer's risk and it is denoted by 'β' or p_c.

Rectifying Inspection Plans: Acceptance-sampling programs requires corrective action when lots are rejected. This takes the form of 100% inspection of rejected lots, with all discovered defective items replaced from a stock of known good items. Such sampling programs are called rectifying inspection programs, because the inspection activity affects the final quality of the outgoing products. The two important points related to rectifying inspection plans are:

Average outgoing quality (AOQ): AOQ is the expected fraction defective, after replacing good items for defective ones in rejected lots and in samples taken from accepted lots, in a lot. It is the average value of lot quality that could be obtained over a long sequence of lots from a process with fraction defective p, that results from the application of the rectifying inspection. Average outgoing quality will vary as the function defective of the incoming lots varies. The curve that plots AOQ against incoming lot quality, is called AOQ curve.

Remark:- The fraction defective (p) of a incoming lot or the quality of a lot before inspection, is termed as '**incoming quality**' of the lot. The fraction defective of the lot after inspection is termed as '**outgoing quality**' of the lot.

Average Outgoing Quality Limit (AOQL) :-

The maximum value of the average outgoing quality (AOQ), the maximum being taken w.r.t. the incoming quality (p), is called the **average outgoing quality limit (AOQL)**. Symbolically,

$$AOQL = \underset{0 \le p \le 1}{\text{Max}} \{AOQ(p)\} .$$

Average Sample Number (ASN): The ASN is the expected value of the sample size required for coming to a decision about the acceptance or rejection in an acceptance-rejection sampling plan. Obviously, it is a function of the incoming lot quality 'p'. The curve that plots ASN against incoming lot quality 'p', is called an ASN curve.

Average Amount of Total Inspection [ATI): Another important measure relative to acceptance-rectifying inspection is the total amount of inspection required by the sampling program. The expected number of items inspected in a lot to arrive at a decision in an acceptance-rectification sampling inspection plan calling for 100% inspection of the rejected lots is called Average Total inspection (ATI). Obviously, ATI is a function of the incoming lot quality (p).

☑ We observe that ——

$$ATI = ASN + (\text{Average size of inspection of the remainder in the rejected lots})$$

Thus, if the lot is accepted on the basis of the sampling inspection plan, then $ATI = ASN$, otherwise $ATI > ASN$. In other words, ASN gives the average number of units inspected per accepted lot.

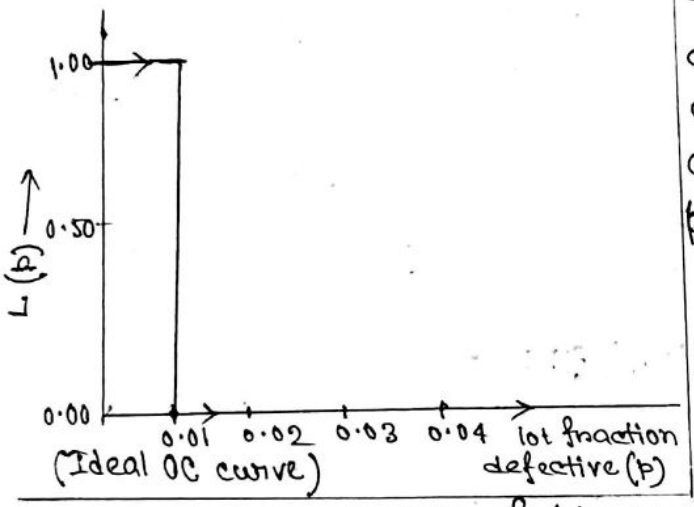
For example, if a single sampling acceptance-rejection plan is used, the number of items inspected from each lot will be the the corresponding sample size n

i.e. $ASN = n$,

and this will be true independently of the quality of the submitted lots.

OC curve: This curve plots the probability of accepting the lot when the fraction defective (or the incoming quality) of the lot is p, $L(p)$, for different values of 'p'. The curve shows that the probability that a lot submitted with a certain fraction defective will be accepted; that is, the OC curve displays the discriminatory power of the sampling plan.

A sampling plan that discriminated perfectly between good and bad lots would have an OC curve that looks like:

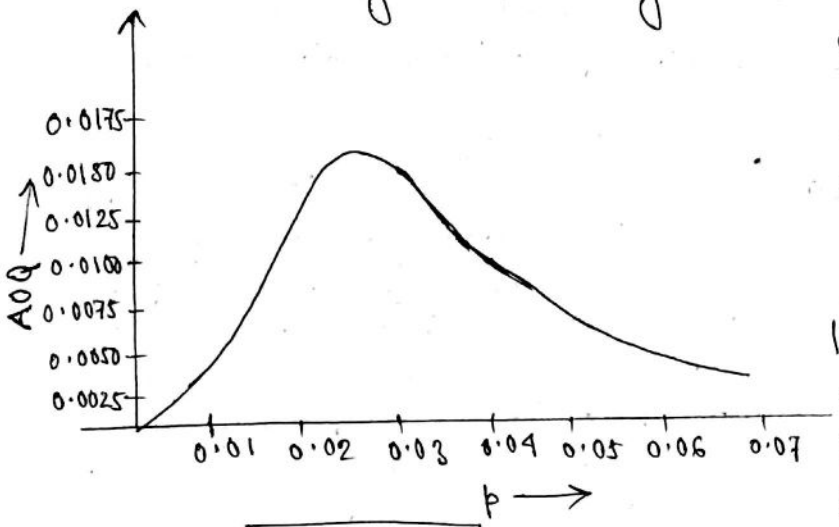


(Ideal OC curve)

The OC curve runs horizontally at a probability of acceptance $L(p) = 1.00$ until a level of quality that is considered 'bad' is reached; at which point the curve drops vertically to a probability of acceptance $L(p) = 0.00$ and then the curve runs horizontally again for all lot fraction defective greater than the undesirable level. In such a sampling plan, if exists, all lots of 'bad' quality could be rejected and all lots of 'good' quality could be accepted.

Unfortunately, the ideal OC curve can almost never be obtained in practice. In theory, it could be realized by 100% inspection, if the inspection were error free. The ideal OC curve shape can be approached, however, by increasing the sample size. Thus, the precision with which a sampling plan differentiates between good and bad lots increases with the size of the sample. The greater is the slope of the OC curve, the greater is the discriminatory power.

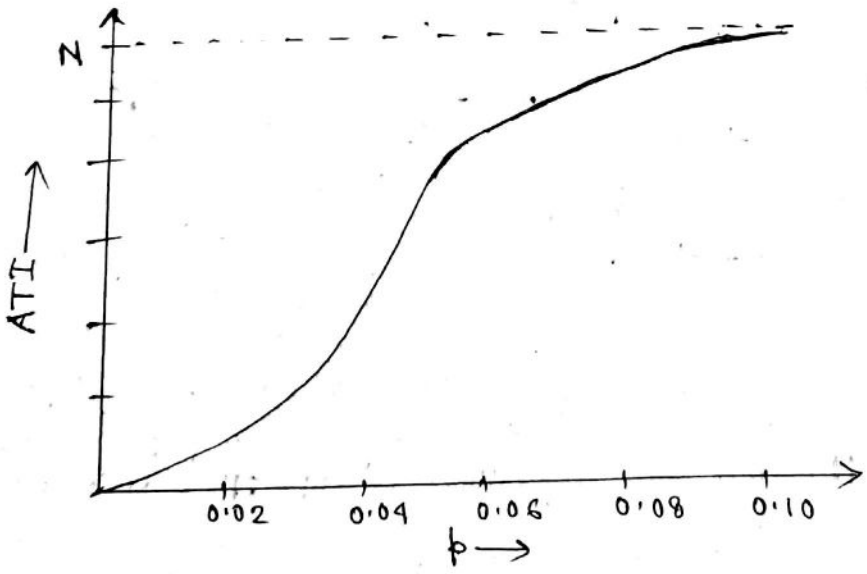
AOQ Curve :- This curve plots the average outgoing quality (AOQ) against incoming lot quality (p). From examining this curve we note that when the incoming quality is very good, the AOQ is very good. In contrast, when incoming quality is very bad, most of the lots are rejected and screened, which leads to a very good level of quality in the outgoing lots. In between these extremes, the AOQ curve rises passes through a maximum, and descends.



AOQ Curve

The maximum ordinate on the AOQ curve represents the worst possible average quality that would result from the rectifying inspection program and this point is called the average outgoing quality limit (AOQL).

ATI Curve :- The average total inspection in a lot is an acceptance-rectification plan is a function of incoming lot quality If p increases, ATI increases. A typical ATI curve looks like:



ATI Curve

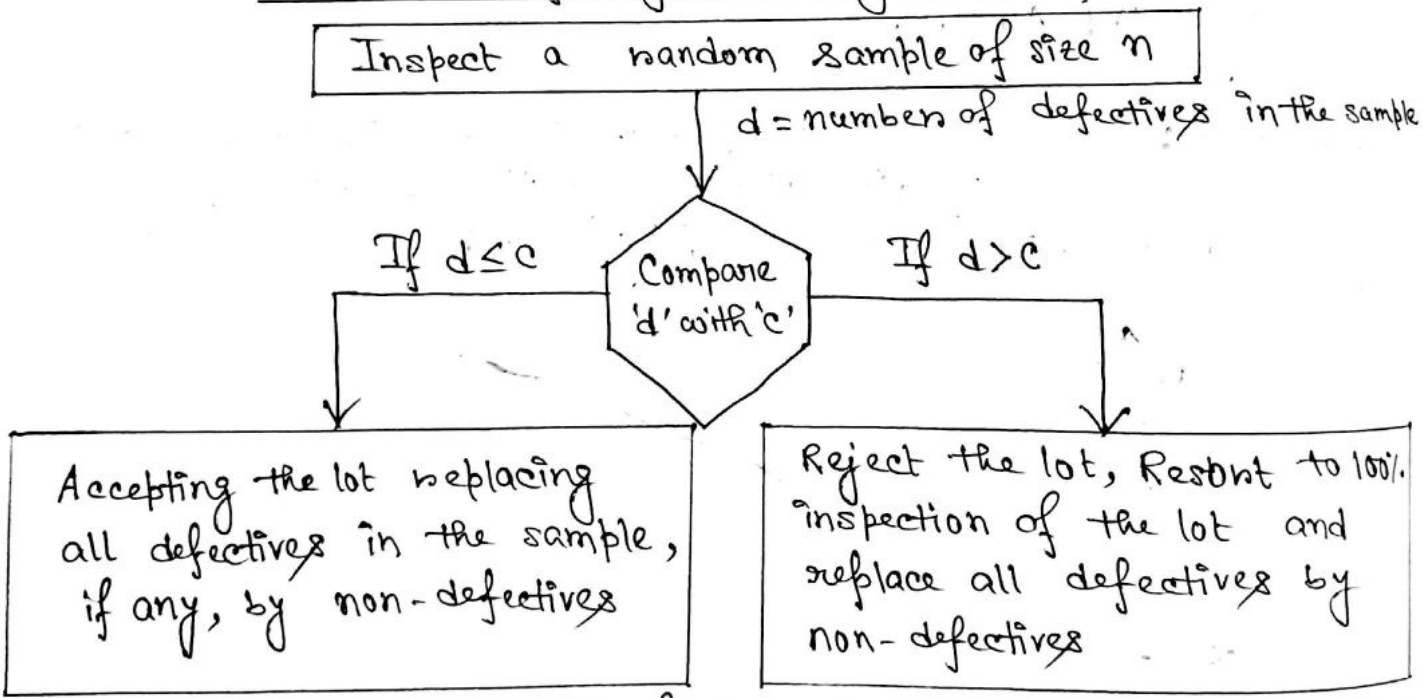
for Attributes

Types of Sampling Plans: ⇝ Acceptance sampling plans are classified according to the sampling methods in three chief methods: Single sampling, Double sampling and sequential sampling.

A. Single-Sampling Plans: — Suppose that a lot of size N has been submitted for inspection. A single sampling plan is defined by the sample size n and the acceptance number c. The procedure would operate as follows: Select n items at random from the lot. If there are 'c' or fewer defectives in the sample, accept the lot, and if there are more than c defective items in the sample, reject the lot. The statistical problem is to determine n and c so as to provide the desired protection.

A single sample acceptance-rectification sampling plan for attributes is described as follows:

Flow chart of single sampling Rectification Plan



OC - Function: ⇝ In a lot of incoming quality 'p', that is, in a lot with fraction defective p, the number of defectives is Np and non-defectives is $N - Np = N(1-p)$. Then, the no. of defectives d in a random sample of size n follows a Hypergeometric distribution with parameters (n, N, p).

Then the probability of accepting a lot of incoming quality p is

$$L(p) = P_a(p) = P[d \le c] = \sum_{d=0}^{c} \frac{\binom{Np}{d}\binom{N-Np}{n-d}}{\binom{N}{n}}$$

AOQ and AOQL :~ If p is the incoming lot quality, there will be no defectives remain in a lot of size N if $d > c$ and if $d \le c$, the number of defectives in a lot of size N is $(Np - d)$. Thus, the mean of the number of defectives remain after sampling inspection is given by :

$$m = \sum_{d=0}^{c} (Np - d) \binom{Np}{d} \binom{N - Np}{n - d} \Big/ \binom{N}{n} + 0$$

The expected fraction defective remains after inspection, i.e. AOQ is given by $AOQ = \tilde{p} = \frac{m}{N} = \sum_{d=0}^{c} \left(p - \frac{d}{N}\right) \cdot \frac{\binom{Np}{d}\binom{N-Np}{n-d}}{\binom{N}{n}}$.

Subject to variation in p, $AOQ(\tilde{p})$ has a maximum value, $\tilde{p}_L$ which is termed as AOQL.

ATI :~ The total amount of inspection consists of two parts :

(1) a sample from each lot, whether it is accepted or rejected.
(2) the rest of the items in the rejected lots. Therefore the average (expected) total inspection in a lot, when the process average is $\bar{p}$ is the sum of (i) the sample size n and (ii) the remainder of rejected lots, $N - n$ multiplied by the probability of obtaining a sample with more defectives than the acceptance number.

If the process average fraction defective in a lot is $\bar{p}$ as claimed by the producer, then the average total inspection (ATI) per lot is :

$$ATI = n + (N - n)\, P[d > c \mid p = \bar{p}]$$

Hence, $ATI = n + (N - n)(1 - L(\bar{p}))$

$= n + (N - n)\{1 - Pa(\bar{p})\}$; where $Pa(\bar{p})$ is the lot acceptance probability when the lot incoming quality is $\bar{p}$.

Plans classified according to Type of Protection :—

1. Lot quality protection or LTPD plan :~ [CU]

Consumer's requirement fixes the values of P_c, the consumer's risk and P_t, the lot tolerance fraction defective, where N is always fixed. If P_t be the lot tolerance fraction defective, the expression for P_c is

$$P_c = P[\text{Accepting a lot of quality } P_t]$$

$$P_c = L(P_t) \text{ or } Pa(P_t)$$

$$\therefore \text{Consumer's risk} = \sum_{d=0}^{c} \binom{NP_t}{d}\binom{N-NP_t}{n-d} \Big/ \binom{N}{n} \quad \text{——(*)}$$

For given values of P_c and P_t, the equation (*) which involves two unknowns n and c is satisfied by various pairs of values of n and c.

If $\bar{P}$ is the producer's process average, the producer's risk is given by $P_p = P[\text{rejecting a lot of quality } \bar{P}]$

$$P_p = 1 - Pa(\bar{P})$$

$$\therefore \text{Producer's risk} = 1 - \sum_{d=0}^{c} \binom{N\bar{P}}{d}\binom{N-N\bar{P}}{n-d} \Big/ \binom{N}{n}.$$

Then ATI is given by,

$$ATI = n + (N-n)\left(1 - Pa(\bar{P})\right) \quad \text{——(**)}$$

To safeguard producer's interest also, out of these possible pairs of (n, c) satisfying (*), one involving the minimum ATI as given by (**) is chosen. The solution, however, is theoretically very difficult to obtain. Dodge and Romig, by applying numerical methods have prepared extensive tables for minimising values of n, c for $P_c = 0.10$ and different values of $\bar{P}$.

Hence, it is possible to design rectifying inspection plan (that is, to find the values of n and c) that gives a specified level of protection (P_c) at the LTPD (p_t) point and that minimizes the ATI for a specified process average ($\bar{p}$).

2. AOQL Plan: — [CU]

Here, the consumer's interests are taken care of by specifying the AOQL, so that no matter how bad the fraction defective is in the coming lots, he will never have a worse quality level on the average than AOQL × 100% defective.

If p be the incoming lot quality of a lot of size N, the AOQ is given by

$$AOQ = \tilde{p}_a = \sum_{d=0}^{c} \left(p - \frac{d}{N}\right)\binom{Np}{d}\binom{N-Np}{n-d}\Big/\binom{N}{n} \qquad \text{---------- (***)}$$

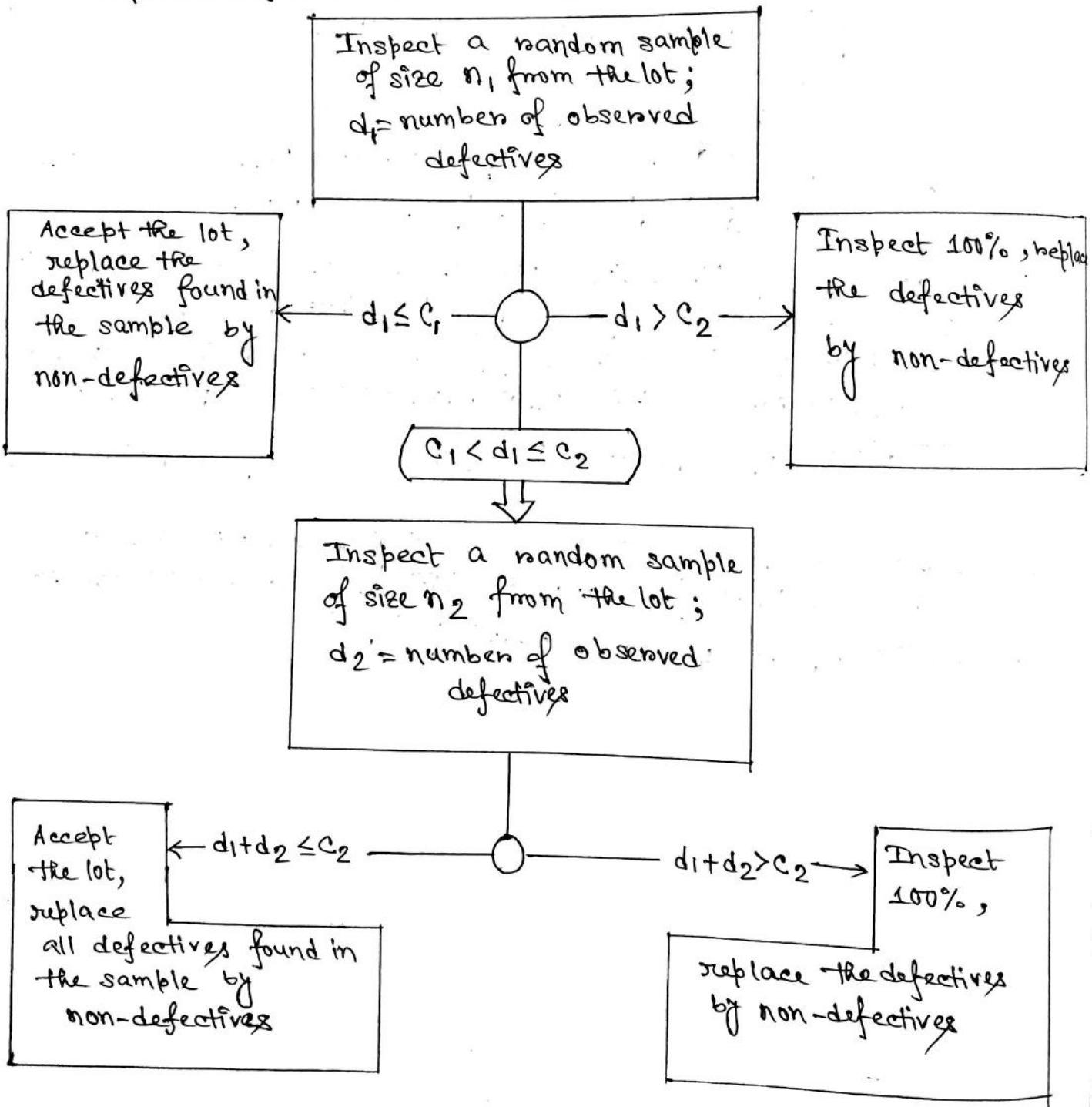
Given N and $\tilde{p}_L$ (AOQL), it is possible to select several pair of values n and c that will give $\tilde{p}$ as defined in (***), having approximately the same value of $\tilde{p}_L$; as a safeguard to producer's interests we select the pair (n, c) which minimizes ATI as defined in (**), i.e. $ATI = n + (N-n)\left(1 - P_a(\bar{p})\right)$, for a specified value of $\bar{p}$.

Hence, it is possible to choose the rectifying sampling plan that has a specified AOQL ($\tilde{p}_L$) and, in addition, yields a minimum ATI at a particular level of process level ($\bar{p}$).

B. Double-Sampling Plans: — A double sampling plan is a procedure in which, under certain circumstances, a second sample is required before the lot can be sentenced. A double sampling plan is defined by four parameters:

n_1 = size of the 1st sample.
c_1 = acceptance number of the first sample.
n_2 = size of the 2nd sample.
c_2 = acceptance number for both samples.

Operation of the double-sampling plan: Acceptance-Rectification



The OC curve : ~ If $P_a(p)$ denotes the probability of acceptance on the combined samples, and $P_a^{I}(p)$, $P_a^{II}(p)$ denote the probability of acceptance on the 1st and 2nd samples, respectively, of a lot of incoming quality p, then

$$\begin{aligned} P_a(p) &= P_a^{I}(p) + P_a^{II}(p) \\ &= P[d_1 \le c_1] + P[c_1 < d_1 \le c_2,\ d_1+d_2 \le c_2] \\ &= \sum_{d_1=0}^{c_1} f(d_1, p) + \sum_{d_2=0}^{c_2-d_1} \sum_{d_1=c_1+1}^{c_2} f(d_1, p)\, g(d_2, p \mid d_1) \end{aligned}$$

where, $f(d_1, p)$ is the probability of getting 'd_1' defectives in the 1st sample and $g(d_2, p \mid d_1)$ is the conditional probability of finding d_2 defectives in the second sample under the condition that d_1 defectives have already appeared in the 1st sample.

thus $$f(d_1, p) = \binom{Np}{d_1}\binom{N-Np}{n_1-d_1} \Big/ \binom{N}{n_1} \quad \text{and}$$

$$g(d_2, p \mid d_1) = \binom{Np-d_1}{d_2}\binom{N-n_1-(Np-d_1)}{n_2-d_2} \Big/ \binom{N-n_1}{n_2}$$

Hence,

$$P_a(p) = \sum_{d_1=0}^{c_1} \frac{\binom{Np}{d_1}\binom{N-Np}{n_1-d_1}}{\binom{N}{n_1}} + \sum_{d_2=0}^{c_2-d_1} \sum_{d_1=c_1+1}^{c_2} \frac{\binom{Np}{d_1}\binom{N-np}{n_1-d_1}\binom{Np-d_1}{d_2}\binom{N-n_1-Np+d_1}{n_2-d_2}}{\binom{N}{n_1}\binom{N-n_1}{n_2}}.$$

Consumer's risk & Producer's risk : —

The consumer's risk is $P_c = P[\text{accepting a lot of quality } p_t]$

$$= P_a(p_t);$$

the producer's risk is $P_p = 1 - P_a(\bar{p})$.

ATI: Since (i) only n_1 items will be inspected if $d_1 \le c_1$ and the probability is $P_a^{I}(p)$.

(ii) (n_1+n_2) items will be inspected if the lot is accepted on the basis of the second sample and its probability is $P_a^{II}(p)$, and (iii) the entire lot of N items will be inspected if the lot is rejected and the probability of this is $\{1 - P_a(p)\}$

Then, the average total inspection (ATI) is given by

$$\begin{aligned} ATI &= n_1 P_a^{I}(p) + (n_1+n_2) P_a^{II}(p) + N\{1 - P_a(p)\} \\ &= n_1 + n_2\{1 - P_a^{I}(p)\} + (N-n_1-n_2)\{1 - P_a(p)\}, \end{aligned}$$

using $P_a(p) = P_a^{I}(p) + P_a^{II}(p)$

[In an acceptance-rejection double sampling plan, the number of items inspected for a lot is either n_1, when the lot is accepted or rejected on the basis of the 1st sample, or (n_1+n_2) when a 2nd sample of size n_2 is drawn. Thus the expected sample size for a decision is given by

$$ASN = n_1 P_1 + (n_1+n_2)(1-P_1) = n_1 + n_2(1-P_1),$$

where, P_1 is the probability of a decision (acceptance or rejection of the lot) on the basis of the 1st sample

$$= P(d_1 \le c_1 \text{ or } d_1 > c_2) = 1 - P[c_1 < d_1 \le c_2]$$

$$= 1 - \sum_{d_1 = c_1+1}^{c_2} \frac{\binom{Np}{d_1}\binom{N-Np}{n_1-d_1}}{\binom{N}{n_1}} . \ ]$$

AOQ :-

$$AOQ = \frac{\left[P_a^{I}(p)\cdot\{N-n_1\} + P_a^{II}(p)\{N-n_1-n_2\}\right]p}{N}$$

The maximum value of this AOQ with respect to p is the AOQL in the double sampling plan.

Designing Double-Sampling Plans :-

It is often necessary to be able to design a double sampling plan that has a specified OC-curve — the values to be determined here are n_1, n_2, c_1 and c_2. There are two approaches for determining these values — LTPD plan, or, AOQL plan.

The Dodge-Romig tables give double sampling plans that have either a specified p_t or a specified AOQL and yield minimum ATI at the given values for the process average.

Comparison of Double Sampling and Single Sampling plans :~

(1). The principal advantage of a double-sampling plan w.r.t. single sampling is that it may reduce the total amount of required inspection. Suppose that the 1st sample taken under a double-sampling plan that offers the consumer the same protection. In all cases, then, in which a lot is accepted or rejected on the first sample, the cost of inspection will be lower for double sampling than it would be for single sampling. It is also possible to reject a lot without complete inspection of the second sample (this is called curtailment on the 2nd sample). Consequently, the use of double sampling can often result in lower total inspection cost.

(2). Furthermore, in some situations, double-sampling plan has the psychological advantage of giving a lot a second chance. This may have some appeal to the vendor but there is no real advantage to double sampling plans can be chosen so that they have the same OC curve.

(3). Unless curtailment is used on the 2nd sample, under some circumstances double sampling may require more total inspection that would be required in a single sampling plan that offers the same protection.

(4). The double-sampling is administratively more complex than a single-sampling, which may increase the opportunity for the occurrence of inspection errors.

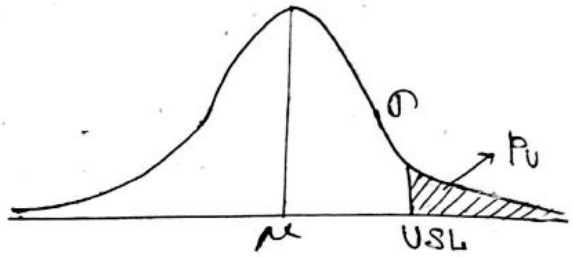
Sampling inspection by Variables :

Consider a variables sampling plan to control the lot or process fraction non-conforming. Since the quality characteristic is a variable, there will exist either a LSL, an USL or both, than define the acceptable values of this parameter.

Let the quality characteristic x is $N(\mu, \sigma^2)$.

(i) If only the USL is given, then an item is considered non-conforming iff $x > USL$.
Then the fraction defective in the lot is $p_U = P[x > USL]$

$$= 1 - \Phi\left(\frac{USL - \mu}{\sigma}\right)$$



(ii) If only LSL is given, then an item is considered defective iff $x < LSL$, and the lot fraction defective is

$$p_L = P[x < LSL] = \Phi\left(\frac{LSL - \mu}{\sigma}\right).$$

(iii) When there are double specification limits, then an item is considered defective if $x < LSL$ or $x > USL$ and the lot fraction defective is

$$p_U + p_L = 1 - \Phi\left(\frac{USL - \mu}{\sigma}\right) + \Phi\left(\frac{LSL - \mu}{\sigma}\right)$$

Sampling inspection provides us with estimates of p_L and p_U or equivalently, of μ and σ, if, estimates $\hat{p}_L$ or $\hat{p}_U$ exceed a specified maximum value M, reject the lot otherwise accept it.

Case I : Variable inspection with known s.d.(σ) :—

When σ is known, there exists MVUEs of p_U and p_L, viz.

$$\hat{p}_U = 1 - \Phi\left(\sqrt{\frac{n}{n-1}}\left(\frac{USL - \bar{x}}{\sigma}\right)\right) \text{ and } \hat{p}_L = \Phi\left(-\sqrt{\frac{n}{n-1}}\left(\frac{\bar{x} - LSL}{\sigma}\right)\right)$$

(i) If only USL is given, the lot is accepted if the estimate $\hat{p}_U$ is small, i.e., if $\hat{p}_U \le M$ (say) $\Leftrightarrow \frac{USL - \bar{x}}{\sigma} \ge K$
$\Leftrightarrow \bar{x} + K\sigma \le USL$. Note that, M is a quantity determined in accordance with the specified prob. of type I error and

$$K = \sqrt{\frac{n-1}{n}}\, t_M .$$

(ii) If only LSL is given, the lot is accepted iff

$$\hat{p}_L \le M \Leftrightarrow \text{iff } \frac{\bar{x} - LSL}{\sigma} \ge K \Leftrightarrow \text{iff } \bar{x} - K\sigma \ge L.$$

(iii) If both specification limits are given, then the lot will be accepted iff $\hat{p}_U + \hat{p}_L \leq M$; otherwise, it will not be accepted.

The values of K, corresponding to the lot size, the sample size and specified acceptance quality level (with prob. of wrong rejection $\alpha = 0.05$), are given in tables A and K of Bowker and Goode's book (1).

Case II: Variable inspection with unknown s.d. (σ) :—

Let $s^2 = \frac{1}{n-1}\sum_i (x_i - \bar{x})^2$ is the sample variance.

(i) For the upper specification limit, the lot is accepted iff

$$\hat{p}_U \leq M \Leftrightarrow \frac{USL - \bar{x}}{s} \geq K^* \Leftrightarrow \bar{x} + K^* s \leq USL,$$

here K^* being a more complicated than the K in the previous case.

(ii) For a given LSL, the lot is accepted iff $\hat{p}_L \leq M$, $\Leftrightarrow \frac{\bar{x} - L}{s} \geq K^*$

$$\Leftrightarrow \bar{x} - K^* s \geq L.$$

(iii) For two-sided specification, the lot will be accepted iff

$$\hat{p}_U + \hat{p}_L \leq M.$$

The value of K for given lot size, sample size and acceptable quality level (with $\alpha = 0.05$), is obtainable from Table A and B of Bowker and Goode's book (1).

☒ Advantages of Variable Sampling :—

1. The variable acceptance-sampling plan that has the same protection as an attribute acceptance-sampling plan would require less sampling. The measurements data required by a variables sampling plan would probably cost more per observation than the collection of attributes data. However, the reduction in sample size obtained may more than off set this increased cost. When destructive testing is employed, variables sampling is particularly useful in reducing the costs of inspection.

2. A second advantage is that measurement data usually provide more information about the manufacturing process than do attributes data.

3. A final point to be emphasized is that when acceptable quality levels are very small, the sample sizes required by attributes sampling plan are very large. Under these circumstances there may be significant advantages in switching to variables measurement.

☒ Disadvantages of Variable Sampling :—

1. Primary disadvantage is that the distn. of the quality characteristic must be known.
2. Most standard variables acceptance-sampling plans assume that the distn. of the quality characteristic is normal but the quality characteristic may not have normal distn..
3. For each quality characteristic, a seperate sampling plan must be employed.

QUESTION & ANSWERS (C.U. PAPER)

4.(a) Derive in details double sampling plan for attributes and derive the quantity by which you protect consumer from inferior product under plan. (10)

(b) S.T. in a single sampling inspection plan by attribute, under suitable assumption, OC curve is given by

$$e^{-np}\sum_{x=0}^{c}(np)^x/x!$$

where, n denotes the sample size, c acceptance number & p lot fraction defective.

ANS:- (b)

$$P_a(p)=\sum_{x=0}^{c}\frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}}=\sum_{x=0}^{c}P[X=x]$$

$$P[X=x]=\binom{n}{x}p^x q^{n-x}, \quad \text{as } N\to\infty.$$

$q=(1-p)$ [Binomial approximation to Hypergeometric distn.]

now, if $n\to\infty, p\to 0$

$$P[X=x]=\frac{e^{-np}.np^x}{x!}$$ [Poisson approximation to Binomial distn.]

Hence, the OC curve is given by,

$$e^{-np}\sum_{x=0}^{c}\frac{(np)^x}{x!}, \text{ where } N\to\infty, n\to\infty, p\to 0,$$

(c) (i) Determine the probability limit 0.1 (i.e. the prob. is 0.1 that without the change in the universe a point will fall above the UCL or fall below the LCL) for $\bar{X}$ and R charts assuming the parent population to be N(1,2) and sample size to be 2.

(ii) Suppose the samples are actually being taken from the N(1.2, 2.4) population. In that case, find the expected number of samples to be drawn to reach the conclusion that the process is not in control w.r.t. either of the quality characteristic (as soon as a sample point goes outside the control limits, you conclude the process is not in control).

ANS: (i) Let (X_1, X_2) be a r.s. from N(1,2).

Let 0.1 probability limits for $\bar{X}$ chart are $L_{\bar{x}}$ and $U_{\bar{x}}$.

Then $$0.1 = 1 - P\left[L_{\bar{x}} < \bar{X} < U_{\bar{x}}\right]$$

$$= 1 - P\left[\frac{L_{\bar{x}} - 1}{1} < \frac{\bar{X} - 1}{1} < \frac{U_{\bar{x}} - 1}{1}\right]$$

$$= 1 - \left\{\Phi\left(U_{\bar{x}} - 1\right) - \Phi\left(L_{\bar{x}} - 1\right)\right\}$$

$$\therefore \Phi\left(U_{\bar{x}} - 1\right) - \Phi\left(L_{\bar{x}} - 1\right) = 0.9$$

[as $X \sim N(1,2)$ $\therefore \bar{X} \sim N(1, \frac{2}{2})$]

Let $U_{\bar{x}}$ and $L_{\bar{x}}$ are symmetric about $E(\bar{X}) = 1$.

Then $U = 1 + k$

$L = 1 - k$

$$\therefore \Phi(k) - \Phi(-k) = 0.9$$

$$\Rightarrow 2\Phi(k) = 1.9$$

$$\Rightarrow \Phi(k) = 0.95$$

$$\Rightarrow k = z_{0.05} = 1.65 \text{ (from table)}$$

Hence the 0.1 prob. limits for $\bar{x}$ are $1 \pm k = -0.65$ and $+2.65$.

$$\text{Range} = |X_1 - X_2| = R$$

$$X_1 - X_2 \sim N(0, 2)$$

$$\Rightarrow \frac{X_1 - X_2}{2} \sim N(0, 1)$$

Now, $P[0 < R < U_R] = 0.9$

$$\Rightarrow P[-U_R < X_1 - X_2 < U_R] = 0.9$$

$$\Rightarrow P\left[-\frac{U_R}{2} < \frac{X_1 - X_2}{2} < \frac{U_R}{2}\right] = 0.9$$

$$\Rightarrow \Phi\left(\frac{U_R}{2}\right) - \Phi\left(-\frac{U_R}{2}\right) = 0.9$$

$$\Rightarrow \Phi\left(\frac{U_R}{2}\right) = 0.95$$

$$\Rightarrow \frac{U_R}{2} = 1.65 \quad \therefore U_R = 3.3$$

$\therefore$ 0.1 prob. limits for R are 0 and 3.3.

(ii) Consider a sample point lies outside either of the control chart as a success.

Let Z be the no. of success require to get the 1st success.

p = Probability of success,

$$= 1 - P[-0.65 < \bar{X} < 2.65,\ 0 < R < 3.3],$$

$$= 1 - P[-0.65 < \bar{X} < 2.65]\, P[0 < R < 3.3],$$

$$= 1 - P[-0.65 < \bar{X} < 2.65] \cdot P[|X_1 - X_2| < 3.3],$$

[For normal sample $\bar{X}$ and $S = \frac{1}{2}|X_1 - X_2|$ are independently distributed as $\bar{X} \sim N\left(1.2, \frac{2.4}{2}\right)$ and $X_1 - X_2 \sim N(0, 4.8)$.]

$$\therefore p = 1 - \left\{\Phi\left(\frac{2.65 - 1.2}{\sqrt{1.2}}\right) - \Phi\left(\frac{-0.65 - 1.2}{\sqrt{1.2}}\right)\right\}\left\{2\Phi\left(\frac{3.3 - 0}{\sqrt{4.8}}\right) - 1\right\}$$

$$= \text{\_\_\_\_\_}, \quad [\text{Use table}]$$

Here $Z \sim$ Geometric(p)

$\therefore$ Required expected no. of samples $= E(Z) = \frac{1}{p} =$ _____.

2.(a) In connection with deriving optimum sampling inspection plan define the following terms (Illustrate your answers with an example)
(i) OC , (ii) AOQL.

(b) Derive a double sampling inspection plan by variable. Describe the usefulness of the plan.

(c) Describe the uses of Indian Standard Sampling Inspection Plans.

ANS:- (c) Use of IS sampling plans:—

These sampling plans have been prepared by the Bureau of Standards, New Delhi and are being widely used.

i) These plans are intended primarily for a continuing series of lots sufficient to allow the switching rules to be applied which provide for (a) an automatic protection to the consumer should a deterioration occur by tightened inspection or discontinuance of inspection, (b) an incentive to reduce inspection costs should consistently good quality be achieved.

ii) These plans may also be used for lots in isolation but in this case the OC curves should be consulted to find a plan to yield the desired protection. Sample sizes are designated by code letters for particular lot size and the prescribed inspection levels. Three types of plans— single, double and multiple are available.

3. Large batches of screws are subject to a single sampling plan with $n=60, c=2$. If the process average $\bar{p}=0.01$, does this lot accept batches of high quality with high probability? ($p \le \bar{p}$) If the proportion of defectives in a batch is 0.05, what is the chance of accepting the batch?

Ans:- Since the batch size is large, n is large, then provided we restrict values of p, we can express the OC curve in terms of

$$L(p) = \sum_{d=0}^{2} e^{-60p} \frac{(60p)^d}{d!}$$

$$= e^{-60p}\left[1 + 60p + \frac{(60p)^2}{2!}\right] \quad \text{——— (*)}$$

Substituting $p=\bar{p}=0.01$, $L(\bar{p}) = 0.977$

Thus, the plan accepts batches of high quality ($p \le \bar{p}$) with high probability (≥ 0.977)

Now, substituting $p = 0.05$ in (*) gives

$$L(0.05) = 0.423.$$

Thus if a batch contains 5% defective screws, the chance of it being accepted is only 0.423.

5. The life time of electric bulbs in a large batch is $N(600, 2500)$ (in hours) when the manufacturing process is operating under the specified norms (under control). The retailers considers the bulbs defective if the mean life-time is less than 500 hours.

(a) Calculate the percentage of defective bulbs in a large batch produced when the manufacturing process is under control.

(b) A random sample of size 20 is taken. Find the value of acceptability constant which ensures that such a batch of bulbs (with mean life time 600hours) should be accepted with probability 0.95.

(c) For the value of n and k in (b), calculate the chance of accepting a bulb containing 5% defective bulbs.

ANS:- (a) $p = \Phi\left(\frac{L-\mu}{\sigma}\right) = \Phi\left(\frac{500-600}{50}\right) = \Phi(-2)$

$= 0.0228$

Thus, 2.28% of the bulbs are defective when the process is under control.

(b) When $L = 500$, $p = 0.0228$, $z_p = -2$, then

$L(p) = \Phi\left(-\sqrt{n}\,(k + z_p)\right) = P(\text{batch is accepted} \mid p)$

$= \Phi\left(-\sqrt{20}\,(k-2)\right)$

$= 0.95$

$\therefore = \Phi(1.645)$

Hence $k = 1.632$

(c) When $p = 0.05$,

$0.05 = \Phi(z_p) = \Phi(-1.645)$

$\therefore z_p = -1.645$.

So, $L(p) = \Phi\left(-\sqrt{20}\,(1.632 - 1.645)\right)$

$= \Phi(-0.0581)$

$= 0.4769$

$\therefore p = 0.477$

Hence, the chance of accepting a batch with 5% defective bulbs is 0.477.

SPC (Statistical Process Control)

Calculation for Control Limits

- **Notations:**

UCL — Upper Control Limit
LCL — Lower Control Limit
CL — Central Line
n — sample size
PCR — Process Capability Ratio $= \frac{USL-LSL}{6\sigma}$
σ — Process standard deviation

$\bar{x}$ — Average of Measurements
$\bar{\bar{x}}$ — Average of Averages
R — Range
$\bar{R}$ — Average of Ranges
USL — Upper Specification Limit
LSL — Lower Specification Limit

- **Variables Data ($\bar{x}$ and R and S charts):**

$\bar{x}$ control chart:

$UCL = \bar{\bar{x}} + A_2\bar{R}$ / $\bar{\bar{x}} + A_3\bar{s}$
(standard not given) $CL = \bar{\bar{x}}$, $LCL = \bar{\bar{x}} - A_2\bar{R}$, where $A_2 = \frac{3}{d_2\sqrt{n}}$, $A_3 = \frac{3}{c_4\sqrt{n}}$

(stan. given) $UCL = \mu_0 + A\sigma_0$, $CL = \mu_0$, $LCL = \mu_0 - A\sigma_0$, where $A = \frac{3}{\sqrt{n}}$

R control chart:

(standard not given) $UCL = \bar{R}D_4$, $CL = \bar{R}$, $LCL = \bar{R}D_3$
where, $D_3 = \left(1 - \frac{3d_3}{d_2}\right)$, $D_4 = \left(1 + \frac{3d_3}{d_2}\right)$

(standard given) $UCL = D_2\sigma_0$, $CL = \sigma_0$, $LCL = D_1\sigma_0$
where, $D_1 = d_2 - 3d_3$, $D_2 = d_2 + 3d_3$

S control chart:

(Standard not given) $UCL = B_4\bar{s}$, $CL = \bar{s}$, $LCL = B_3\bar{s}$
where, $B_3 = 1 - \frac{3}{c_4}\sqrt{1-c_4^2}$, $B_4 = 1 + \frac{3}{c_4}\sqrt{1-c_4^2}$

(Standard given) $UCL = B_6\sigma_0$, $CL = c_4\sigma_0$, $LCL = B_5\sigma_0$
where, $B_5 = c_4 - 3\sqrt{1-c_4^2}$, $B_6 = c_4 + 3\sqrt{1+c_4^2}$.

- **Attribute Data (p, np, c and u control chart):**

	p (fraction)	np (no. of defectives)	c (count of defefectives)	u (average no. of defects per unit)
CL	$\bar{p}$	$n\bar{p}$	$\bar{c}$	$\bar{u}$
UCL	$\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c} + 3\sqrt{\bar{c}}$	$\bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$
LCL	$\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c} - 3\sqrt{\bar{c}}$	$\bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$
Notes	If n varies, use $\bar{n}$ or individual n_i	n must be a constant	n must be a constant	If n varies, use $\bar{n}$ or individual n_i.

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Appendix G

Table of Control Chart Constants

				$\bar{X}$ and R Charts			$\bar{X}$ and S Charts		
n	d_2	d_3	C_4	A_2	D_3	D_4	A_3	B_3	B_4
2	1.128	0.8525	0.7979	1.880	—	3.267	2.659	—	3.267
3	1.693	0.8884	0.8862	1.023	—	2.574	1.954	—	2.568
4	2.059	0.8798	0.9213	0.729	—	2.282	1.628	—	2.266
5	2.326	0.8798	0.9400	0.577	—	2.114	1.427	—	2.089
6	2.534	0.8480	0.9515	0.483	—	2.004	1.287	0.030	1.970
7	2.704	0.8332	0.9594	0.419	0.076	1.924	1.182	0.118	1.882
8	2.847	0.8198	0.9650	0.373	0.136	1.864	1.099	0.185	1.815
9	2.970	0.8078	0.9693	0.337	0.184	1.816	1.032	0.239	1.761
10	3.078	0.7971	0.9727	0.308	0.223	1.777	0.975	0.284	1.716
11	3.173	0.7873	0.9754	0.285	0.256	1.744	0.927	0.321	1.679
12	3.258	0.7785	0.9776	0.266	0.283	1.717	0.886	0.354	1.646
13	3.336	0.7704	0.9794	0.249	0.307	1.693	0.850	0.382	1.618
14	3.407	0.7630	0.9810	0.235	0.328	1.672	0.817	0.406	1.594
15	3.472	0.7562	0.9823	0.223	0.347	1.653	0.789	0.428	1.572
16	3.532	0.7499	0.9835	0.212	0.363	1.637	0.763	0.448	1.552
17	3.588	0.7441	0.9845	0.203	0.378	1.662	0.739	0.466	1.534
18	3.640	0.7386	0.9854	0.194	0.391	1.607	0.718	0.482	1.518
19	3.689	0.7335	0.9862	0.187	0.403	1.597	0.698	0.497	1.503
20	3.735	0.7287	0.9869	0.180	0.415	1.585	0.680	0.510	1.490
21	3.778	0.7272	0.9876	0.173	0.425	1.575	0.663	0.523	1.477
22	3.819	0.7199	0.9882	0.167	0.434	1.566	0.647	0.534	1.466
23	3.858	0.1759	0.9887	0.162	0.443	1.557	0.633	0.545	1.455
24	3.895	0.7121	0.9892	0.157	0.451	1.548	0.619	0.555	1.445
25	3.931	0.7084	0.9896	0.153	0.459	1.541	0.606	0.565	1.435

STATISTICAL PROCESS CONTROL

Product :- An article or substance that is manufactured or refined for sale

Examples:- Automobiles, Refrigerators, music systems, computer, etc.

Services:- A system supplying a public need such as transport, communications, or utilities such as electricity and water.

Examples:- Public Transport system, banking, railways, etc.

Quality :- Definitions :-

⟨1⟩ Fitness for use & Conformance to specifications/requirements.

⟨2⟩ (ISO 9000 Quality Management System)
The totality of features and characteristics of a product or service that bears its ability to satisfy stated or implied needs.

⟨3⟩ (Modern or Japanese approach)
Quality is inversely proportional to variability. The best quality product or service is the one with minimum variation in the performance or the one which gives uniform performance.

⟨4⟩ (Taguchi's Definition)
Quality is the loss to the society caused by a product after being shipped. According to Taguchi the best quality product is the one which causes minimum loss to the society at any time, everytime, till the end of time.

Quality Improvement means continuously reduce variation. Irrespective of carefully maintained or correctly designed every process have a certain amount of natural or inherent variability caused by combined effect of many small, essentially unavoidable causes.

Objective of SQC :- Quickly detect the occurrence of assignable causes so that Corrective actions may be undertaken before Unacceptable products are manufactured.

Control charts:- An on-line process monitoring technique used for statistical process control. Eventual goal is elimination of variability in process. May not be possible to completely eliminate variability but Control charts are very effective in reducing variability.

Stable or in control process :- A process operating with only chance cause of variation.
A process operating in the presence of assignable causes is out of control or unstable.

Control Chart:-
- A tool to ensure that process is stable or in control.
- A tool to detect the presence of assignable causes in the process
- Graphical display of a quality characteristic that has been measured or computed from sample versus the sample number or time.

A graphical tool with three horizontal lines
1. Lower Control Limit (LCL)
2. Center Line (CL)
3. Upper Control Limit (UCL)

In Control Chart:- (Walter A Shewhart)
- Central line represents the average value of the characteristic corresponding to in control state.
- Control limits are chosen such that if the process is in control nearly all the sample points will fall between them.
- As long as the points plot within the control limits, the process is assumed to be in control and no action is necessary.
- A point that plots outside of the control limits is interpreted as evidence that the process is out of control.
- Generally the plotted points in a control chart are joined with straight line segments to easily visualize how the process has evolved over time.
- Even if all points plot inside the control limits, if there is a systematic or non-random pattern, that could be an indication of out of control.
- If the process is in control, all the plotted points essentially have a random pattern.

Types of Control Charts:-

Variable Control Chart: Used for monitoring variable quality characteristic. Variable characteristics can be conveniently describe using a measure of central tendency & variability. These are called Variable Control charts.

Attribute Control Chart:- Used for monitoring attribute quality characteristics. When the product is judged as conforming or non-confirming to requirements or when the count of non-conformities appearing in a product or unit is considered. Control charts for such characteristics are called attribute control charts.

Major reasons for the popularity of control charts:-

1. Improves productivity: Reduces scrap and rework so productivity increases, cost decreases and production capacity increases.
2. Prevents Defects: Helps to keep the process in control indicating do it right the first time. It is cheaper to build it right initially than sort out good units from bad later.
3. Prevent unnecessary process adjustments: Distinguishes between natural and abnormal variation. Unnecessary adjustments can deteriorate the process performance.
4. Control charts improves the process. Generally process don't operate in a state of statistical control. Use of control charts will identify assignable causes. Eliminating the causes will reduce variability & will improve process.

Out-of-control-Action-Plan (OCAP):- A flow chart or document describing the sequence of activities to be undertaken once assignable causes are detected.

Choice of Control charts:- (Use Normal Distribution)

For Normal Distn. between $\mu \pm 1\sigma$: 68.26% of values will lie
$\mu \pm 2\sigma$: 95.46% " " " "
$\mu \pm 3\sigma$: 99.73% " " " "

If characteristic x is normally distributed with mean μ & s.d. σ then $P(\mu - L\sigma \le X \le \mu + L\sigma) = \alpha$.

So, we choose $L = 3$.

$\therefore P(\mu - 3\sigma \le X \le \mu + 3\sigma) = 0.9973$.

$UCL = \mu + 3\sigma$; $CL = \mu$; $LCL = \mu - 3\sigma$.

Some Useful Definitions:-

Estimate: A numerical value of an estimator.
Estimator: A statistic corresponding to the parameter.
Point Estimator: A statistic that produces a single numerical value as estimate for the unknown population parameter.

- Point estimator should be unbiased (the expected value of the estimator should be same as the parameter value) & should have minimum variance.

NOTE:- Sample mean ($\bar{x}$) is the unbiased estimator of population mean & sample variance (s^2) is the unbiased estimator of population variance. But Sample standard deviation is not an unbiased estimator of population standard deviation.

$E(\bar{x}) = \mu$, $E(s^2) = \sigma^2$.

Random Sampling:- (i) Variation within the items in a subgroup will be maximum.
(ii) Variation between items in different subgroups will be minimum

Rational Sampling:- (i) Variation within the items in a subgroup will be minimum.
(ii) Variation between items in different subgroups will be maximum.

Individual x & Moving Range chart (x – MR Charts)

- Control chart with subgroup size 1 ($n=1$).
- Sample consists of an individual unit only.

Uses:- 1. When automated inspection & measurement is used. Every unit manufactured is measured so basis for rational subgrouping.

2. When the production rate is very slow. The long interval between observations will cause problems with rational subgrouping.

3. When the variation within the subgroup is almost negligible. The repeat measurements differ only because of laboratory or measurement errors.

4. Multiple measurements are taken on the same unit.

Requirements:- The quality characteristic must be normally distributed. The process variability is estimated using MR.

Along with individual x chart, generally a control chart for moving range is also constructed.

Moving Range:- The range between two successive observations

$$MR_i = |x_i - x_{i-1}|$$

For x chart:-

$$UCL = \mu + 3\sigma$$
$$CL = \mu$$
$$LCL = \mu - 3\sigma$$

where, $\hat{\mu} = \bar{x} = \frac{x_1 + x_2 + \cdots + x_m}{m}$, $\hat{\sigma} = \frac{\overline{MR}}{d_2}$,

$$\overline{MR} = \frac{MR_1 + MR_2 + \cdots + MR_m}{m}.$$

∴ For individual x chart, the limits are:

$$UCL = \bar{x} + \frac{3}{d_2}\overline{MR}$$
$$CL = \bar{x}$$
$$LCL = \bar{x} - \frac{3}{d_2}\overline{MR}, \text{ for } n=2,\ d_2 = 1.128.$$

For MR chart :–

$$UCL = \overline{MR} + 3d_3\frac{\overline{MR}}{d_2} = D_4\overline{MR}$$
$$CL = \overline{MR} = \overline{MR}$$
$$LCL = \overline{MR} - 3d_3\frac{\overline{MR}}{d_2} = D_3\overline{MR}$$

where, $D_3 = \left(1 - \frac{3d_3}{d_2}\right)$ and $D_4 = \left(1 + \frac{3d_3}{d_2}\right)$.

$\overline{X}$ & R Chart

Suppose a quality characteristic is normally distributed with mean μ and standard deviation σ. If $x_1, x_2, \ldots, x_n$ is a sample of size n then the sample mean $\overline{x} = \frac{x_1 + \cdots + x_n}{n}$ is also normally distributed with mean μ & standard deviation $\sigma/\sqrt{n}$.

Methodology :–

1. Collect a sample of size m (m is at least 20 to 25).
2. Each sample contain n observations of quality characteristic (typically n is small 4, 5 or 6). n is called sub-group size.
3. Let $\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_m$ be the subgroup averages.
4. Let $R_1, R_2, \ldots, R_m$ be the subgroup ranges.
5. The $\overline{X}$ chart is for subgroup averages.
6. The R chart is for subgroup ranges.

For $\overline{X}$ chart :–

$$UCL = \mu + \frac{3\sigma}{\sqrt{n}} = \overline{\overline{x}} + \frac{3}{d_2\sqrt{n}}\overline{R} = \overline{\overline{x}} + A_2\overline{R}$$
$$CL = \mu = \overline{\overline{x}} = \overline{\overline{x}}$$
$$LCL = \mu - \frac{3\sigma}{\sqrt{n}} = \overline{\overline{x}} - \frac{3}{d_2\sqrt{n}}\overline{R} = \overline{\overline{x}} - A_2\overline{R}$$

where, $\overline{\overline{x}}$ is the unbiased estimator of μ, given by

$$\overline{\overline{x}} = \frac{\overline{x}_1 + \overline{x}_2 + \cdots + \overline{x}_m}{m}.$$

Average Range is given by $\overline{R}$, $\overline{R} = \frac{R_1 + \cdots + R_m}{m}$.

Relative Range $W = \frac{R}{\sigma}$ and $E(W) = d_2$.

An unbiased estimator of σ is given by $\frac{\overline{R}}{d_2}$.

Also, $A_2 = \frac{3}{d_2\sqrt{n}}$ is available for different sample sizes in table of control chart constants.

For R chart:- Relative range $W = \frac{R}{\sigma}$ has mean $E(W) = d_2$, $Var(W) = d_3$.

Then $\hat{\mu}_R = \bar{R}$, $\sigma_R = d_3\sigma$, $\hat{\sigma}_R = d_3 \cdot \frac{\bar{R}}{d_2}$ is an unbiased estimator of σ_R.

$$UCL = \hat{\mu}_R + 3\hat{\sigma}_R = \bar{R} + 3 d_3 \cdot \frac{\bar{R}}{d_2} = D_4\bar{R}$$
$$CL = \hat{\mu}_R = \bar{R} = \bar{R}$$
$$LCL = \hat{\mu}_R - 3\hat{\sigma}_R = \bar{R} - 3 d_3 \cdot \frac{\bar{R}}{d_2} = D_3\bar{R}$$

where, $D_3 = \left(1 - \frac{3d_3}{d_2}\right)$ and $D_4 = \left(1 + \frac{3d_3}{d_2}\right)$ are tabulated for different values of n.

$\bar{X}$ & s Chart

When subgroup size n is moderately large (say $n > 10$ or 12), Range may not be a good measure of variation. It is desirable to estimate variation using standard deviation.

Sample variance s^2 is an unbiased estimator of popln var. σ^2

where, $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

We also have $E(s) = c_4\sigma$, c_4 is a constant depends on 'n'.

$V(s) = \sigma^2(1 - c_4^2)$, $\sigma_S = \sigma\sqrt{1 - c_4^2}$.

For $\bar{X}$ chart:-

$$LCL = \hat{\mu} - \frac{3\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} - \frac{3}{c_4\sqrt{n}}\bar{s} = \bar{\bar{x}} - A_3\bar{s}$$
$$CL = \hat{\mu} = \bar{\bar{x}} = \bar{\bar{x}}$$
$$UCL = \hat{\mu} + 3\frac{\hat{\sigma}}{\sqrt{n}} = \bar{\bar{x}} + \frac{3}{c_4\sqrt{n}}\bar{s} = \bar{\bar{x}} + A_3\bar{s}$$

where, $\hat{\mu} = \bar{\bar{x}} = \frac{\bar{x}_1 + \cdots + \bar{x}_m}{m}$ is an unbiased estimator of μ,

$\hat{\sigma} = \frac{\bar{s}}{c_4}$, where $\bar{s} = \frac{s_1 + s_2 + \cdots + s_m}{m}$, is an unbiased estimator of σ.

Also, $A_3 = \frac{3}{c_4\sqrt{n}}$ is available for different sample sizes in the table of control chart constants.

For S chart:- Estimate of mean, $\hat{\mu}_S = \bar{S}$

Standard deviation of S, $\sigma_S = \sigma\sqrt{1-c_4^2}$

$\therefore \hat{\sigma} = \frac{\bar{S}}{c_4}$ is an unbiased estimator of σ.

$$LCL = \hat{\mu}_S - 3\hat{\sigma}_S = \bar{S} - \frac{3\bar{S}}{c_4}\sqrt{1-c_4^2} = B_3\bar{S}$$

$$CL = \hat{\mu}_S = \bar{S} = \bar{S}$$

$$UCL = \hat{\mu}_S + 3\hat{\sigma}_S = \bar{S} + \frac{3\bar{S}}{c_4}\sqrt{1-c_4^2} = B_4\bar{S}$$

where, $B_3 = \left(1 - \frac{3}{c_4}\sqrt{1-c_4^2}\right)$ & $B_4 = \left(1 + \frac{3}{c_4}\sqrt{1-c_4^2}\right)$.

Note:-

For $\bar{X}$ & R chart:- Process mean $= \bar{\bar{x}}$

Process s.d. $= \frac{\bar{R}}{d_2}$

For $\bar{X}$ & S chart:-

Process mean $= \bar{\bar{x}}$

Process SD $= \frac{\bar{S}}{c_4}$

For X & MR chart:- Process mean $= \bar{x}$

Process SD $= \frac{\overline{MR}}{d_2}$

Scrap $= P(X \leq LSL)$; Rework $= P(X \geq USL)$

Non-conforming = Scrap + Rework.

Control charts for Attributes

- Many cases quality characteristic are not numeric.

Ex:- Classification of each item inspected as either conforming or non-conforming (defectives) to the specifications or requirements.

- Types of Control charts for Attributes:-

1. Control charts for nonconforming units (defectives)
2. Control charts for nonconformities (defects)

i.e. 1. { Number of defectives chart (np chart)
Control charts for fraction non-conforming (p-chart)
: fraction defective charts.

2. { Number of defects (c chart)
Defects per unit chart (u chart)

☑ Usage of 1:- np chart is generally used when the subgroup size n is constant.

p chart is used when the subgroup size n is varying from sample to sample.

☑ Usage of 2:- c chart is generally used when the subgroup size n (total inspected) is constant.

u chart is used when the subgroup size n is varying from sample to sample.

- Control charts for Number of Defectives : np chart

Used when subgroup size is constant.
Based on Binomial Distribution.
Number of Defectives are plotted on the chart.

If a random sample of n units of a product is selected and if D is the number of units of product that are non conforming, then D has a binomial distribution with parameters n and p, $\hat{p} = \frac{D}{n}$.

$$E(D) = np, \quad V(D) = np(1-p),$$
$$SD(D) = \sqrt{np(1-p)}.$$

Control limits are:- $LCL = \mu - 3\sigma = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$

$CL = \mu = n\bar{p}$

$UCL = \mu + 3\sigma = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$

Estimate of $p = \hat{p} = \bar{p} = \sum_{i=1}^{m} Di / mn$, where m is the number of samples.

- Control charts for Fraction Defectives : p chart

Used when subgroup size n is not constant.

Based on Binomial distribution.

Fraction of defectives are plotted on the chart.

If a random sample of n units of a product is selected and if D is the number of units of product that are non-conforming, then D has a binomial distribution with parameter n & p.

$$\hat{p} = \frac{D}{n}.$$

$$E(\hat{p}) = \frac{E(D)}{n} = \frac{np}{n} = p$$

$$V(\hat{p}) = V\left(\frac{D}{n}\right) = \frac{1}{n^2} \cdot np(1-p) = \frac{p(1-p)}{n}.$$

$$SD(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$$

Control limits are:-

$$LCL = \mu - 3\sigma = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$CL = \mu = \bar{p}$$

$$UCL = \mu + 3\sigma = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}},$$

Estimate of $p = \hat{p} = \bar{p} = \sum_{i=1}^{m} Di \Big/ \sum_{i=1}^{m} ni$, where m is the number of samples.

- **Control charts for Number of Defects : c chart**

Used when subgroup size is constant.
Based on Poisson Distribution.
Number of defects are plotted on the chart.

If a random sample of size n units of a product is selected and if X is the number of non-conformities, then X has a poisson distribution with parameter c.

$$E(X) = c$$
$$V(X) = c$$
$$SD(X) = \sqrt{c}.$$

Control limits are :-

$$LCL = \mu - 3\sigma = \bar{c} - 3\sqrt{\bar{c}}$$
$$CL = \mu = \bar{c}$$
$$UCL = \mu + 3\sigma = \bar{c} + 3\sqrt{\bar{c}}$$

Estimate of $c = \hat{c} = \bar{c} = \sum_{i=1}^{m} x_i / m$, where m is the number of samples.

- **Control charts for Defects per unit : u chart**

Used when sample size is not constant.
Based on Poisson Distribution.
Defects per unit (X/n) are plotted on the chart.

If a random sample of n units of a product is selected and if X is the number of non-conformities, then X has a Poisson Distribution with parameter c.
Nonconformities per unit (X/n) is denoted by u.

Estimate of $u = \hat{u} = \bar{u} = \sum_{i=1}^{m} X_i / \sum_{i=1}^{m} n_i$, where m is the number of samples.

$$E\left(\frac{X}{n}\right) = \frac{c}{n} = u.$$
$$V\left(\frac{X}{n}\right) = \frac{1}{n^2} \cdot c = \frac{u}{n}.$$
$$SD\left(\frac{X}{n}\right) = \sqrt{\frac{u}{n}}$$

Control limits are:-

$$UCL = \mu + 3\sigma = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$$
$$CL = \mu = \bar{u}$$
$$LCL = \mu - 3\sigma = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}.$$

- **Some more out of control cases:—**
  1. 9 consecutive values are in one side of center line.
  2. 6 consecutive values are steadily increasing or decreasing.
  3. 2 out of 3 values > 2SD from center line (same side)
  4. 4 out of 5 values > 1SD from center line (same side).

Note:- The basic methods of SPC and capability analysis have been in use for over 50 years.
The basic methods of SPC are called Shewhart Control charts.
Motivated by the success of basic techniques, increased emphasis on
— variability reduction,
— yield enhancement,
— process improvement,
lead to development of many new techniques for SPC.

- **Disadvantages of Shewhart control charts:-**
  1. At any point of time, the decision is made only based on the last point plotted on the chart.
  2. Generally ignores the information given by the entire sequence of plotted points.
  3. This makes Shewhart control charts relatively insensitive to small shifts in the process — on order of 1.5σ or less.

Alternatives to Shewhart Control charts are:—
- Cumulative sum (CUSUM) control charts
- Exponentially weighted moving average (EWMA) control charts

STATISTICAL PROCESS CONTROL

Definition of SPC :-

A powerful collection of problem solving tools useful for achieving process stability and reducing variability.

Two types of Variation:-

1. **Chance cause of variation:-**
- Variations of small magnitude
- Difficult to identify
- Difficult to eliminate
- Integral part of the process
- known as natural or allowable cause of variation.

2. **Assignable cause of variation :-**
- Variations of large magnitude
- Represents an unacceptable level of process performance
- Known as special cause of variation
- possible to identify -
- possible to eliminate.

[Att:- Prob. of occurance is very low but it appears.]

SPC

$\bar{X}$-R chart Exercise

Q. **Xbar - R chart : Example :-**

The table below presents 9 subgroups of four measurements on inside diameter (ID) of a part processed in a turning machine? Set up Xbar and R charts on this process. Verify that the process is in statistical control?

Sample No.	Hour	X_1	X_2	X_3	X_4	Mean $\bar{x}$	Range R
1	8.00	5.00	5.01	4.98	5.00	4.998	0.03
2	9.00	5.01	4.98	5.00	5.00	4.998	0.03
3	10.00	5.02	5.01	5.00	5.00	5.008	0.02
4	11.00	5.00	5.00	5.00	5.00	5.00	0.00
5	12.00	4.98	4.98	5.01	4.99	4.990	0.03
6	13.00	5.02	4.99	5.00	4.98	4.998	0.04
7	14.00	4.99	4.99	4.98	4.98	4.985	0.01
8	15.00	5.00	5.01	5.02	5.00	5.008	0.02
9	16.00	4.98	5.00	5.01	4.98	4.993	0.03

Process: Turning
Sample Size (N) : 9

Here $\bar{\bar{x}} = 4.997$
$\bar{R} = 0.023$

Characteristic: Diameter
Subgroup Size(n) : 4

R chart:- $UCL = D_4\bar{R} = 0.023 \times 2.282$, $n=4$.

$CL = \bar{R} = 0.023 \quad | = 0.053$

$LCL = D_3\bar{R} = 0$, since $D_3 = 0$

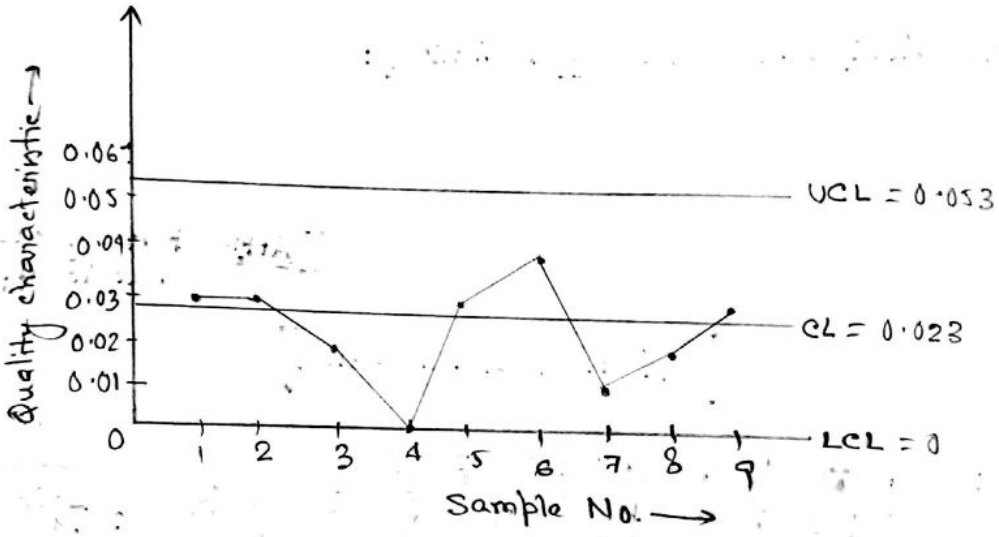
$\bar{X}$ chart:-

$LCL = \bar{\bar{x}} - A_2\bar{R} = 4.98$

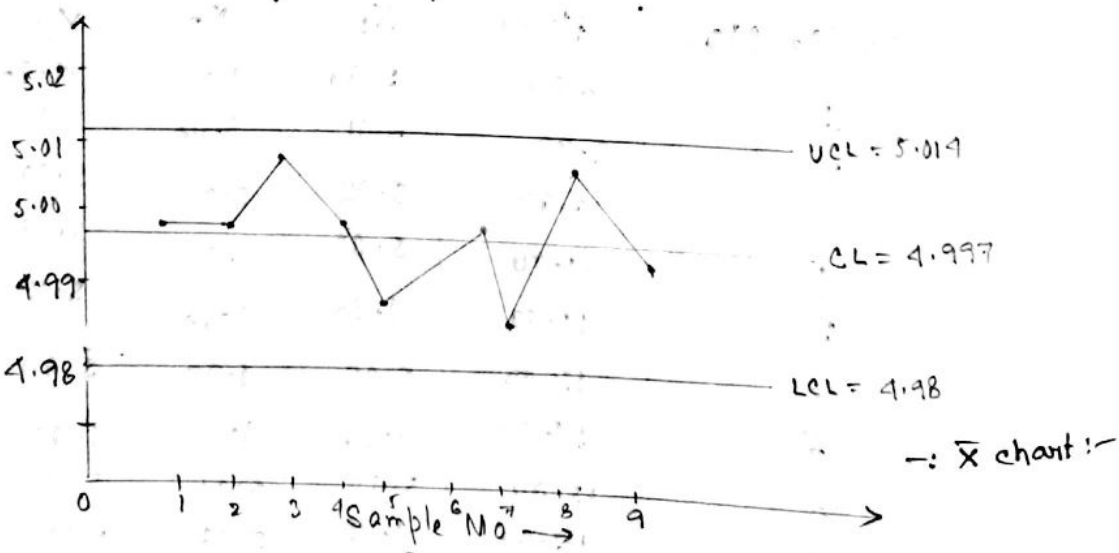
$CL = \bar{\bar{x}} = 4.997$

$UCL = \bar{\bar{x}} + A_2\bar{R}$, since $A_2 = 0.729$

$= 5.014$



—: Graph of R-chart :—



—: $\bar{X}$ chart :—

Since, All the points in $\bar{X}$ & R chart lie within control limits, so the process is in control.

N.P.:- Control charts control stability but does not control characteristic.

$\bar{X}$-R Chart Exercise

Q. Sample of size $n=6$ items are taken from a manufacturing process at regular intervals. A quality characteristic is measured and $\bar{x}$ and R value are calculated for each sample. After 50 samples, we have

$$\sum_{i=1}^{50} \bar{x}_i = 2000 \quad \text{and} \quad \sum_{i=1}^{50} R_i = 200.$$

Assume that the quality characteristic is normally distributed.

(a) Compute control limits for the $\bar{X}$ & R control charts.

(b) Assume both charts exhibit control. Estimate the process mean & s.d.

(c) If the specification limits are 41 ± 5.0. What are your conclusions regarding the ability of the process to produce items within these specification.

(d) Assuming that if an item exceeds upper specification limit it can be reworked and if it is below lower specification limit it must be scraped. What is the % of scrap & rework?

Solution:- (a) Control chart of R chart:- $\bar{R} = \frac{200}{50} = 4$, $n=6$.

So, $D_3 = 0$, $D_4 = 2.004$.

So, $UCL = D_4 \bar{R} = 4 \times 2.004 = 8.016$

$CL = 4$

$LCL = 0$

Control chart for $\bar{X}$ chart:- $\bar{\bar{x}} = \frac{2000}{50} = 40$, $\bar{R} = 4$.

$A_2 = 0.483$.

$UCL = \bar{\bar{x}} + \bar{R}A_2 = 40 + 4 \times 0.483 = 41.932$

$CL = 40$

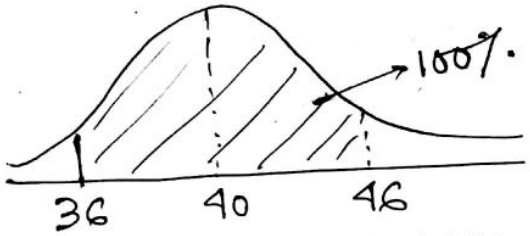
$LCL = 40 - 1.932 = 38.068$

(b) Process Mean $= \bar{\bar{x}} = 40$.

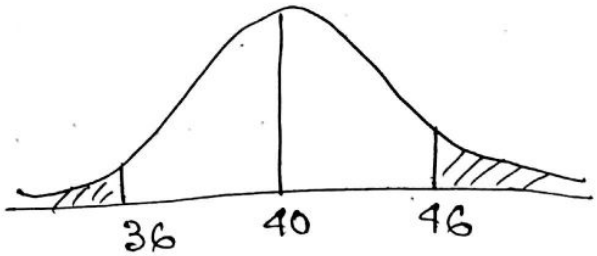
Process SD $= \frac{\bar{R}}{d_2} = 1.578$

(c) $USL = 46$

$LSL = 36$



If the area is 100%. then we meet 100% customer satisfaction.



If there is some area gap, then it's not 100% satisfactory for customer.

P(the process produces items within the specifications)

$= P(36 \le X \le 46) = P(X \le 46) - P(X \le 36)$

$= P\left(\frac{X-\mu}{\sigma} \le \frac{46-40}{1.578}\right) - P\left(\frac{X-\mu}{\sigma} \le \frac{36-40}{1.578}\right)$

$= P(Z \le 3.8023) - P(Z \le -2.534)$

$= 0.9993 - 0.0057$

$= 0.99423$

i.e. 99.42% are under the limit of specifications.

(d) $\text{Scrap} = P(X \le 36) = 0.57\%$

$\text{Rework} = 1 - P(X \le 46) = 0.007\%$.

$\bar{X}$ - s chart Exercise

Q. Samples of $n=4$ items are taken from a manufacturing process of regular intervals. A normally distributed quality characteristic is measured and $\bar{X}$ & s values are calculated from each sample. After 50 subgroups have been analysed, we have

$$\sum_{i=1}^{50} \bar{x}_i = 1000, \quad \sum_{i=1}^{50} S_i = 72$$

(a) Compute the control limits for the $\bar{X}$ & S control charts.
(b) Assume that all points on both the control charts plot within the control limits, estimate the process mean & s.d..
(c) If the specification limits are 19 ± 4.0. Estimate the fraction non-conforming.
(d) Assume that if an item exceeds the USL it can be reworked & if it is below LSL it must be scrapped, then what's % of scrap & rework ?.
(e) If the process is centred at $\mu = 19$, what'd be the effect on % scrap & rework.

Solution:- (a) $\bar{\bar{x}} = \frac{1000}{50} = 20, \quad \bar{S} = \frac{72}{50} = 1.44$

For S - chart:- $n=4, B_4 = 2.266, B_3 = 0$

$$UCL = B_4 \bar{S} = 3.2313$$
$$CL = 1.44$$
$$LCL = 0$$

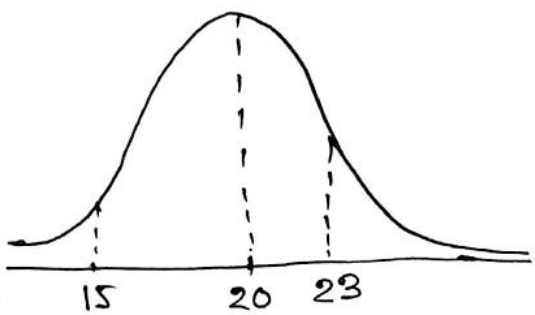
For $\bar{X}$ - chart:- $n=4, A_3 = 1.628$

$$UCL = \bar{\bar{x}} + A_3 \bar{S} = 22.344$$
$$CL = \bar{\bar{x}} = 20$$
$$LCL = \bar{\bar{x}} - A_3 \bar{S} = 17.655$$

(b) Process mean, $\hat{\mu} = \bar{\bar{x}} = 20$,

$$SD(\hat{\sigma}) = \frac{\bar{S}}{c_4} = 1.563$$

(c) USL = 23
LSL = 15



$$P(15 \le x \le 23) = P(X \le 23) - P(X \le 15)$$
$$= P\left(\frac{X-\mu}{\sigma} \le \frac{23-\mu}{\sigma}\right) - P\left(\frac{X-\mu}{\sigma} \le \frac{15-\mu}{\sigma}\right)$$
$$= P\left(Z \le \frac{23-20}{1.563}\right) - P\left(Z \le \frac{15-20}{1.563}\right)$$
$$= P(Z \le 1.919) - P(Z \le -3.198)$$
$$= 0.9725 - .00071$$

(d) Scrap $= P(X \le LSL) = 0.00071$

Rework $= 1 - P(X \le 23) = 0.0275$

$\therefore$ Non-conforming = Scrap + Rework
$= 0.02828$

(e) If $\mu = 19$, the $P(X \le 23) = P\left(Z \le \frac{23-19}{1.563}\right)$
$= P(Z \le 2.55918)$
$= 0.99477$

$\therefore$ Rework $= 1 - 0.99477 = .00523$, i.e. $.523\%$

$P(X \ge 15) = P\left(Z \ge \frac{15-19}{1.563}\right) = P(Z \ge -2.56)$
$= 0.00523$

Scrap $= 0.00523$ i.e. 0.523%.

$\therefore$ Non-conforming $= 1.04\%$

Individual X & MR chart

Q. The viscosity of a polymer is measured hourly. Measurements for the last 20 hours are shown as follows:

Test	Viscosity	MR	Test	Viscosity	MR
1	2838		11	3174	304
2	2785	53	12	3102	72
3	3058	273	13	2762	340
4	3064	6	14	2975	213
5	2996	68	15	2719	256
6	2882	114	16	2861	142
7	2878	4	17	2797	64
8	2920	42	18	3078	281
9	3050	130	19	2964	119
10	2870	180	20	2805	159

(a) Set up a control chart on viscosity and a moving range chart. Does the process exhibit statistical control.

(b) Estimate the process mean & standard deviation.

(c) The next five measurements on viscosity are: 3163, 3199, 3054, 3147 and 3158. Do these measurements indicate the process is in statistical control.

Solution:- (a) $\overline{MR} = \frac{2815}{19} = 148.157$

For $n=2$, $d_2 = 1.128$

For MR chart:
$$\begin{cases} UCL = D_4 \overline{MR} = 148.16 \times 3.267 = 484.04 \\ LCL = D_3 \overline{MR} = 0 \\ CL = 148.16 \end{cases}$$

[check chart behind] chart shows all the points are within the control limit.

Control limits for X chart:- $\overline{X} = 2928.9$

$$UCL = \overline{X} + \frac{3}{d_2}\overline{MR} = 3322.934$$

$$CL = 2928.9$$

$$LCL = \overline{X} - \frac{3}{d_2}\overline{MR} = 2535.86$$

Since all points are within the control limit. So the process is in control.

(b) Process mean is $\hat{\mu} = \bar{x} = 2928.9$, $\hat{\sigma} = \frac{\overline{MR}}{d_2} = \frac{148.157}{1.128} = 131.344$

(c) Yes, these 5 points indicate that the process is in statistical control.

$\bar{X}$ - s chart Exercise

Q. The fill volume of soft-drink beverage bottles is an important quality characteristic. The volume is measured (approximately) by placing a gauge over the crown and comparing the height of the liquid in the neck of the bottle against a coded scale. On this scale, a reading of zero corresponds to the correct fill height. Fifteen samples of size $n=10$ have been analysed and given in the table. Set up $\bar{X}$ & s charts for this process.

Sample No.	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	$\bar{X}$	S
1	2.5	0.5	2.0	-1.0	1.0	-1.0	0.5	1.5	0.5	-1.5	0.5	1.3333
2	0	0	0.5	1	1.5	1	-1	1	1.5	-1	0.45	0.9265
3	1.5	1	1	-1	0	-1.5	-1	-1	1	-1	-0.1	1.1255
4	0	0.5	-2	0	-1	1.5	-1.5	0	-2	-1.5	-0.6	1.1738
5	0	0	0	-0.5	0.5	1	-0.5	-0.5	0	0	0	0.4714
6	1	-0.5	0	0	0	0.5	-1	1	-2	1	0	0.9718
7	1	-1	-1	-1	0	1.5	0	1	0	0	0.05	0.896
8	0	-1.5	-0.5	1.5	0	0	0	-1	0.5	-0.5	-0.15	0.8182
9	-2	-1.5	1.5	1.5	0	0	0.5	1	0	1	0.2	1.1832
10	-0.5	3.5	0	-1	-1.5	-1.5	-1	-1	1	-1	-0.15	1.5284
11	0	1.5	0	0	2	-1.5	0.5	-0.5	2	-1	0.3	1.2065
12	0	2	-0.5	0	-0.5	2	1.5	0	0.5	-1	0.4	1.075
13	-1	-0.5	-0.5	-1	0	0.5	0.5	-1.5	-1	1.5	-0.55	0.6852
14	0.5	1	-1	-0.5	-2	-1	-1.5	0	1.5	-1.5	-0.15	1.2483
15	1	0	1.5	-1.5	1	-1	0	1	-2		0.15	1.2704

Solution:- $\bar{\bar{X}} = 0.023$, $\bar{S} = 1.060$

For $\bar{X}$ chart:- $LCL = \bar{\bar{X}} - A_3\bar{S} = -1.0105$

$CL = \bar{\bar{X}} = 0.023$

$UCL = \bar{\bar{X}} + A_3\bar{S} = 1.0565$, $A_3 = 0.975$ for $n=10$.

For s chart:- $LCL = B_3\bar{S} = 0.301$; $B_3 = 0.284$, $B_4 = 1.716$

$CL = \bar{S} = 1.060$

$UCL = B_4\bar{S} = 1.819$

So, the process is in statistical control.

EXERCISE | np Chart

Q. 1. **np Chart:-** (used when subgroup size n is constant)

Inspection results of video of the month shipment to customers for 10 consecutive days are given in table. The number of inspection each day is constant and is equal to 1000. Construct np chart to control the defectives?

Sample Number	Number of defectives
1	47
2	42
3	48
4	58
5	32
6	38
7	53
8	68
9	45
10	37

Solution:- Subgroup size, $n = 1000$.
Sample size, $m = 10$.

$$\bar{p} = \frac{\text{Sum of defectives}}{\text{Total checked}} = \frac{\Sigma Di}{mn} = \frac{468}{10000} = 0.0468$$

np Control chart:-

$$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})} = 66.84$$
$$= 1000 \times 0.0468 = 46.8$$
$$CL = n\bar{p}$$
$$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})} = 26.76.$$

Since one point is out of control limits, so the process is out of control.
Now remove that point.

Recalculate control limits for np chart:- $n = 1000$, $m = 9$.

$$\bar{p} = \frac{\Sigma Di}{mn} = \frac{400}{9000} = 0.0444$$

$UCL = 63.995$
$CL = 44.444$
$LCL = 24.894$

P chart

Exercise:- The daily inspection results for electric carving knives are given below. Construct a control chart to monitor the process :

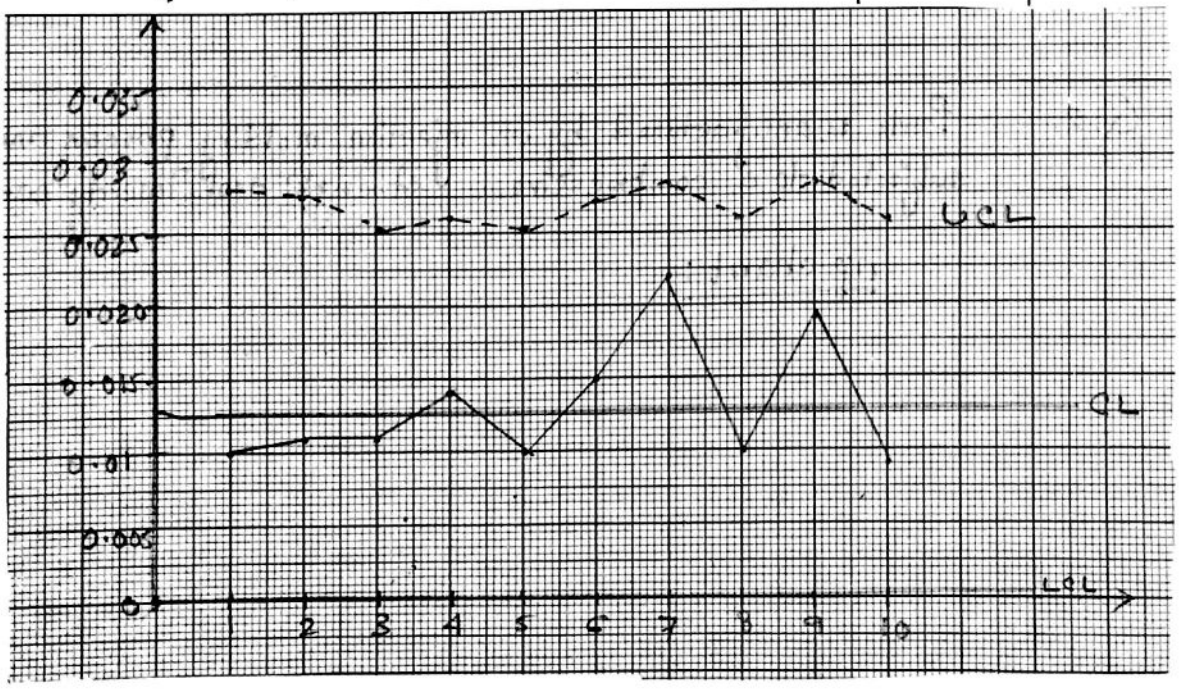
Sample No.	Number Inspected	Number of Defectives	$SD = \sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$	LCL	UCL
1	500	5	0.00502	0	0.0278
2	550	6	0.00479	0	0.0271
3	700	8	0.00424	0.00003	0.0255
4	625	9	0.00449	0	0.0262
5	700	7	0.00424	0.00003	0.0255
6	550	8	0.00479	0	0.0271
7	450	10	0.00529	0	0.0286
8	600	6	0.00458	0	0.0265
9	475	9	0.00515	0	0.0282
10	650	6	0.00440	0	0.0260

$$\bar{p} = \frac{\sum d_i}{\sum n_i} = \frac{74}{5800} = 0.0127586. = CL$$

Now, we have to calculate fraction defectives $= \frac{\#\text{ of defectives}}{\#\text{ of inspected}}$

Then plot the fraction defectives in their corresponding limits.

Sample No.	1	2	3	4	5	6	7	8	9	10
Fraction Defectives	0.010	0.011	0.011	0.014	0.010	0.015	0.022	0.010	0.019	0.009



c chart

Q. 100 product levels are inspected everyday for surface nonconformities. The data for the past 20 days is given below. Construct a suitable control chart to monitor the non-conformities:

Day	Number of Nonconformities	Day	Number of Nonconformities
1	22	11	15
2	29	12	10
3	25	13	33
4	17	14	23
5	20	15	27
6	16	16	15
7	34	17	17
8	11	18	17
9	31	19	19
10	29	20	22

Solution:- Sample size = 20, sub-group size = 100

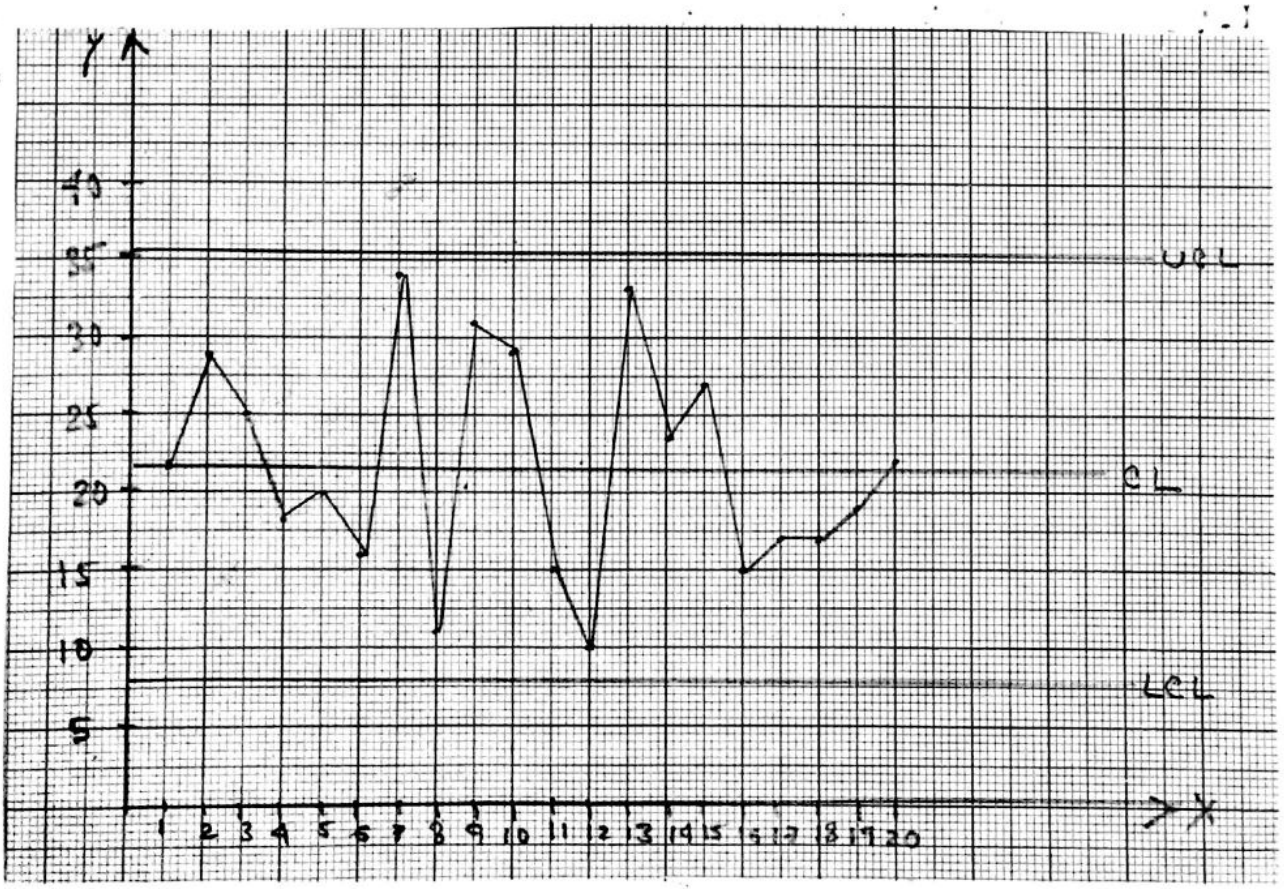
Mean = $\bar{c} = 21.6$

SD = $\sqrt{\bar{c}} = 4.647$

$\therefore$ UCL = $\bar{c} + 3\sqrt{\bar{c}} = 35.55$

CL = 21.6

LCL = $\bar{c} - 3\sqrt{\bar{c}} = 7.66$



u chart

The inspection results for the surface finish of rolls of white paper for 10 dots is given below. Construct a control chart to monitor the process.

Lot Number	Number Inspected (n_i)	Number of Defects (x_i)	$\sqrt{\frac{\bar{u}}{n_i}}$
1	10	45	0.605
2	10	51	0.605
3	10	36	0.605
4	9	48	0.637
5	10	42	0.605
6	10	5	0.605
7	10	33	0.605
8	8	27	0.676
9	8	31	0.676
10	8	22	0.676

Sol. $\bar{u} = \text{Mean} = \frac{\text{Sum of Defects}}{\text{Total inspected}} = \frac{340}{93} = 3.6559$

$SD = \sqrt{\frac{\bar{u}}{n}}$, $CL = 3.6559$; $UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$, $LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$

Sample Number	1	2	3	4	5	6	7	8	9	10
UCL	5.47	5.47	5.47	5.568	5.47	5.47	5.47	5.68	5.68	5.68
LCL	1.842	1.842	1.842	1.744	1.842	1.842	1.842	1.628	1.628	1.628

Plot Defects per unit $\left(\frac{X}{n}\right)$ in the control chart.

(u) Defects per unit	4.50	5.10	3.60	5.33	4.20	0.50	3.30	3.38	3.88	2.75

Assuming that the process has improved, so one point is below LCL. Same limits will be used for future control.

PROCESS CAPABILITY ANALYSIS

Ex.1. 20 data on acid content (mm) is given in the table below. If the specification on acid content is 0.70 ± 0.2 mm. Calculate Process capability?

0.85	0.75	0.80	0.65	0.75	0.6	0.8	0.7	0.75	0.6
0.8	0.75	0.7	0.7	0.75	0.75	0.85	0.6	0.5	0.65

Solution:- USL = 0.9, LSL = 0.5 ; (Units are in mm)

Mean = 0.715, SD = 0.092 ;

$C_p = 0.725 = \frac{USL - LSL}{6\sigma} = \frac{0.90 - 0.50}{6 \times 0.092} = 0.72$

$C_{pl} = 0.78 = \frac{\mu - LSL}{3\sigma}$

$C_{pu} = 0.671 = \frac{USL - \mu}{3\sigma}$

$C_{pk} = 0.671 = \min(C_{pu}, C_{pl})$

The process has not the potential and it is not capable. So, We need to reduce the variation to make C_p & C_{pk} greater than 1.

Ex.2. The specification on coating thickness of powder coated panels is 80 microns ± 5 microns. A sample of 50 powder coated panels are randomly selected and thickness are measured. The data is given below.

Check whether the process is capable of meeting the specification? If the coating thickness below the lower specification, then the panel can be reworked at a cost of $5. Similarly if the coating thickness is more than upper specification, then also the panel can be reworked by removing the point & recoating it at a cost of $20.

Suppose a batch of 120 panels are powder coated. Estimate the rework cost?

Thickness				
81.4	77.9	83.1	82.8	79.7
83.7	84.2	79	80.9	80.8
82.3	81.7	78.9	81.1	84.9
79.8	80.1	80	82.1	79.1
79.5	79	80.2	79.3	82.4
81.8	82.8	81.7	80.2	82.7
82.8	79.2	81.2	82.4	81.4
80.6	81.7	82.3	80.6	79.4
82.6	81.8	82	80.6	82.4
81.9	82.9	82.5	82.4	83.2

Solution:- USL = 85
LSL = 75
Mean = 81.34
SD = 1.551
$C_p = 1.074$, the process has the potential.
$C_{pl} = 1.362$
$C_{pu} = 0.786$
$C_{pk} = 0.786$, but it is not capable to meet specification.

Below LSL = $P(x < LSL) = P\left(Z < \frac{LSL - \mu}{\sigma}\right) = P\left(Z < \frac{75 - 81.34}{1.551}\right)$

$= P(Z < -4.08)$

$= \Phi(-4.08)$

$= 0.00002 = 0.002\%$

Above USL $= 1 - P\left(Z > \frac{USL - \mu}{\sigma}\right)$

$= 1 - P(Z > 2.36)$

$= 1 - 0.99086 = 0.00914 = 0.914\%$

Rework cost of 120 panels

	Number	Cost	Total cost
<LSL	0.0026	5	0.012941
>USL	1.0965	20	21.92992

Total Rework cost = 21.94.

Methods:-

1. **Histogram Method:-**

- Collect large sample of at least 100 observations on the quality characteristic under study.
- Draw Histogram.
- Judge based on Histogram whether the quality characteristic is normally distributed.
- If yes, Estimate process mean, $\hat{\mu} = \bar{x}$, $\hat{\sigma} = s$ is the estimated s.d.
- Estimate C_p and C_{pk}.

2. Control Chart Method:-

- Collect sample data in sub-groups.
- Construct $\bar{x}-R$ and $\bar{x}-s$ chart and check the stability of the process.
- Estimate process mean μ & s.d. σ from control charts

$$\hat{\mu} = \bar{\bar{x}} \quad , \quad \hat{\sigma} = \frac{\bar{R}}{d_2} \text{ or } \frac{\bar{s}}{c_4}.$$

- Estimate C_p & C_{pk}.

Example:- A high voltage power supply should have a nominal output voltage of 350V. A sub group of four units is selected each day and tested for process control purposes. The subgroup averages and ranges are computed and given in the next slide.

1. Set up xbar and R charts on this process. Is the process in statistical control?
2. Estimate the process mean and standard deviation?
3. If specifications are at 350V ± 2V, Estimate the process capability?
4. Assuming that if an item exceeds upper specification limit it can be reworked and if it is below lower specification limit it must be scraped, what is the percentage scrap and rework?

Sample	xbar	Range	Sample	xbar	Range
1	351.00	0.9	11	351.25	0.8
2	350.78	0.7	12	350.98	0.7
3	350.75	0.5	13	351.33	0.7
4	350.90	0.7	14	351.05	0.6
5	350.98	0.6	15	351.10	0.9
6	351.08	0.2	16	351.25	0.5
7	351.08	0.8	17	350.98	0.4
8	350.65	0.6	18	351.08	0.8
9	350.90	0.5	19	350.88	0.6
10	351.35	0.6	20	351.33	0.4

Solution:- 1. $\bar{R} = \frac{125}{20} = 0.625$

$D_3 = 0$, $D_4 = 2.282$

Rchart:-

$UCL = D_4 \bar{R} = 1.42625$

$LCL = D_3 \bar{R} = 0$

$CL = 0.625$

All points of R-chart are within control limits.

$\bar{\bar{X}}$ chart:- $\bar{\bar{X}} = 351.04$, $A_2 = 0.729$

$UCL = \bar{\bar{X}} + A_2\bar{R} = 351.49$

$CL = \bar{\bar{X}} = 351.04$

$LCL = \bar{\bar{X}} - A_2\bar{R} = 350.58$

2. The process is in control.

Process mean, $\hat{\mu} = \bar{\bar{X}} = 351.04$

Process SD, $\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{0.625}{2.059} = 0.3035$

3. Now, given specifications are:

$USL = 352$

$LSL = 348$

So, $C_p = \frac{USL - LSL}{6\sigma} = \frac{4}{6 \times 0.304} = 2.193$

$C_{pu} = \frac{USL - \mu}{3\sigma} = \frac{352 - 351.04}{3 \times 0.304} = 1.053$

$C_{pL} = \frac{\mu - LSL}{3\sigma} = \frac{351.04 - 348}{3 \times 0.304} = 3.333$

$C_{pk} = \min\{1.053, 3.333\} = 1.05 > 1$

So, it has potential to produce and is capability of doing it.

4. Scrap:- $P(x < LSL) = P\left(\frac{x-\mu}{\sigma} < \frac{LSL-\mu}{\sigma}\right)$

$= P\left(Z < \frac{348 - 351.04}{0.304}\right)$

$= 0$

Rework:- $P(x > USL) = P\left(\frac{x-\mu}{\sigma} > \frac{USL-\mu}{\sigma}\right)$

$= P\left(Z > \frac{352 - 351.04}{0.304}\right)$

$= 0.00079$

$= .079\%$

∴ No scrap work, but 0.079% rework is there.

3. Probability Plot Method:- Used when the sample size is small or not sufficient to construct histogram.

- Collect sample data on the quality characteristic under study
- Construct the normal probability plot
- If the plotted points fall approximately on a straight line, then conclude that the quality characteristic follows normal distribution.
- Estimate process mean μ & s.d. σ from Normal Probability Plot as follows

$$\mu = 50^{th} \text{ percentile}$$
$$\sigma = 84^{th} \text{ percentile} - 50^{th} \text{ percentile}$$

- Compute C_p & C_{pk}.

Example:- The performance of the claims reimbursement process of finance department of a company is judged based on time (days) taken to reimburse employee expenses claims. The company wants to settle the claims within 25 days of submitting the documents. The data on cycle times (in days) of 30 randomly selected employee expense claims is given below. Check whether the process is capable of meeting the requirement?

5	5	16	17	14	12
8	13	6	12	11	10
18	18	13	12	19	14
17	16	11	22	13	16
10	18	12	12	12	14

Solution:-

Step-1:- Arrange the data in the ascending order.

Step-2:- Rank (i) the observations.

Step-3:- Compute the empirical cumulative d.f. $F(x) = \frac{i - 0.5}{n}$, where n is the total number of samples.

Step 4:- Plot X versus $F(x)$ in a Normal Probability paper. If the plotted points fall approximately on a straight line, then the quality characteristics follows Normal Distn.

Step 5:- Compute the standard normal score z corresponding to $F(x)$ using normal distn. tables as shown below:

Cycle Time(x)	i	F(x)	z
5	1	0.017	−2.12
5	2	0.05	−1.65
6	3	0.0833	−1.38
8	4	0.1166	−1.20
10	5	0.15	−1.04
10	6	0.183	−0.90
11	7	0.217	−0.78
11	8	0.25	−0.67
12	9	0.283	−0.57
12	10	0.317	−0.47
12	11	0.35	−0.38
12	12	0.383	−0.30
12	13	0.417	−0.21
12	14	0.45	−0.12
13	15	0.483	−0.04
13	16	0.517	0.05
13	17	0.55	0.13
14	18	0.583	0.21
14	19	0.617	0.30
14	20	0.65	0.39
16	21	0.683	0.48
16	22	0.717	0.57
16	23	0.75	0.68
17	24	0.783	0.88
17	25	0.813	0.89
18	26	0.85	1.04
18	27	0.883	1.19
18	28	0.917	1.39
19	29	0.95	1.65
22	30	0.983	2.12

Step-6:- Plot X vs Z in an ordinary graph paper.
If the plotted points fall approximately on a straight line, then the quality characteristic follows normal distribution.

Note:- Try to draw the straight line connecting 25^{th} and 75^{th} percentiles.

Estimate process mean μ and s.d. σ from the normal plot as follows

$\mu = 50^{th}$ Percentile

$\sigma = 84^{th}$ Percentile $- 50^{th}$ Percentile

Percentile	F(x)	Z	Corresponding value (Y
50	0.5	0	13
84	0.84	0.99	17

$\mu = 13$

$\sigma = 17 - 13 = 4.$

USL = 25.

Since LSL is not defined. So we can't calculate C_p.

For $C_{pk} = \min\{C_{pu}, C_{pl}\}$

As C_p is not available, $C_{pk} = C_{pu} = 1.$

$$C_{pu} = \frac{USL - \mu}{3\sigma} = \frac{25-13}{3\times 4} = 1.$$

So, the process is capable of meeting the requirements.

Process Capability Analysis

- An engineering study to estimate the capability of the process or to check whether a process is capable of meeting customer requirements.
- Expressed as Process Capability Indices or Ratios.

Common Process Capability Indices:-

1. Process Potential Index C_p (potential capability of the process).
2. Process Performance Index C_{pk} (actual capability of the process).

Process Potential Index C_p:- A methodology to check whether the process have the potential to meet the customer requirements.
Generally customer requirements are given as specification on product characteristics.

Example:- Specification on Heat treatment process:
Hardness should be within 55 ± 5 HRC

Customer requirements mean Variation allowed by the customer or Variation acceptable to customer.

The above example means that as long as Hardness of the heat treated jobs are between 50 HRC to 60 HRC, customer is satisfied. So, Lower Specification Limit (LSL) = 50 HRC
Upper Specification Limit (USL) = 60 HRC

C_p: A process have the potential to meet customer requirement, if Total or natural variation in process < Allowed variation.

Process capability means Natural Variation in the process.

Definition of C_p:- If the quality characteristic is normally distributed with mean μ and standard deviation σ, then

Total variation: $\mu \pm 3\sigma$

Eg:- Suppose surface hardness achieved of induction hardened piston is normally distributed with mean 55HRC and SD 1 HRC.

$\mu = 55$HRC, $\sigma = 1$HRC

$\therefore$ Total Variation $= 55 - 3\times1$ to $55+3\times1$
$= 52$HRC to 58HRC

Definition:- Ratio of allowed variation to Total variation,

$$C_p = \frac{\text{Allowed Variation}}{\text{Total variation}} = \frac{USL - LSL}{(\mu+3\sigma)-(\mu-3\sigma)}$$

$$= \frac{USL - LSL}{6\sigma}$$

A process has the potential to meet customer requirements if

total variation < allowed variation

$$6\sigma < (USL - LSL)$$

$$\therefore C_p > 1$$

Process Potential Index C_p : Issues

- C_p checks only whether the process has the potential to meet the requirements.
- C_p never checks whether the process is actually meeting requirements.

Example:- Process : Heat Treatment characteristic : Hardness
Specification: 55 ± 5 HRC

	Process 1	Process 2	Process 3
Mean (μ)	55	52	58
SD (σ)	1	1	1
USL − LSL	10	10	10
6σ	6	6	6
C_p	1.66	1.66	1.66

$\therefore C_p = 1.66$ for all 3 processes.

So all 3 process have the potential to meet customer requirement but only Process 1 is meeting customer requirement. Hence process performance index is developed.

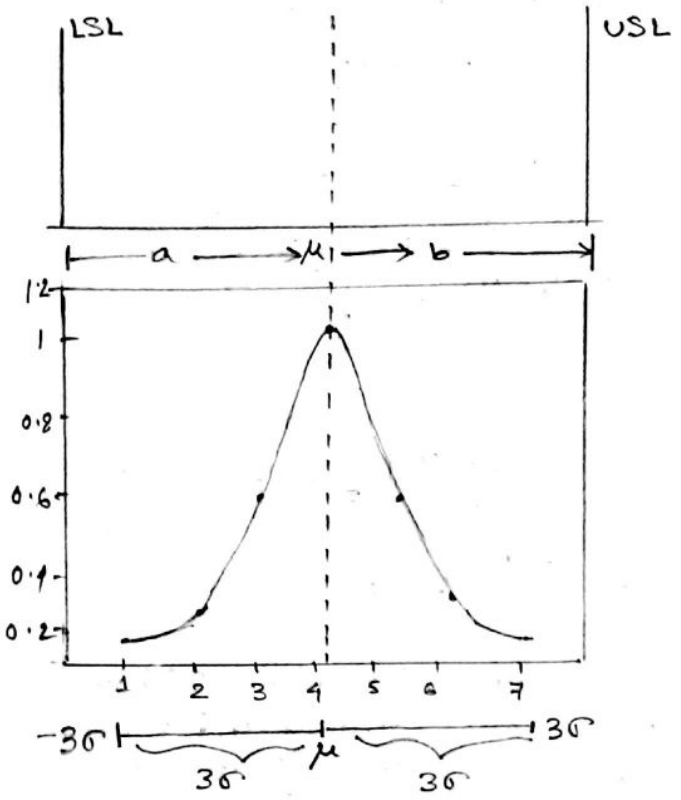
Process Performance Index, C_{pk} : Definition :-

$$C_{pk} = \text{Min}[C_{pl}, C_{pu}], \quad C_{pl} = \frac{\mu - LSL}{3\sigma}, \quad C_{pu} = \frac{USL - \mu}{3\sigma}.$$

C_{pk} checks whether the process is centered at the middle of specification.

$$C_{pk} < 1 \Rightarrow \text{Performance is not OK.}$$

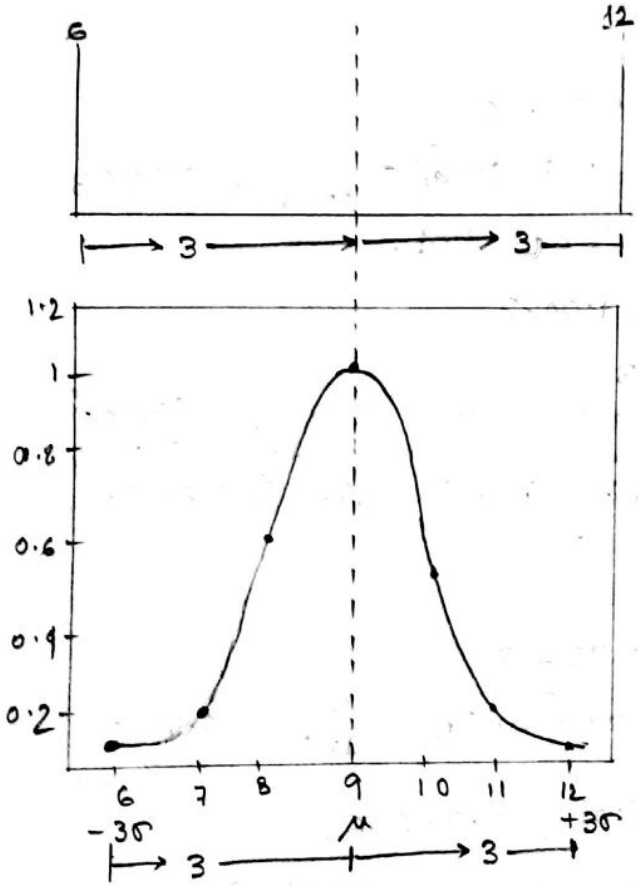
Graphical Representation:-



Example:-

$$C_{pl} = \frac{\mu - LSL}{3\sigma} = \frac{a}{3\sigma}$$

$$C_{pu} = \frac{USL - \mu}{3\sigma} = \frac{b}{3\sigma}.$$



Example:-

USL = 12, LSL = 6

$\mu = 9$ $\sigma = 1$

$C_{pu} = 3/3 = 1$

$C_{pl} = 3/3 = 1$

$C_{pk} = \text{Min}\{C_{pu}, C_{pl}\} = 1.$

$C_p = \frac{(USL - LSL)}{6\sigma} = \frac{6}{6} = 1.$

When Mean is at middle of specification $[(USL + LSL)/2]$ then $C_{pu} = C_{pl} = C_{pk} = C_p$

Otherwise, $C_{pk} < C_p$

When $C_{pk} < C_p$, performance is not optimum.

Process Capability of Non-Normal Characteristics - Approximate Method

Useful for large sample with size > 300

Uses the relationship between C_{pk} and fraction non-conforming

Fraction non-conforming above USL =

$$P(x > USL) = P\left[\frac{x-\mu}{\sigma} > \frac{USL-\mu}{\sigma}\right] = P[Z > Z_U]$$
$$= 1 - P[Z < Z_U]$$

where, $C_{pu} = \frac{USL-\mu}{3\sigma}$, $Z_U = \frac{USL-\mu}{\sigma}$; $C_{pu} = \frac{Z_U}{3}$.

Fraction non-conforming below LSL =

$$P(x < LSL) = P\left[\frac{x-\mu}{\sigma} < \frac{LSL-\mu}{\sigma}\right] = P[Z < Z_L]$$

where, $C_{pl} = \frac{\mu-LSL}{3\sigma}$, $Z_L = \frac{LSL-\mu}{\sigma}$; $C_{pL} = \frac{-Z_L}{3}$.

Ex.1. A company has to process every invoice within 24 hours. A random sample of 1200 invoices are selected and measured the time to process the invoice. The data shows that 2 out of 1500 invoices has taken more than 24 hours to process. Calculate the process capability?

Solution:- Sample size = 1500

No. of non-conforming > USL = 2

Fraction non-conforming $= P(Z > Z_u) = \frac{2}{1500} = 0.00133$

$P(Z < Z_u) = 0.9987$ $\therefore Z_u = 3.01$.

$\therefore C_{pu} = \frac{Z_u}{3} = \frac{3.01}{3} = 1.0033$

$\therefore C_{pk} = C_{pu} = 1.0033$

Ex.2. A back office wants to process at least 90 transactions hourly. The productivity for 1200 hours are measured and found that 3 out of 1200 cases, the productivity is below 90. Calculate process capability?

Solution:- Sample size = 1200

No. of non-conforming < LSL = 3

$P(Z < Z_L)$ = fraction non-conforming $= \frac{3}{1200} = 0.0025$

$\therefore Z_L = -2.81$

So, $C_{pL} = \frac{-Z_L}{3} = \frac{2.81}{3} = 0.93666$.

So, the office is not capable of producing.

MEASUREMENT SYSTEM ANALYSIS

Methodology to evaluate the capability of the measurement system.
Generally any activity involving measurements
— some variability will be inherent in the units or items measured.
— Remaining variability will result from the measurement system.

Example:- Human body temperature using a thermometer.

Day	Body Temperature	
	Self	Friend
1	98.6	97.5
2	97.6	97.9
3	102.1	101.8
4	102.2	102.1

Methodology to evaluate the capability of the measurement system:-

Major components are:-
1. Instrument or gauge used for measurement
2. Operators who use the instrument to measure the items

Objective:-
1. Determine how much of the total observed variability is due to the gauge or instrument.
2. Isolate the components of variability in the measurement system.
3. Access whether the instrument or gauge is capable.

- Variation in the data have two components:-
  - Variation in the process/product
  - Variation in the measurement system

Total Variation = Product variation + Measurement System Variation

$$\sigma^2_{total} = \sigma^2_{product} + \sigma^2_{gauge}$$

Gauge Repeatability and Reproducibility (Gauge R & R):-

This is a methodology to estimate the measurement system variation.

Gauge R & R has two components:-
- Variation caused by operator (appraiser variation - AV)
- Variation caused by instrument (equipment variation - EV)

Repeatability (EV) :- • The variation due to the measuring instrument.
• The variation observed when the same operator measures the same sample repeatedly with the same instrument.

Reproducibility (AV):- • The variation due to the measurement system.
• The variation observed when different operators measure the same sample using the same instrument.

$$\text{So, } \sigma^2_{\text{Gauge}} = \sigma^2_{\text{Reproducibility}} + \sigma^2_{\text{Repeatability}}$$

Gauge R & R: Data collection:- (i) Collect at least 10 samples
(ii) Choose at least 2 operators for study
(iii) Allow each operator to measure each sample at least twice.

Methods:- • $\bar{X}$ - R chart method.
• ANOVA method.

$\bar{X}$ - R chart method:- Number of Operators : 2 = n
Number of Parts : 10 = n

	Operator 1			
Part	1	2	Mean	Range
1	21	20	20.5	1
2	24	23	23.5	1
3	20	21	20.5	1
4	27	27	27.0	0
5	19	18	18.5	1
6	23	21	22.0	2
7	22	21	21.5	1
8	19	17	18.0	2
9	24	23	23.5	1
10	25	23	24.0	2
			21.9 = $\bar{X}_1$	1.2 = $\bar{R}_1$

	Operator 2			
Part	1	2	Mean	Range
1	20	20	20	0
2	24	24	24	0
3	19	21	20	2
4	28	26	27	2
5	19	18	18.5	1
6	24	21	22.5	3
7	22	24	23	2
8	18	20	19	2
9	25	23	24	2
10	26	25	25.5	1
			$\bar{X}_2 = 22.35$	$\bar{R}_2 = 1.5$

Trials	K_1	K_2
2	.8862	.7071
3	.5908	.5231

$$\bar{R} = \frac{\bar{R}_1 + \bar{R}_2}{2} = 1.35$$

Repeatability $(EV) = K_1 \times \bar{R} = 0.8862 \times 1.35 = 1.19681$

Overall variation between operators: $|\bar{X}_2 - \bar{X}_1| = 0.45 = D$

Reproducibility $(AV) = \sqrt{(D \times K_2)^2 - (EV^2/nn)}$

$$= \sqrt{(0.45 \times 0.7071)^2 - (1.19681^2/10 \times 2)}$$

$$= 0.1739.$$

Total Gauge R & R $= \sqrt{\text{Repeatability}^2 + \text{Reproducibility}^2} = \sqrt{EV^2 + AV^2}$

$$= \sqrt{1.19681^2 + 0.1739^2}$$

$$= 1.2094.$$

Part Variation:-

Part	Operator 1		Operator 2		Mean
	1	2	1	2	
1	21	20	20	20	20.25
2	24	23	24	24	23.75
3	20	21	19	21	20.25
4	27	27	28	26	27.00
5	19	18	19	18	16.00
6	23	21	24	21	17.25
7	22	21	22	24	17.25
8	19	17	18	20	18.50
9	24	23	25	23	23.75
10	25	23	26	25	24.75

Parts	K_3
2	0.7071
3	0.5231
4	0.4467
5	0.4030
6	0.3742
7	0.3534
8	0.3374
9	0.3249
10	0.3146

R_p : Mean max − Mean min = 27 − 16 = 11

Part Variation (PV) = $K_3 \times R_p = 0.3146 \times 11$
$= 3.4606$

Total variation $= \sqrt{(\text{Gage R \& R})^2 + (\text{Part Variation})^2}$
$= \sqrt{(1.2094)^2 + (3.4606)^2}$
$= 3.6658$

$= \frac{\text{GSD} \times 100}{\text{Total SD}}$

Source	SD	GXSD	% Study Var
Repeatability	1.19681	7.18086	32.65
Reproducibility	0.7739	1.0434	4.74
Total Gauge R&R	1.2094	7.2564	32.99
Part Variation	3.4606	20.7636	94.40
Total Variation	3.6658	21.9948	100.00

Guidelines for accepting the Measurement System:—

Gauge R & R	Remark
Under 10%	Gauge system is satisfactory.
10% to 30%	May be acceptable based upon application, cost of gage, cost of repair etc.
Over 30%	Gauge system not satisfactory

ANOVA Method:-

Number of Auditors : 2
Number of calls : 5
Replication : 2

Appraiser

Part	A	B	Sum
1	50	55	215
1	54	56	
2	65	64	264
2	67	68	
3	75	79	308
3	76	78	
4	81	82	324
4	79	82	
5	95	96	381
5	94	96	
Sum	736	756	1492

(i) Interaction Sum table:-

Part	A	B
1	104	111
2	132	132
3	151	157
4	160	164
5	189	192

(ii) Correction factor:-

Grand total $T = 1492$
Total Count $N = 20$
Correction Factor $CF = \frac{1492^2}{20}$
$= 111303.2$

(iii) Sum of Squares:-

$Total = 50^2 + 56^2 + \dots + 96^2 - CF = 3976.8$

$Sample = \frac{215^2 + 264^2 + \dots + 381^2}{4} - CF = 3927.3$

$Column = \frac{736^2 + 756^2}{10} - CF = 20$

$Interaction = \frac{104^2 + 111^2 + \dots + 192^2}{2} - SS_{sample} - SS_{column} - CF$
$= 7.5$

$Within = SS_{total} - SS_{sample} - SS_{columns} - SS_{interaction} = 22.$

(iv) Degree of Freedom:-

Source	Formula	Degree of Freedom
Total	Total count −1	19
Sample	No. of rows − 1	4
Columns	No. of columns − 1	1
Interaction	df of sample × df of Appraisers	4×1 = 4
Within	Total df − sample df − column df − Interaction df	10

(v) ANOVA Table construction:-

Source of Variation	SS	df	MS	F	Fcrit
Sample	3927.3	4	981.825	446.2841	3.47805
Columns	20	1	20	9.0909	4.9646
Interaction	7.5	4	1.875	0.8523	3.478
Within	22	10	2.2		
Total	3976.8	19			

$MS = SS/df$

$F(i) = \frac{MS(i)}{MS_{within}}$

Now, checking whether interaction F $\begin{matrix} > \\ = \\ < \end{matrix}$ Interaction $F_{critical}$

Case I:- Since $F = 0.85 < F_{crit} = 3.478$, Interaction is not significant. Modify ANOVA table:-

$SS_{within} = SS_{within} + SS_{interaction} = 22 + 7.5$

$= 29.5$

$DF_{within} = df_{within} + df_{interaction} = 10 + 4$

$= 14$

Modified table:- $MS_{within} = SS_{within} / df_{within} = \frac{29.5}{14} = 2.107$

Source of Variation	SS	df	MS
Sample	3927.3	4	981.8
Columns	20	1	20
Within	29.5	14	2.107
Total	3976.8	19	

Variances:-

Equipment (EV) = $MS_{within} = 2.107143$

Appraiser (AV) = $(MS_{columns} - MS_{within}) / (\text{no. of parts} \times \text{no. of replications})$

$= \frac{20 - 2.107143}{5 \times 2} = 1.7893$

Part (PV) = $(MS_{simple} - MS_{within}) / (\text{no. of appraisers} \times \text{no. of replications})$

$= \frac{981.825 - 2.107143}{2 \times 2} = 244.9295$

Garge R & R = EV + AV = $2.107143 + 1.7893 = 3.8964$

Total = Gage R & R + PV = $3.8964 + 244.9295 = 248.8259$.

	Variance	SD	GSD	% Study Var = $\frac{GSD \times 100}{GSD\ total}$
Gage R&R	3.8964	1.9739	11.8436	12.5137
EV	2.1071	1.4516	8.7096	9.2024
AV	1.7893	1.3376	8.0259	8.4799
PV	244.9295	15.6502	93.9013	99.2139
Total	248.8259	15.7742	94.6453	

Guidelines to accept gage:-

% Contribution of Gage R&R	Remark
Under 10%	Gage system is satisfactory
10% to 30%	May be acceptable based upon application, cost of gage, cost of repair, etc.
Over 30%	Gage system not satisfactory

Case II:- Assuming SS interaction = 70

i.e. When Interaction F > Interaction F critical

ANOVA Table:-

Source of Variation	SS	df	MS	F	F crit
Sample	3864.8	4	966.2	439.18	3.478
Columns	20	1	20	9.09	4.964
Interaction	70	4	17.5	7.95	3.478
Within	22	10	2.2		
Total	3976.8	19			

Variances:-

Equipment (EV) = $MS_{within} = 2.2$

Interaction (INT) = (MS interaction − MS within) / (no. of replications)

= (17.5 − 2.2) / 2 = 7.65

Appraiser (AV) = (MS columns − MS interaction) / (no. of parts × no. of replications)

$$= \frac{(20 - 17.5)}{5 \times 2} = 0.25$$

Part (PV) = (MS sample − MS interaction) / (no. of appraisers × no. of replications)

$$= \frac{981.825 - 17.5}{2 \times 2} = 237.175$$

Gage R & R = EV + AV + INT = 10.1

Total = Gage R & R + PV = 247.275

Complete calculation:-

	Variance	SD	GSD	% contribution
Gage R & R	10.1000	3.178	19.0683	20.2102
EV	2.2	1.4832	8.8994	9.4324
INT	7.65	2.7659	16.5952	17.589
AV	0.25	0.5	3	3.1797
PV	237.175	15.4	92.4029	97.9369
Total	247.275	15.725	94.3499	

Conclusion:- Since % contribution of gage R & R = 20.21 > 10%, the measurement system may be acceptable.

Measurement System Analysis : Discrete

Example: The transaction monitoring process results for 2 auditors is given below. The results of expert (standard) is also given. Perform MSA and give your conclusions?

Transaction	Auditor 1	Auditor 2	Standard	Transaction	Auditor 1	Auditor 2	Standard
1	Pass	Pass	Pass	13	Pass	Pass	Pass
2	Pass	Pass	Pass	14	Fail	Pass	Pass
3	Fail	Pass	Pass	15	Fail	Fail	Fail
4	Pass	Fail	Pass	16	Fail	Fail	Fail
5	Fail	Fail	Fail	17	Fail	Fail	Fail
6	Fail	Pass	Fail	18	Pass	Pass	Pass
7	Pass	Pass	Pass	19	Pass	Pass	Pass
8	Pass	Pass	Pass	20	Pass	Pass	Pass
9	Pass	Pass	Pass	21	Fail	Pass	Pass
10	Pass	Pass	Pass	22	Pass	Pass	Pass
11	Fail	Fail	Pass	23	Pass	Pass	Pass
12	Pass	Pass	Pass	24	Pass	Pass	Fail
				25	Fail	Fail	Fail

Summarize the data as shocon below:-

		Auditor 2		
		Pass	Fail	Total
Auditor 1	Pass	14	1	15
	Fail	4	6	20
	Total	18	7	25

∴ Observed agreement = Sum of (Pass, Pass & Fail, Fail) cases

= 14 + 6 = 20

Calculation of Expected count :-

Expected Count of cell (1,1) = $\frac{\text{Row 1 sum} \times \text{column 1 sum}}{\text{Total}}$

Expected Count (Pass, Pass) = $\frac{15 \times 18}{25} = 10.8$

Between Auditor Analysis:-

Expected Count table:-

		Auditor 2	
		Pass	Fail
Auditor 1	Pass	10.8	4.2
	Fail	7.2	2.8

∴ Expected agreement = Sum of (Pass, Pass & Fail, Fail) cases

= 10.8 + 2.8

= 13.6

Calculate kappa,

$$K = \frac{\text{No. Observed Agreement} - \text{No. Expected Agreement}}{\text{Total Observation} - \text{No. Expected Agreement}}$$

$$= \frac{20 - 13.6}{25 - 13.6}$$

$$= 0.5614$$

Kappa	Strength of Agreement
< 0.00	Poor or None
0.00 – 0.20	Slight
0.21 – 0.40	Fail
0.41 – 0.60	Moderate
0.61 – 0.80	Substantial
0.81 – 1.00	Almost perfect

ACCEPTANCE SAMPLING

Inspection of raw material, semi finished products and finished products are part of quality assurance activity.

Acceptance Sampling :- A sampling procedure to accept or reject products based on inspection.

Example:- 1. A company receives a shipment of product from a vendor.

2. A sample is taken from the lot or shipment and certain quality characteristic of the units in the sample is inspected.

3. Based on the results of inspection, a decision is made either to accept or reject the lot or shipment (lot sentencing)

4. Accepted lots are put into production & rejected lots may be returned to the vendor or subjected some other lot disposition action.

Acceptance Sampling :-

1. The purpose is to accept or reject the lot not to estimate quality.
2. Donot provide any direct quality control, Simple accepts or rejects lots.
3. Even if all lots are of same quality, sampling will accept some lots and reject others.

Approaches for lot acceptance or rejection —

1. Accept with no inspection
2. 100% inspection
3. Acceptance Sampling

Advantages:- 1. Less expensive because there is less inspection.

2. Less handling of products hence reduced damage.
3. Number of personnel required inspection is less.
4. Often reduces inspection errror.
5. Return of entire lots not just the defectives in the sample often provides stronger motivation to the vendor to improve quality.

Disadvantages:- 1. Risk of accepting bad lots and rejection of good lots.

2. Requires documentation of planning and documentation of sampling procedure.
3. Not much information about the quality of the vendor's process is gained.

Single Sampling Plan:- Defined by sample size n and acceptance number c.

- **Procedure**:-

1. Select n items at random from the lot containing N items
2. Inspect the n items in the sample and count the defective items d.
3. If $d > c$, reject the lot. Otherwise accept the lot of N items.

- **Example**:- $N = 10000$, $n = 89$, $c = 2$.

1. Select 89 items randomly from the 10000 items in the lot.
2. Inspect and count the number of defectives d.
3. If $d > 2$, reject the entire lot of 10000. Otherwise accept the lot.

- **Operating Characteristic Curve**:- Measures the performance of a sampling plan.

— Plot the probability of accepting a lot P_a versus the lot fraction defective p (lot quality).
— Shows the discriminatory power of sampling plan.
— Shows the chance that a lot submitted with certain fraction defective will be accepted or not.

Construction of OC curve:-

1. Vary the lot fraction defective (p) from 0 to 1.
2. Compute the probability of acceptance of the lot P_a.
3. Plot p versus P_a in the graph.

Ex:- Let p be the fraction defective in a lot. Let a sample of size n is selected and inspected from the lot. The probability of getting d defectives is

$$P\{d \text{ defectives}\} = \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d}$$

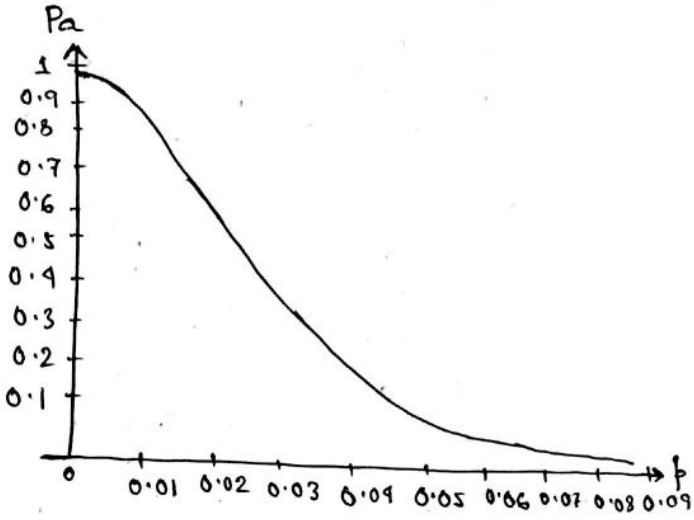
Lot is accepted if $d \le c$.

Probability of acceptance $= P_a = P\{d \le c\} = \sum_{d=0}^{c} \frac{n!}{d!(n-d)!} p^d (1-p)^{n-d}$

Example:- Suppose a product is shipped in lots of size $N = 5000$. The receiving inspection procedure used is single sampling plan with $n = 50$ and $c = 1$. Construct the OC curve of the plan? Compute AQL and LTPD for a producer's risk of $\alpha = 0.005$ and consumer's risk of $\beta = 0.1$?

Solution:- $N = 5000$, $n = 50$, $c = 1$

Fraction Defective (p)	Prob. of Acceptance (Pa)
0.005	0.9739
0.01	0.9106
0.02	0.7358
0.03	0.5553
0.04	0.4005
0.05	0.2794
0.06	0.1900
0.07	0.1265
0.08	0.0827
0.09	0.0532



→ OC curve :-

$\alpha = 0.005$, $Pa = 0.995$

$\beta = 0.1$, $Pa = 0.1$

Identify AQL (fraction defective p) corresponding to $Pa = 0.995$ from graph.

Identify LTPD (fraction defective p) corresponding to $Pa = 0.01$ from graph.

- **Average Quality Level (AQL) :-**

A percent defective that is the base line requirement for the quality of the producer's product.

The producer could like to design a sampling plan such that there is a high prob. of accepting a lot that has a defect level less than or equal to the AQL.

Producer's Risk :- (α)

This is the prob. for a given (n, c) sampling plan, of rejecting a lot that has a defect level equal to the AQL.

The producer suffers when this occurs, because a lot with acceptance quality was rejected.

P	Pa
0	1
0.005	0.9739
0.0072	0.9494
0.01	0.9106
0.02	0.7358
0.03	0.5553
0.04	0.4005
0.05	0.2794
0.06	0.1900
0.07	0.1265
0.08	0.0827
0.09	0.0532
0.1	0.0338
0.11	0.0212
0.12	0.0131

AQL = 0.0072

LTPD = 0.013

- Lot tolerance Percent Defective (LTPD):- A designated high defect level that would be unacceptable to the customer. Consumer would like the sampling plan to have a low probability of accepting a lot with a defect level as high as the LTPD.

Consumer's Risk:- (β) This is the probability, for a given (n, c) sampling plan, of accepting a lot with a defect level equal to the LTPD.
The consumer suffers when this occurs, because a lot with unacceptable quality was accepted.

Rectified Inspection Program:-

The process of screening (100%) inspection of rejected lot, reworking or replacing the defective items with good ones. This is usually done by the suppliers or vendors.
The quality of outgoing lots with rectifying inspection will be better than that of incoming lot quality or the quality of the lot submitted for inspection.

- Average Outgoing Quality (AOQ):- Quality of the lot resulting from rectifying inspection.

Average value of lot quality that would be obtained over a long sequence of lots with rectifying inspection.
Suppose a lot of size N is subjected to acceptance sampling with a fraction defective p.
Let a sample of size n is randomly selected, inspected and counted the number of defectives d and all the defectives d will be replaced with good ones.
If $d > c$, then the lot is rejected.
If lot is rejected, then the remaining $N-n$ items are also inspected and all the defectives are replaced with good ones.
If lot is rejected, then all the N items in the lot will be good. No defectives after the inspection.

Average Outgoing Quality (AOQ) Curve:- If the lot is accepted then N-n items not inspected can contain defectives.

Since fraction defectives is p, the estimated number of defectives after inspection = $p(N-n)$.

If P_a is the chance of accepting the lot, then expected number of defectives after rectifying inspection $= P_a \cdot p(N-n)$

Average Outgoing Quality, $AOQ = \frac{P_a p(N-n)}{N}$

When N is large compared to n, then $AOQ \approx P_a p$.

Plot AOQ Vs. Incoming quality or fraction defective p.

Average Outgoing Quality Level (AOQL):- Highest outgoing fraction defective. Worst possible average quality result from rectifying inspection.

Average Total Inspection (ATI):- Let P_a be the probability of accepting a lot submitted for inspection. If the lot is accepted, then the number of items inspected is n.

If the lot is rejected, then the remaining N-n items also inspected.

Then the chance of inspecting N-n items (rejecting the lot) is $(1-P_a)$.

Average Total Inspection, $ATI = n + (1-P_a)(N-n)$

Average Total Inspection Curve:- Plot of ATI versum incoming quality or fraction defective of lot submitted for inspections p for specific N.

When incoming quality is very good then the lot will generally will be accepted and number of defectives is N-n uninspected items also will be low. Hence, outgoing quality also will be very good.

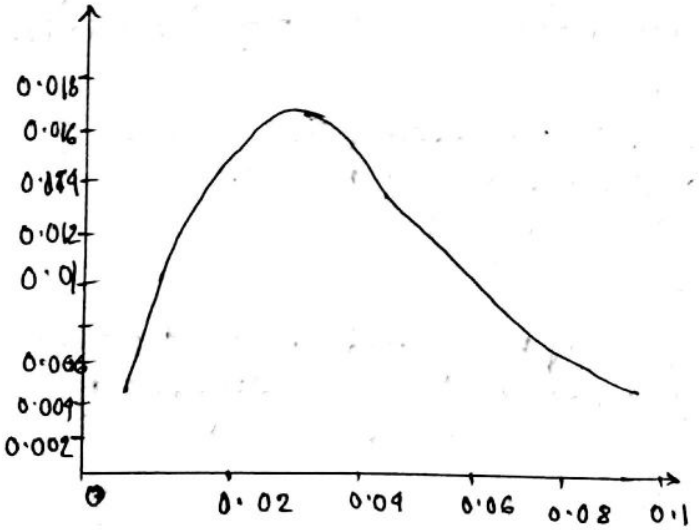
When incoming quality is very bad then the lots will generally be rejected and all defectives will be replaced with good ones. Hence outgoing quality will again be very good.

When incoming quality is average then the lot may or may not be accepted. Outgoing quality will be average and outgoing fraction defective will be reasonably high.

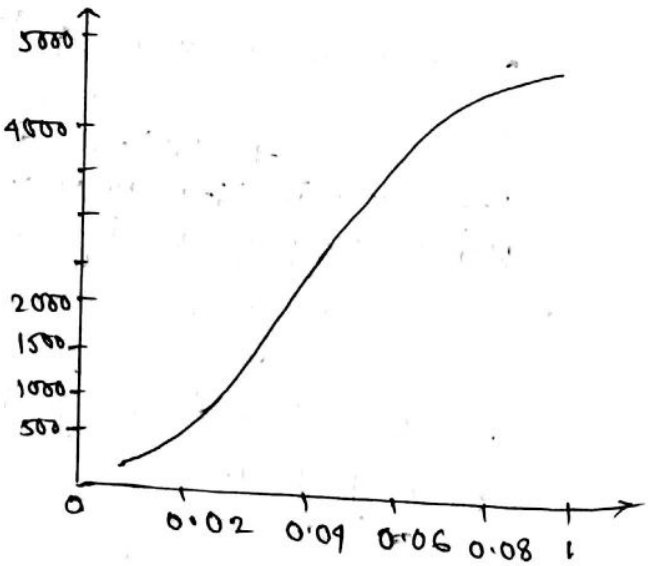
Ex. Suppose a product is shipped in lots of size $N = 5000$. The receiving inspection procedure used is single sampling plan with $n = 50$ and $c = 1$.
(i) Construct AOQ plot. (ii) Construct the ATI plot.

Solution:-

Fraction defective (p)	Prob. of Acceptance (P_a)	P_aP	ATI
0.005	0.9739	0.0049	179
0.01	0.9106	0.0091	493
0.02	0.7358	0.0147	1358
0.03	0.5553	0.0167	2251
0.04	0.4005	0.016	3018
0.05	0.2794	0.014	3617
0.06	0.19	0.0114	4060
0.07	0.1265	0.0089	4379
0.08	0.0827	0.0066	4591
0.09	0.0532	0.0048	4737



-: AOQ curve:-



-: ATI Curve:-

Double Sampling Plan:- Decision to accept or reject the lot is taken based on two samples.

Procedure:- Let lot of size N is submitted for inspection.

1. Take a sample of size n_1 from the lot and count the defectives d_1 in the sample.
2. If $d_1 \le c_1$, the lot is accepted.
3. If $d_1 > c_2$, the lot is rejected.
4. If $c_1 < d_1 \le c_2$, then another sample of size n_2 is taken & count the defectives d_2 in sample 2.
5. If $d_1 + d_2 \le c_2$, then lot is accepted.
6. If $d_1 + d_2 > c_2$, then lot is rejected.

Notations:- N: lot size
n_1: sample size on first sample
c_1: acceptance number of the first sample
n_2: sample size on second sample
c_2: acceptance number for both samples

Advantages:-

1. Reduces total amount of inspection, if decision is taken with first sample itself.
2. If lot is accepted or rejected based on first sample, the cost of inspection will be less.
3. Having a psychological advantage that a lot is given a second chance.

OC curve:- Measures the performance of a double sampling plan. Plots the prob. of accepting a lot P_a versus the lot fraction defective p.
It shows that a lot submitted with certain fraction defective will be accepted or not.

$$P_a = P_a^{I} + P_a^{II}$$

P_a^{I} = Prob. of acceptance on the 1st sample $= P(d_1 \le c_1)$

$$= \sum_{d_1=0}^{c_1} \frac{n_1!}{d_1!\,(n_1-d_1)!} p^{d_1} (1-p)^{n_1-d_1}$$

P_a^{II} = Prob. of acceptance on the 2nd sample $= P[d_1 + d_2 \le c_2]$

where p is the fraction defective in the lot.

$$P_a^{I} = \sum_{d_1=0}^{c_1} \frac{\binom{Np}{d_1}\binom{N-Np}{n_1-d_1}}{\binom{N}{n_1}} \; ; \; P_a^{II} = \sum_{d_2=0}^{c_2-d_1} \sum_{d_1=c_1+1}^{c_2} \frac{\binom{Np}{d_1}\binom{N-Np}{n_1-d_1}\binom{Np-d_1}{d_2}\binom{N-n_1-Np+d_1}{n_2-d_2}}{\binom{N}{n_1}\binom{N-n_1}{n_2}}$$

Example:- Calculate the probability of acceptance for a double sampling plan for a lot of size $N = 5000$ with fraction defective $p = 0.05$. The parameters of the sampling plan are $n_1 = 50$, $c_1 = 1$, $n_2 = 100$, $c_2 = 4$?

Solution:- $P_a = P_a^{I} + P_a^{II}$

$$P_a^{I} = P(d_1 \le c_1) = 0.2794$$

$$P_a^{II} = P(c_1 < d_1 + d_2 \le c_2)$$

$$= P(d_1 = 2, d_2 \le 2) + P(d_1 = 3, d_2 \le 1) + P(d_1 = 4, d_2 = 0)$$

$$= P(d_1 = 2) \times P(d_2 \le 2) + P(d_1 = 3) P(d_2 \le 1) + P(d_1 = 4) P(d_2 = 0)$$

$$= (0.2611 \times 0.1183) + (0.2199 \times 0.0370) + (0.1360 \times 0.0059)$$

$$= 0.0308 + 0.0082 + 0.0008 = 0.0398$$

$\therefore P_a = 0.3192$

$$ASN = n_1 P_1 + (n_1 + n_2)(1 - P_1) \; ; \quad P_1 = P(d_1 \le c_1) + P(d_1 > c_2)$$

$$= 1 \times 0.383 + 5(1 - 0.383) \qquad = P(d_1 \le 1) + P(d_1 > 4)$$

$$= 3.468 \qquad = 0.2794 + 0.1036$$

$$= 0.383$$

$$AOQ = \frac{[P_a^{I}(N - n_1) + P_a^{II}(N - n_1 - n_2)]p}{N} = 0.0157$$

$$ATI = n_1 P_a^{I} + (n_1 + n_2) P_a^{II} + N(1 - P_a)$$

$$= 3424$$

Average Sample Number (ASN):-

Number inspected = n_1, if lot is accepted on rejected in first sample
Number inspected = n_1+n_2, if a second sample is needed.

$$ASN = n_1 P_1 + (n_1+n_2)(1-P_1) \text{ ; where,}$$

$$P_1 = P\{\text{lot is accepted in 1st sample}\} + P\{\text{lot is rejected in 1st sample}\}$$
$$= P(d_1 \le c_1) + P(d_1 > c_2)$$

ASN curve:- Compute ASN for various values of fraction defective p and plot ASN versus p.

Double Sampling plan – Rectifying Inspection:-

The rejected lots are screened 100%, all defectives are replaced with good ones and accept the lot.

Fraction defective p in the accepted lot after inspection.
Suppose a lot of size N with fraction defective p is submitted for inspection.

If the lot is rejected and subjected to rectifying inspection, then the fraction defective after inspection is 0.

If the lot is accepted in the first sample, then the uninspected $(N-n_1)$ may contain $(N-n_1)p$ defectives.

If the lot is accepted in the second sample, then the uninspected $(N-n_1-n_2)$ may contain $(N-n_1-n_2)p$ defectives.

The rejected lots are screened 100%, all defectives are replaced with good ones and accept the lot.

Average Outgoing Quality (AOQ):-

$$AOQ = \frac{[P_a^{I}(N-n_1) + P_a^{II}(N-n_1-n_2)]p}{N}$$

AOQ curve:- Plot of AOQ vs. Various fraction defective p.

Average Total Inspection (ATI):-

1. If lot is rejected the entire lot N is inspected.
2. If lot is accepted in first sample, then n_1 items are inspected.
3. If lot is accepted in second sample, then n_1+n_2 items are inspected.

$$ATI = n_1 P_a^{I} + (n_1+n_2) P_a^{II} + N(1-P_a)$$

Multiple Sampling Plan:-

A multiple-sampling plan is an extension of double-sampling in that more than two samples can be required to sentence a lot. Example:-

Cumulative-Sampling Size	Acceptance No.	Rejection No.
20	0	3
40	1	4
60	3	5
80	5	7
100	8	9

Advantage:- The principal advantage of multiple-sampling plans is that the samples required at each stage are usually smaller than those in single or double sampling. So it is economical.

Procedure:- If, at the completion of any stage of sampling, the number of defective items is less than or equal to the acceptance number, the lot is accepted.

If, during any stage, the number of defectives equals or exceeds the rejection number, the lot is rejected; otherwise the next sample is taken.

The multiple-sampling procedure continues until the fifth sample is taken, at which time a lot disposition decision can be made.

MILITARY STANDARD 105E Sampling Schemes

Developed during World War II
Same as ISO 2859 of International Organisation for Standardization (ISO).
Provides single, double and multiple sampling plans.

Type of Inspection:- 1. Normal Inspection
2. Tightened Inspection
3. Reduced Inspection

MIL STD 105E Sampling Schemes:-

Normal Inspection : Used at the start of the inspection activity
Tightened Inspection : When supplier's recent quality history has deteriorated.
Acceptance requirements under tightened inspection are more stringent.
Reduced Inspection : When supplier's recent quality history has been exceptionally good.
Sample size under reduced inspection is generally less than that under normal inspection.

Focal point is AQL.
Sample size is determined by lot size & inspection level.

Inspection Levels:- Level-II is designated as normal.
Level-I requires about one half the amount of inspection as that of level II and used when less discrimination is needed.
Level-III requires about twice as much inspection as that of level II and used when more discrimination is needed.

Special Inspection Levels:- S-1, S-2, S-3 and S-4
Used when small sample sizes are necessary and when large risks can be tolerated.

Switching Rules:-

Normal to tightened:- When two out of five lots have been rejected on original submission.

Tightened to Normal:- When five consecutive lots have been accepted on original submission.

Normal to reduced :- When all four of the following conditions are satisfied:
(a) Preceeding 10 lots have been in normal inspection & none of them has been rejected.
(b) Total number of defectives from the preceeding 10 lots is less than or equal to the applicable limit number specified in the standard.

(c) Production is at steady state, no issues like machine breakdown, material shortage, etc.
(d) Desirable by the authority.

Reduced to Normal:- When any of the following four conditions are satisfied:
(a) A lot is rejected
(b) When procedure terminated with neither acceptance or rejection criteria has been met.
(c) Production is irregular or delayed
(d) Other conditions warrant normal inspection.

Discontinue Inspection:- When 10 consecutive lots remain on tightened inspection. Action should be taken at supplier level to improve the quality.

MIL STD 105E – Procedure:-

1. Choose the AQL
2. Choose the inspection level (normally level II)
3. Determine lot size
4. Find appropriate sample code from sample size code letter table.
5. Determine appropriate type of sampling plan to use (single, double, multiple)
6. Enter the appropriate table to find the type of plan to be used.
7. Determine the corresponding tightened and reduced plans to be used when required.

Ex.1. A supplier ships a component in lots of size $N = 3000$. The AQL has been established for this product at 1%. Find the normal, tightened and reduced single sampling plans for this situation from MIL STD 105E, assuming that general inspection level II is appropriate?

Solution:- $N = 3000$, AQL = 1%, Level : II

Sample code level = K.

	c	n
Normal	3	125
Reduced	2	125
Tightened	1	50

Ex.2. A product is supplied in lots of size N = 10,000. The AQL has been specified at 0.10%. find the normal, tightened and reduced single sampling plans from MIL STD 105E, assuming general inspection level II?

Solution:- N = 10,000

AQL = 0.1%

Sample code level = L

	n	c
Normal	200	0
Reduced	80	0
Tightened	200	0

DODGE-ROMIG Sampling Plans

Dereloped by H.F. Dodge and H.G. Romig.
Plans are available for single & double sampling.

Types of Sampling Plans:-

1. Plans for Lot Tolerance Percent Defective (LTPD) plans
2. Plans for Average Outgoing Quality Level (AOQL) plans

- LTPD Plans :- Plans are available for LTPD values of 0.5%, 1%, 2%, 3%, 4%, 5%, 7% and 10%.
Knowledge of process average - average fraction defective (non-conforming) of the incoming product is necessary.

Procedure:-

1. Choose required LTPD
2. Determine lot size
3. Determine process average (fraction non-conforming)
4. Based on the lot size and process average, choose the sample size n and acceptance number c from the corresponding LTPD table.

Ex. A product is shipped in lots of size $N = 2000$. Find a Dodge-Romig single sampling plan for which the LTPD = 1%, assuming that the process average is 0.25% defective?

Solution:- $N = 2000$.

LTPD = 5%.

Process average = 0.25%

$n = 75$, $c = 1$, AOQL = 1.0

- AOQL Plans:- Plans are available for AOQL values of 0.1%, 0.25%, 0.5%, 0.75%, 1%, 1.5%, 2%, 2.5%, 3%, 4%, 5%, 7% and 10%.

Knowledge of process average - average fraction defective of the incoming product is necessary.

Only applicable when rectifying inspection is used.

Procedures:-

1. Choose required AOQL
2. Determine lot size
3. Determine process average (fraction non-conforming)
4. Based on the lot size and process average, choose the sample size n and acceptable number c from the corresponding AOQL table.

Ex. A company wish to find a single sampling plan for a situation where lots of size $N = 8000$ are shipped from a supplier. The supplier's process operates at a fallout level of 0.50% defective. The company want the AOQL from inspection activity to be 3%. Suggest the appropriate Dodge-Romig plan?

Solution:- $N = 8000$. P = process average = 0.50%.

AOQL = 2.0%

$n = 55$, $c = 2$, from Dodge-Romig plan.

//

Pre-Control Chart :- (Setup Approval Chart)

- A technique used to detect shifts or upsets in the process which will result in producing non-conforming products or parts.
- Conventional control charts are used to detect shifts in process due to assignable causes or to ensure stability of the process.
- Based on Normal Distribution.
- Useful when $C_p \geq 1$, and $C_p = C_{pk}$.
- It is easy to reset the process or adjust the process mean.

The pre-control chart has two additional limits called Upper and Lower pre-control limits (UPCL and LPCL).

Let X be normally distributed quality characteristic with process mean μ and process standard deviation σ. Then

$$UCL = \mu + 3\sigma$$
$$UPCL = \mu + 1.5\sigma$$
$$CL = \mu$$
$$LPCL = \mu - 1.5\sigma$$
$$LCL = \mu - 3\sigma$$

Approximately, 86% of process output will lie inside $\mu \pm 1.5\sigma$ limits and 7% will lie in each region between PC and Control limits.

Special Case :- $C_p = C_{pk} = 1$.

$$UCL = USL$$
$$UPCL = \frac{USL + \mu}{2}$$
$$CL = \mu$$
$$LPCL = \frac{\mu + LSL}{2}$$
$$LCL = LSL$$

- Working Rules :-

1. Start the process and check 1st product or item. If the 1st item is outside the control limits, reset the process.
2. If an item is inside the control limits but outside the PC line, then check the next item.
3. If the 2nd item is also outside the same PC line, reset the process.
4. If 2nd item is inside the PC line then continue.
5. If one item is outside a PC line & the next item is outside the other PC line, then the process variability is out of control. Investigate and take necessary corrective actions.

6. When five consecutive points are inside the PC line, shift to sampling.
7. During sampling donot adjust the process unless an item fall outside PC lines.
8. If the point is outside control charts, reset the process and proceed as in step 6.
9. If the points is within control limit but outside the PC line, then check the next item as in step 4.
10. When a process is reset, five consecutive items must fall within PC lines before changing to sampling.

Control Charts for multi stream process (MSP):-

Data at any point of time consists of measurements from several sources or streams, sources of streams are assumed to be identical.

It is possible to monitor and adjust each of the streams individually or in small groups.

Use group control chart:- Plot only the largest and smallest $\bar{x}$ on $\bar{x}$ chart and only largest range is plotted on R chart.

Out of control cases:- 1. Output of one stream (or a few streams) has shifted off target.

2. Output of all streams has shifted off target.

Group Control Charts:- Suppose that the process has s streams and each stream has same target value and inherent variability. Distribution of measurement is well approximated by the normal. Sampling is prefered. Suppose sample size is n.

This process is repeated until m subgroups of samples have been taken.

So, $\bar{\bar{x}} = \frac{\sum\sum x_{ij}}{m \times s}$, $\bar{R} = \frac{\sum\sum R_{ij}}{m \times s}$.

$UCL = \bar{\bar{x}} + A_2\bar{R}$, $LCL = \bar{\bar{x}} - A_2\bar{R}$ for the $\bar{x}$ chart

$UCL = D_4\bar{R}$, $LCL = D_3\bar{R}$ for the R chart.

It is useful to examine the stream numbers on the chart.

ADVANCED CONTROL CHARTS

- **Cumulative Sum Control chart (CUSUM chart) :—**
  - Used cohen small shifts are important.
  - Uses all informations in the sequence of sample values
  - Highly effective for subgroup of size of $n = 1$.
  - Ensures the quality characteristics will be always on or close to the target.
  - Mostly used in chemical and process industries cohere subgroup size is often 1.
  - Highly suitable for modern industries with automated inspection or on line control.
  - Plots the cumulative sum of the deviation of sample values from the target value.

Working Rules:– 1. If the process shifts or drifts off the target value, then cusum will signal.
2. An adjustment is made to some control factors to bring the process back on target.

CUSUM Control Chart: Logic :—

1. Let the quality characteristic X has a normal distribution with mean μ and standard deviation σ.
2. Let μ_0 be the target value of X.
3. Accumulate the deviation from μ_0 that are above target with one statistic C^+.
4. Accumulate the deviation from μ_0 that are below target with one statistic C^-.

$$C_i^+ = \max\left[0,\ x_i - (\mu_0 + K) + C_{i-1}^+\right]$$

$$C_i^- = \max\left[0,\ (\mu_0 - K) - x_i + C_{i-1}^-\right]$$

where $C_0^+, C_0^- = 0$.

5. Reference value or allowance value K is chosen halfway between target value μ_0 and maximum allowed shift value μ_1, $K = \dfrac{|\mu_0 - \mu_1|}{2}$
6. Plot C^+ an C^- on the chart.
7. If either C^+ or C^- is beyond the decision interval H, reset the process $H = 5\sigma$.

Example:- The data on molecular weight taken hourly from a chemical process is given below:

Sample	x	sample	x
1	1045	11	1139
2	1055	12	1169
3	1037	13	1151
4	1064	14	1128
5	1095	15	1238
6	1008	16	1125
7	1050	17	1163
8	1087	18	1188
9	1125	19	1146
10	1146	20	1167

The target value of molecular weight is 1050. Design a cusum to quickly detect a shift of about 1.5σ.

Solution:-

Sample	x	Moving Range
1	1045	—
2	1055	10
3	1037	18
4	1064	27
5	1095	31
6	1008	87
7	1050	42
8	1087	37
9	1125	38
10	1146	21
11	1139	7
12	1169	30
13	1151	18
14	1128	23
15	1238	10
16	1125	13
17	1163	38
18	1188	25
19	1146	42
20	1167	21

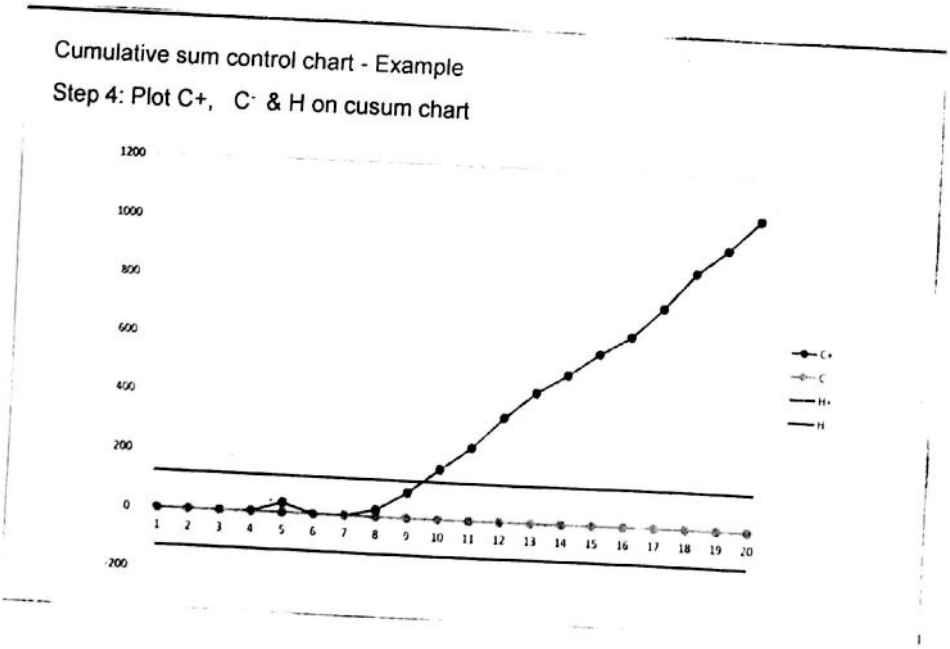
MR Chart	
$\overline{MR}$	28.316
UCL	92.508
CL	28.316
LCL	0.000
σ	25.10

Compute μ_1, K & H :-

σ	25.10
μ_0	1050
μ_1	1075.10
K	12.55
H	125.5

Sample	x	$x_i - (\mu_0 + K) + C_{i-1}^+$	C^+
1	1045	-17.5500	0.00
2	1055	-7.5500	0.00
3	1037	-25.5500	0.00
4	1064	1.4500	1.45
5	1095	33.9000	33.90
6	1008	-20.6500	0.00
7	1050	-12.5500	0.00
8	1087	24.4500	24.45
9	1125	86.9000	86.90
10	1146	170.3500	170.35
11	1139	246.8000	246.80
12	1169	353.2500	353.25
13	1151	441.7000	441.70
14	1128	507.1500	507.15
15	1138	582.6000	582.60
16	1125	645.0500	645.05
17	1163	745.5000	745.50
18	1188	870.9500	870.95
19	1146	954.4000	954.40
20	1167	1058.8500	1058.85

Sample	x	$(\mu_0 - K) - x + C_{i-1}^-$	C^-
1	1045	-7.55	0.00
2	1055	-17.55	0.00
3	1037	0.45	0.45
4	1064	-26.10	0.00
5	1095	-57.55	0.00
6	1008	29.45	29.45
7	1050	16.90	16.90
8	1087	-32.65	0.00
9	1125	-87.55	0.00
10	1146	-108.55	0.00
11	1139	-101.55	0.00
12	1169	-131.55	0.00
13	1151	-113.55	0.00
14	1128	-90.55	0.00
15	1138	-100.55	0.00
16	1125	-87.55	0.00
17	1163	-125.55	0.00
18	1188	-150.55	0.00
19	1146	-108.55	0.00
20	1167	-129.55	0.00



Note:- Except CUSUM chart all other charts have memory less property.

- **Exponentially Weighted Moving Average Control chart :~**
  - Very effective against small process shifts.
  - Uses all information in the sequence of ~~small~~ sample values.
  - Highly effective for subgroup size of $n = 1$.
  - Ensures the quality characteristic will always be on or close to the target.
  - Mostly used in chemical and process industries where sub-group size is often 1.
  - Highly suitable for modern industries with automated inspection or on line control.
  - EWMA is used extensively in time series modelling & forecasting.

EWMA Control Chart: Logic: ~

Let x_i are independent random variables with variance σ^2, then exponentially weighted moving average z_i is

$$z_i = \lambda x_i + (1-\lambda) z_{i-1}.$$

where, $0 \le \lambda \le 1$, $z_0 = \mu_0$, the process target variance of z_i,

$$\sigma_{z_i}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right]$$

In EWMA charts, z_i is plotted against sample number i.

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

$$CL = \mu_0$$

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}$$

Generally. $\lambda = 0.2$ and $L = 2.962 \simeq 3$.

- **Slopping Control Charts:-** Many process are subject to tool wear. As tool wears out, there will be a drift or trend in the process mean.
As $\bar{X}$ chart will show the process out of control and assignable cause is tool wear.
Tracking this assignable cause or replacing the tool very often is expensive.
If the process highly capable ($C_p, C_{pk} > 1$) then slopping control chart can be used to detect other assignable causes and the tool can be used till its useful life or till it produce non-conforming units/products.

- **Assumptions:-** — The process is highly capable.
  — The tool wears out more or less at uniform rate.
  — Set the process such that the mean is close to LSL.
  — Collect sample data on quality characteristic at regular

- **Steps:-** — Compute $\bar{X}$ and R
  — Construct R chart and ensure the process variation is in control.
  — Estimate the process standard deviation σ.
  — Take the time (h) corresponding to middle sample as 0 so that there will be equal number of samples on either side of zero.
  — Plot $\bar{X}$ versus h and Fit a line.
  — Construct slopping control chart with fitted line as CL and fitted value $\pm A_2\bar{R}$ as control limits.
  — Set the process initially with $\bar{X} = LSL + L\sigma$
  Reset the process when $\bar{X}$ reaches $USL - L\sigma$.

- **Control Charts for short production runs:-**

Use deviations from nominal or target value instead of measured variable on the control chart.
Also known as DNOM (Deviations from Nominal) chart.

- **Steps:-** 1. Let m_i, $i = 1, 2, \ldots, k$ be the values of quality characteristics with target values t_i.
Compute $x_i = m_i - t_i$, $i = 1, 2, \ldots, k$.

2. Compute $\bar{X}$ and Range for x_i.

3. Construct $\bar{X}$ and R chart.

Example of EWMA Control Chart:-

Exponentially Weighted Moving Average Control Chart

Example: Bath concentration are measured hourly in a chemical process. Data (in ppm) for the last 32 hours shown below. The process target is $\mu_0 = 175$ ppm

Sample No.	Data	Sample No.	Data	Sample No.	Data	Sample No.	Data
1	160	9	180	17	190	25	206
2	158	10	195	18	189	26	210
3	150	11	179	19	185	27	216
4	151	12	184	20	182	28	212
5	153	13	175	21	181	29	211
6	154	14	192	22	180	30	202
7	158	15	186	23	183	31	205
8	162	16	197	24	186	32	197

a. Estimate the process standard deviation?
b. Set up EWMA control chart

Set up MR chart & Estimate Process sigma:-

Sample No	Data	MR	Sample No	Data	MR
1	160		17	190	7
2	158	2	18	189	1
3	150	8	19	185	4
4	151	1	20	182	3
5	153	2	21	181	1
6	154	1	22	180	1
7	158	4	23	183	3
8	162	4	24	186	3
9	180	18	25	206	20
10	195	9	26	210	4
11	179	16	27	216	6
12	184	5	28	212	4
13	175	9	29	211	1
14	192	17	30	202	9
15	186	6	31	205	3
16	197	11	32	197	8

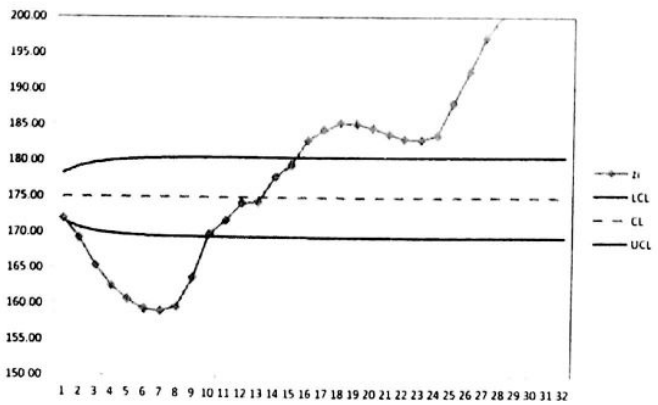
$UCL = 20.761$
$CL = 6.355$
$LCL = 0.000$
$\sigma = 5.634$

Compute z_i's and control limits

Sample No	Data	zi	SD	LCL	UCL	Sample No	Data	zi	SD	LCL	UCL
1	160	172.00	0.20	171.66	178.34	17	190	184.41	0.33	169.44	180.56
2	158	169.20	0.26	170.73	179.27	18	189	185.33	0.33	169.44	180.56
3	150	165.36	0.29	170.22	179.78	19	185	185.26	0.33	169.44	180.56
4	151	162.49	0.30	169.93	180.07	20	182	184.61	0.33	169.44	180.56
5	153	160.59	0.31	169.74	180.26	21	181	183.89	0.33	169.44	180.56
6	154	159.27	0.32	169.63	180.37	22	180	183.11	0.33	169.44	180.56
7	158	159.02	0.33	169.56	180.44	23	183	183.09	0.33	169.44	180.56
8	162	159.61	0.33	169.52	180.48	24	186	183.67	0.33	169.44	180.56
9	180	163.69	0.33	169.49	180.51	25	206	188.14	0.33	169.44	180.56
10	195	169.95	0.33	169.47	180.53	26	210	192.51	0.33	169.44	180.56
11	179	171.76	0.33	169.46	180.54	27	216	197.21	0.33	169.44	180.56
12	184	174.21	0.33	169.45	180.55	28	212	200.17	0.33	169.44	180.56
13	175	174.37	0.33	169.45	180.55	29	211	202.33	0.33	169.44	180.56
14	192	177.89	0.33	169.44	180.56	30	202	202.27	0.33	169.44	180.56
15	186	179.52	0.33	169.44	180.56	31	205	202.81	0.33	169.44	180.56
16	197	183.01	0.33	169.44	180.56	32	197	201.65	0.33	169.44	180.56

Choose λ and L, $\lambda = 0.2$, $\mu_0 = 175$ and $L = 2.962$

Step 4: Construct EWMA control chart



Example of Slopping Control Charts :-

The specifications on a critical dimension of a process subject to tool wear is 0.644 ± 0.004. Thirteen samples of subgroup size 5 are collected at every half an hour interval and the xbar and range computed. The data is given below.

1. Construct a slopping control chart to monitor the process
2. Estimate the duration and number of samples after which the process need to be reset?
3. How much should be the reset value?

Sample	xbar	R	Time
1	0.6417	0.0011	-6
2	0.6418	0.0016	-5
3	0.6424	0.001	-4
4	0.6431	0.0015	-3
5	0.6433	0.0009	-2
6	0.6437	0.001	-1
7	0.6433	0.0014	0
8	0.6436	0.0004	1
9	0.6441	0.0006	2
10	0.6444	0.0011	3
11	0.6456	0.0009	4
12	0.6457	0.0007	5
13	0.6457	0.0009	6

$\overline{\overline{X}} = 0.6437$

$\overline{R} = 0.0010$

R Chart :-

UCL = 0.0021

CL = 0.0010

LCL = 0.00

Capability Analysis:-

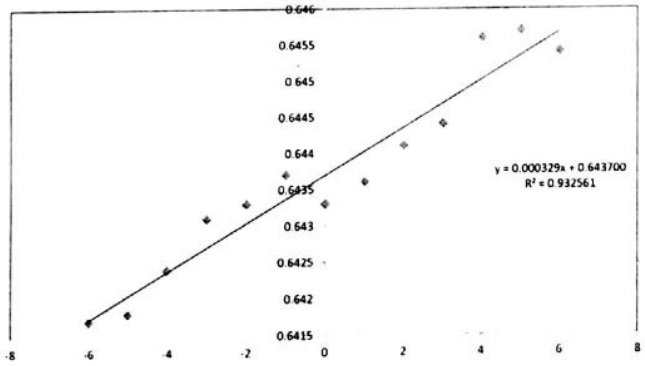
Mean = 0.6437

SD = 0.0004

USL = 0.6480

LSL = 0.6400

$C_p = 3.078 > 1.$



xbar = 0.6437+ 0.000329h

Initial set up point :- $\overline{X}_{initial} = LSL + L\sigma = LSL + 4\sigma = 0.6417$

Reset point :- $\overline{X}_{final} = USL - 4\sigma = 0.6463$

Interval between resets :- $(\overline{X}_{final} - \overline{X}_{initial}) / \text{slope}$

$= \dfrac{0.6463 - 0.6417}{0.000329} = 14$

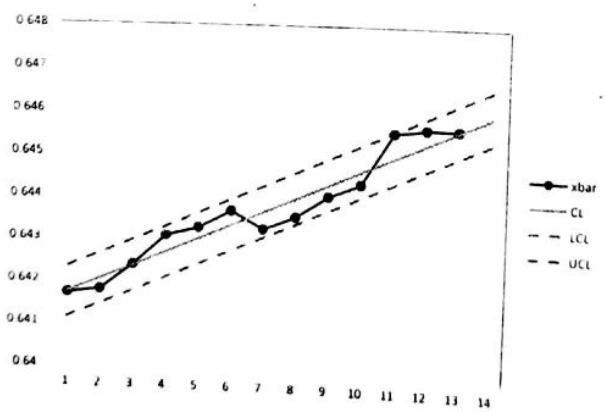
Conclusion:- Reset the process after 14 subgroups to initial set up point of $\overline{X} = 0.6417$. Since sampling frequency is half an hour reset the process at every 7 hrs.

Time h corresponds to 'initial set up point', $h = \dfrac{\overline{X}_{initial} - a}{b} = \dfrac{0.6417 - 0.6437}{0.000329} = -6$

Time h corresponds to Reset point

$$h = \frac{\bar{x}_{final} - a}{b} = \frac{0.6463 - 0.6437}{0.000329} = 7 .$$

Time	Sample	xbar	CL (Model)	LCL	UCL
-6	1	0.6417	0.6417	0.6411	0.6423
-5	2	0.6418	0.6421	0.6415	0.6427
-4	3	0.6424	0.6424	0.6418	0.6430
-3	4	0.6431	0.6427	0.6421	0.6433
-2	5	0.6433	0.6430	0.6424	0.6436
-1	6	0.6437	0.6434	0.6428	0.6440
0	7	0.6433	0.6437	0.6431	0.6443
1	8	0.6436	0.6440	0.6434	0.6446
2	9	0.6441	0.6444	0.6438	0.6450
3	10	0.6444	0.6447	0.6441	0.6453
4	11	0.6456	0.6450	0.6444	0.6456
5	12	0.6457	0.6453	0.6447	0.6459
6	13	0.6457	0.6457	0.6451	0.6463
7	14		0.6460	0.6454	0.6466



Example of Short Production Runs:-

Example

Use the following data to set up short run xbar and R charts using DNOM approach. The target dimensions for each part are $T_A = 100$, $T_B = 60$, $T_C = 75$ and $T_D = 50$

Sample	Part Type	m1	m2	m3	Sample	Part Type	m1	m2	m3
1	A	105	102	103	11	C	77	75	74
2	A	101	98	100	12	C	75	72	79
3	A	103	100	99	13	C	74	75	77
4	A	101	104	97	14	C	73	76	75
5	A	106	102	100	15	D	50	51	49
6	B	57	60	59	16	D	46	50	50
7	B	61	64	63	17	D	51	46	50
8	B	60	58	62	18	D	49	50	53
9	C	73	75	77	19	D	50	52	51
10	C	78	75	76	20	D	53	51	50

Compute $x_i = m_i - t_i$,

Sample	Part Type	m1	m2	m3	Sample	Part Type	m1	m2	m3
1	A	5	2	3	11	C	2	0	-1
2	A	1	-2	0	12	C	0	-3	4
3	A	3	0	-1	13	C	-1	0	2
4	A	1	4	-3	14	C	-2	1	0
5	A	6	2	0	15	D	0	1	-1
6	B	-3	0	-1	16	D	-4	0	0
7	B	1	4	3	17	D	1	-4	0
8	B	0	-2	2	18	D	-1	0	3
9	C	-2	0	2	19	D	0	2	1
10	C	3	0	1	20	D	3	1	0

Control charts for short production runs

Compute xbar and R

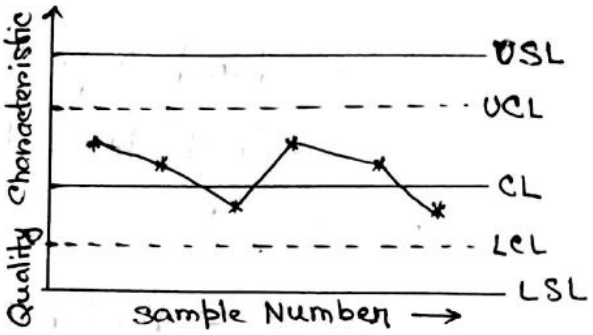
Sample	m1	m2	m3	xbar	Range	Sample	m1	m2	m3	xbar	Range
1	5	2	3	3.333	3	11	2	0	-1	0.333	3
2	1	-2	0	-0.333	3	12	0	-3	4	0.333	7
3	3	0	-1	0.667	4	13	-1	0	2	0.333	3
4	1	4	-3	0.667	7	14	-2	1	0	-0.333	3
5	6	2	0	2.667	6	15	0	1	-1	0.000	2
6	-3	0	-1	-1.333	3	16	-4	0	0	-1.333	4
7	1	4	3	2.667	3	17	1	-4	0	-1.000	5
8	0	-2	2	0.000	4	18	-1	0	3	0.667	4
9	-2	0	2	0.000	4	19	0	2	1	1.000	2
10	3	0	1	1.333	3	20	3	1	0	1.333	3

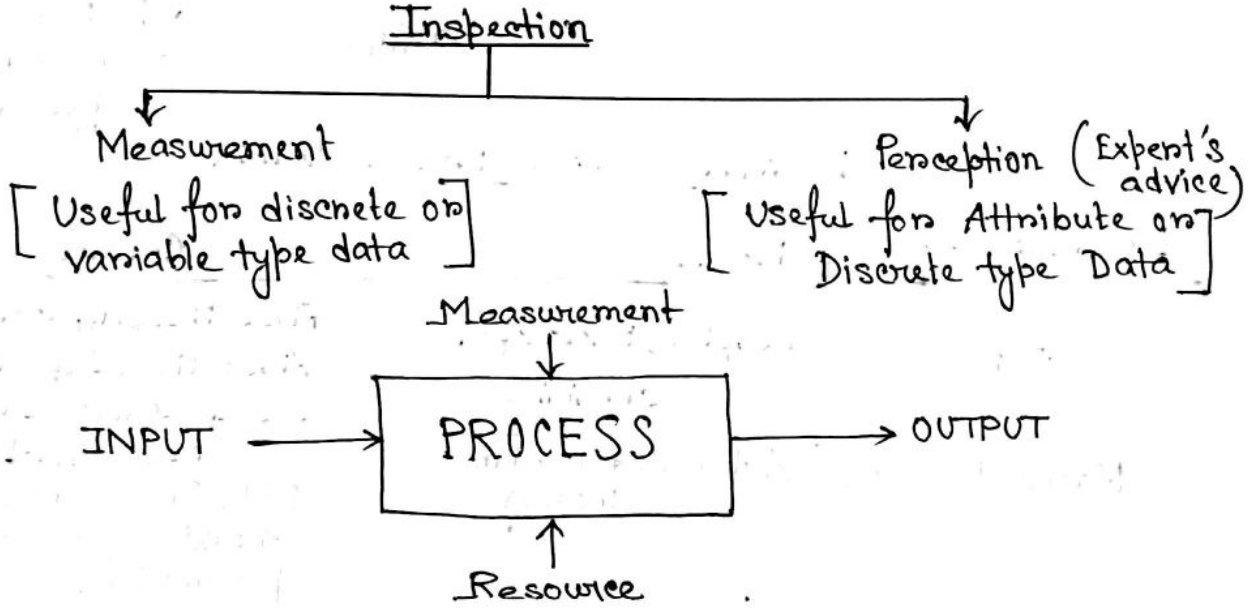
Statistical Process Control 2

— SPC used to maintain a process at a particular level of performance cohere the process will at least meet specifications.

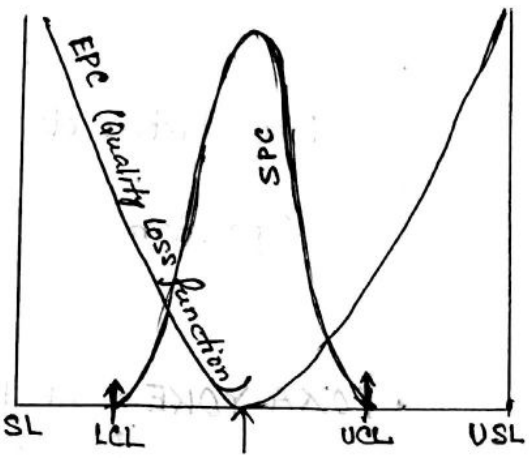
— Control Chart is a technique for defect prevention.

— Defect is a particular product characteristics not meeting specifications. Defects denote the points below LSL and above USL.





- A process is a seaquence of activities converting input into output.
- SPC is implemented for stable and capable process cohere sometimes assignable causes may come and we may control it
- EPC: Engineering Process Control ; here no action is taken as long as parts are within specifications.
- Quality means hitting the target with minimum variability around it.
- Capability means how well the process is meeting the tolerance levels.
- Process Capability is the total variation due to chance causes.
- Assignable Cause has prob. of occurance is very low but it occurs.



Schemes of Process Control:-

- Set up Approval / First piece Inspection
- In process Inspection <
  - 100%
  - Sampling
  - SPC
- Final Inspection
- Last piece Inspection

Levels of Process Control ?

Quality = Variability ⟶ Causes
- Prevent the cause from occuring. (POKA YOKE)
- Detect the cause as and when it appears.

Control

Dominance System – Process Control

Dominated by	Process	Control
Set up	Stamping, Photocopy, Machining	First Piece Approval (FPA) First Piece Inspection (FPI) Set up Control Chart (Pre-control Chart)
Machine Parameter	Automated machine Process	FPI, Freq. check, checking of process parameter.
Raw material	Any product produced by using natural raw material	Incoming inspection of critical parameter
Machine	Machining Process — Turning, Press Parts	FPI, Control Chart
Process Qualification	Welding, Painting, Riveting, Plating, Heat treatment	Monitor the process parameter and control them (Operation Qualification)
Tools, Fixture, etc	Press Operation	FPA, LPI, Tool Maintainance, SPC
Operator	Manual (Assembly operations)	Operator training & Qualification, POKA YOKE (Mistake Proofing)

- **POKA YOKE:** At product design; At process design; During inspection in the same process; Design next process usage

- **Implementing Control Chart :-**

1. Calculate Process Capability
2. Process Monitoring (Control)

Calculation of Process Capability :-

— Select the product characteristic

— collect data

→ collect continuous chronological data, then divide it into subgroups

→ collect data in subgroup format with adequate time interval between them.

— Check for normality by using Normality Probability Paper.

— Carry out Control limit calculation

— Check stability of the process.

— If found stable, calculate Process Capability (C_p, C_{pk})

$$C_p = \frac{USL - LSL}{6\hat{\sigma}}, \quad \hat{\sigma} = \frac{\bar{R}}{d_2} \text{ or } \frac{\bar{MR}}{d_2}$$

$$C_{pk} = \min\left\{\frac{\bar{X} - LSL}{3\hat{\sigma}}, \frac{USL - \bar{X}}{3\hat{\sigma}}\right\}$$

- **Ex.** Show that $C_{pk} \leq C_p$ and illustrate equality case.

Solution:-

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{USL - \bar{X} + \bar{X} - LSL}{6\hat{\sigma}}$$

$$= \frac{1}{2}\left[\frac{USL - \bar{X}}{3\hat{\sigma}} + \frac{\bar{X} - LSL}{3\hat{\sigma}}\right]$$

$$\geq \min\left\{\frac{USL - \bar{X}}{3\hat{\sigma}}, \frac{\bar{X} - LSL}{3\hat{\sigma}}\right\} = C_{pk}$$

When target is at centre, $\mu = \frac{USL + LSL}{2}$.

So, $C_{pk} = \min\left\{\frac{USL - \mu}{3\hat{\sigma}}, \frac{\mu - LSL}{3\hat{\sigma}}\right\}$

$$= \min\left\{\frac{USL - LSL}{6\hat{\sigma}}, \frac{USL - LSL}{6\hat{\sigma}}\right\}, \text{ putting value of } \mu.$$

$$= \frac{USL - LSL}{6\hat{\sigma}} = C_p.$$

Ex. Derive a strategy to achieve a C_{pk} value of 1.33.

Sol. To achieve $C_{pk} = 1.33$, one should have $C_p = 1.33$.

$$\frac{\text{Tolerance}}{6\delta} = 1.33$$

$$\frac{6\delta}{T} = 0.75$$

$T = 0.1$ mm, 75% of $T = 0.075$ mm

$6\delta = 0.075$ mm (max)

$\therefore \delta_{max} = 0.0125$

then $\bar{x} = 5$

$$\frac{\bar{x} - LSL}{3\delta} = 1.33 \Rightarrow \bar{x} - LSL = 4\delta$$

$$\frac{USL - \bar{x}}{3\delta} = 1.33 \Rightarrow USL - \bar{x} = 4\delta$$

So, $\bar{x} - 4\delta > LSL$ and $\bar{x} + 4\delta < USL \ \forall \ (\bar{x}, \delta)$

Ex. Prove that $C_{pk} = (1-K)C_p$, where $K = \dfrac{\left|\frac{USL+LSL}{2} - \mu\right|}{\frac{USL-LSL}{2}}$;

Sol. For $T > \mu$; $T = \frac{USL+LSL}{2}$;

$|T - \mu| = T - \mu$.

$$K = \frac{\frac{USL+LSL}{2} - \mu}{\frac{USL-LSL}{2}}$$

$$\Rightarrow K = \frac{\left(\frac{USL+LSL-2\mu}{6\sigma}\right)}{\frac{USL-LSL}{6\sigma}}$$

$$\therefore K.C_p = \frac{(USL-\mu) - (\mu - LSL)}{6\sigma} = \frac{|(USL-\mu) - (\mu - LSL)|}{6\sigma};$$

since $K C_p > 0$

Note:- Control chart always gives short term capability.

Q. Why rational subgrouping used in control chart?

$$\Rightarrow kC_p = \frac{|(USL-\mu)-(\mu-LSL)|}{6\sigma} + \left[\frac{(USL-\mu)}{6\sigma} + \frac{(\mu-LSL)}{6\sigma}\right] - \left[\frac{(USL-\mu)}{6\sigma} + \frac{(\mu-LSL)}{6\sigma}\right]$$

$$\Rightarrow \left\{\left(\frac{USL-\mu}{6\sigma}\right) + \left(\frac{\mu-LSL}{6\sigma}\right) - \left|\frac{(USL-\mu)}{6\sigma} - \frac{(\mu-LSL)}{6\sigma}\right|\right\}$$

$$= \left(\frac{USL-\mu}{6\sigma} + \frac{\mu-LSL}{6\sigma}\right) - kC_p$$

$$\Rightarrow \frac{1}{2}\left\{\left(\frac{USL-\mu}{3\sigma}\right) + \left(\frac{\mu-LSL}{3\sigma}\right) - \left|\frac{(USL-\mu)}{3\sigma} - \frac{(\mu-LSL)}{3\sigma}\right|\right\}$$

$$= \frac{USL-LSL}{6\sigma} - kC_p$$

$$\Rightarrow \min\left\{\left(\frac{USL-\mu}{3\sigma}\right), \left(\frac{\mu-LSL}{3\sigma}\right)\right\} = C_p - kC_p$$

$$\Rightarrow C_{pk} = (1-k)C_p.$$

Same can be shown for $T < \mu$. Hence the proof.

- Use Control Chart for monitoring :-

1. Select the process.
2. Select the product/product characteristic.
3. Select the most appropriate control charts to implement.
4. Carry out brainstorming to identify the possible/likely assignable causes and their counter measures (OCAP).
5. Collect data.
6. Carry out initial study and capability study.
7. If process found both stable and capable, use the control limit for process control in future.

Questions:- 1. Difference between Process Capability and Machine capability and how to calculate.

2. Difference between long term and short term capability.

3. Find out different formulae of process capability index and present them with example when
(i) Target at centre
(ii) Target not at centre.

Taguchi Capability Index

The process capability ratio C_{pk} was initially developed because C_p does not adequately deal with the case of a process with mean μ that is not centered between the specification limits. However, C_{pk} alone is still an inadequate measure of centering. For any fixed value of μ in the interval from LSL to USL, C_{pk} depends inversely on σ and becomes large as σ approaches zero. This characteristic can make C_{pk} unsuitable measure of centering.

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{1}{2} \cdot \frac{USL - LSL}{3\sigma} = \frac{d}{3\sigma}$$

$$C_{pk} = \frac{\min\{USL - \mu, \mu - LSL\}}{3\sigma} = \frac{d - |\mu - T|}{3\sigma}$$

where, $d = \frac{USL - LSL}{2}$, $T = \frac{USL + LSL}{2}$

$$\tau^2 = E(X-T)^2 = E(X-\mu)^2 + (\mu - T)^2 = \sigma^2 + (\mu - T)^2$$

Define $C_{pm} = \frac{USL - LSL}{6\tau} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$

$$= \frac{C_p}{\sqrt{1 + \xi^2}}, \quad \xi = \frac{\mu - T}{\sigma}$$

$C_{pk} = 0$ when $\mu \geq USL$ and $\mu \leq LSL$

$C_{pm} \xrightarrow{a} 0$ as $|\mu - T| \to \infty$.

$$\therefore C_{pm} < \frac{USL - LSL}{6|\mu - T|}$$

$C_{pm} = 1$, necessary condition is $|\mu - T| < \frac{USL - LSL}{6}$

$C_{pm} = 1 \Rightarrow \mu$ lies in the middle third of specification range.

$C_{pm} = 4/3 \Rightarrow \mu$ lies within the middle fourth of the specification range.

These statements provide a concrete interpretation of C_{pm} as a measure of process centering.

Group Control Chart

Example:- A machine has four heads. Samples of $n = 3$ units are selected from each head, and the R and $\bar{x}$ values for an important quality characteristic are computed. Set up group control chart for this process.

[Page :- 488 10.4. Montgomery]

Sample No	Head 1 $\bar{x}$	R	2 $\bar{x}$	R	3 $\bar{x}$	R	4 $\bar{x}$	R
1	53	2	54	1	56	2	55	3
2	51	1	55	2	54	1	54	1
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								
17								
18								
19								
20								

* **Rational Subgrouping:-** It is a method of collecting data where variation within subgroups is minimum but variation between subgroups is maximum.

Solution:- Given $s = 4$,

R-Chart:- $\bar{R} = \frac{\sum\sum R_{ij}}{20 \times s}$; s = no. of stream = 4.
total sample number = 20
sample size = $n = 3$

$= \frac{187}{20 \times 4}$

$= 2.3375$

$UCL_{\bar{R}} = D_4 \bar{R} = 2.574 \times 2.3375$, for $n = 3$, $D_4 = 2.574$
$= 6.016$

$CL_{\bar{R}} = 2.3375 = \bar{R}$

$LCL_{\bar{R}} = 0 = D_3 \bar{R}$

The minimum and maximum of R for all the samples fall within the control limits, so process is in control.

$\bar{\bar{X}}$ chart:- $\bar{\bar{X}} = \frac{\sum\sum \bar{X}_{ij}}{20 \times s} = \frac{4239}{80} = 52.9875$

$UCL = \bar{\bar{X}} + A_2 \bar{R} = 55.3788$

$CL = \bar{\bar{X}} = 52.9875$

$LCL = \bar{\bar{X}} - A_2 \bar{R} = 50.2962$

Now, 16 points go out of the control limits, so we have to find out assignable variation in the process.

From 1st machine, only one point goes out.

From other 3 machines 5 points go out from each 3.

So, machine 1 is at better condition.

Rule:- Maximum no. of points than can be removed is 20% from the subgroups. If more than 20% points come, then the process is not stable, no need of control chart.

Group Control Chart:- There is a group of M/c doing similar process. The characteristic of the product is same. Steps are: Data collection
Limit calculation
Plot & Monitor
[Used for Multiple Stream Processes]

Homogenization:- It is a process by which we can remove the assignable cause from control limits.

For R chart:- $$\bar{R} = \frac{\Sigma\Sigma R_{ij}}{\text{sample No} \times \text{No. of heads}}$$

$$= \frac{\Sigma\Sigma R_{ij}}{20 \times 8}$$

$$LCL_{\bar{R}} = D_3 \bar{R}, \quad D_3 \text{ for } n = \text{sample size}$$

$$UCL_{\bar{R}} = D_4 \bar{R}.$$

Plot only the maximum & minimum of a subgroup.
R chart is to detect within subgroup variation.

For $\bar{\bar{X}}$ chart:- $$\bar{\bar{X}} = \frac{\Sigma\Sigma \bar{X}_{ij}}{20 \times 8}$$

$$LCL = \bar{\bar{X}} - A_2 \bar{R}$$
$$UCL = \bar{\bar{X}} + A_2 \bar{R}$$

Here also we plot the maximum and minimum value.
$\bar{\bar{X}}$ chart is to detect between subgroup variation.

Assumption:- Chance cause variation is smaller than assignable cause of variation.
Chance cause of variation is many in number but have little variation.
Assignable cause of variation is small in number but have large variation.

Assumption for Group Control:- There is no significant difference between process capabilities of the machines.
variation less → highly capable
variation high → less capable
If we group then assignable causes for highly capable process comes in control.

Note:- $(\bar{x}_1, s_1)$; $(\bar{x}_2, s_2)$: if there is no significant difference between two samples, we can use pooled variance.

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}.$$

Test statistics $t = \dfrac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim t_{n_1+n_2-2}$.

Group Charts:-

Case-I:- Target is different but tolerance is same.

M/c-1 (10 ± 0.5 mm)	M/c-2 (8 ± 0.5 mm)	M/c-3 (7.5 ± 0.5 mm)
$x_{11}\ x_{12} \dots x_{15}$, $\bar{x}_1 - 10$, R_{11}	$\bar{x}_2 - 8$, R_{12}	$\bar{x}_3 - 7.5$, R_{13}
$x_{20,1}\ x_{20,2} \dots x_{20,5}$ $\bar{x}_{201} - 10$, R_{201}	$\bar{x}_{20,2} - 8$, R_{202}	$\bar{x}_{20,3} - 7.5$, $R_{20,3}$

0 ± 0.5 is common tolerance, the target is transformed to 0.

So, changing the scale, in the place of $\bar{x}$ taking, $\bar{x}-10, \bar{x}-8, \bar{x}-7.5$.

Assumption:- All product target is same.

Objective:- Everything looks alike.

Case-II:- Target and tollerance both are different.

10 ± 0.5 mm $\quad$ 8 ± 1 mm $\quad$ 7.5 ± 0.5 mm

Transformation $= \dfrac{x_{ij} - \text{target}}{\text{tolerance}}$ so, 0 ± 0.5 mm

This is called target $\bar{x}$ chart.

Condition:- Value of c_p, c_{pk} is very much similar between machines.

Short term & long term variability:-

Control chart only gives short term variability.

In short-term variability $\hat{s}$ is given by

$$\hat{s} = \frac{\bar{R}}{d_2}.$$

In long-term variability $\hat{s}$ is given by

$$\hat{s}' = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}.$$

Also, $\hat{s}' > \hat{s}$.

Q. Given specification: 800 ± 20; $\hat{s} = 4$, $n = 4$

Design a control chart, if rejection is 1% then prob. of a point outside $\bar{x}$ chart will be 0.9? Draw the control limit.

→ UCL = 820

LCL = 780

$$\hat{s} = \frac{\bar{R}}{d_2} = \frac{\bar{R}}{2.059} \Rightarrow \bar{R} = 8.236$$

$$P(x > 820) = 0.005$$

$$\Rightarrow P\left(Z > \frac{820 - \bar{\bar{x}}}{s/\sqrt{n}}\right) = 0.005$$

$$\Rightarrow P\left(Z < \frac{820 - \bar{\bar{x}}}{4/\sqrt{4}}\right) = 0995$$

So, $\frac{820 - \bar{\bar{x}}}{2} = 2.58$ (from Normal table)

so, $\bar{\bar{x}} = 814.84$

Machine Capability:- Variation must be less than the process variation.

— variability of the machine,

— To study machine capability, all other factors should be constant

Two way:- 1. Dry run; 2. With component

— Run the machine without manufacturing anything.

— Study of vibration, temperature.

— We can adjust a value by looking at statistics but not on a single value.

— Use control chart. That is adjust the process when there is some assignable causes are present.

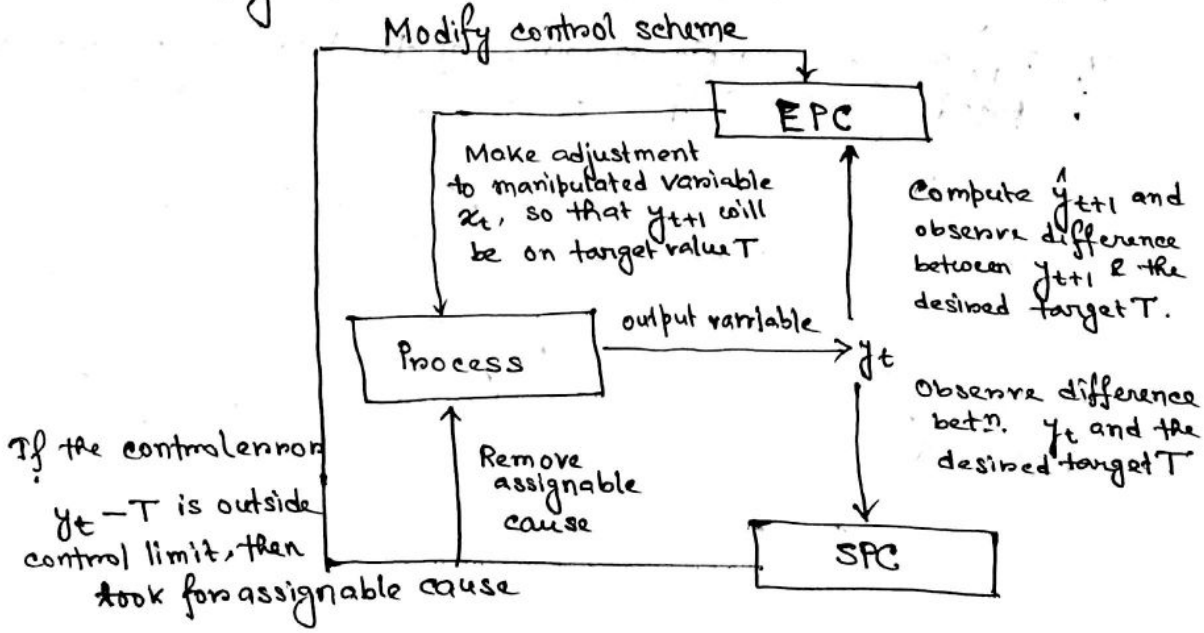
Q. How to implement a control chart for small batch production?

→ – M/c is fixed.
Part No + Prog. change.
– Of cycle time is very high or no. of component produced is very less.
– then we can collect data individually or in subgroups.
– standardize the data because data on different type of product will be available.
– Check the normality.
– IMR chart can be used for short term variation.

Combining SPC and EPC:- Engineering control theory is based on the idea that if we can
1. predict the next observation on the process,
2. have some other variable that we can manipulate in order to affect the process output,
3. know the effect of the manipulated variable.

Note that, this is in sharp contrast with SPC, where 'control action' on a process adjustment is taken only when there is statistical evidence that the process is out of control. On the other hand, EPC makes no attempt to identify an assignable cause that may impact the process. All EPC schemes do react to process upsets; they don't make any effort to remove the assignable causes.

Ex:- Consider the process of driving a car, with the objective of keeping it in the center of the right hand lane. The driver can adjust the process at any time without using statistical control chart.



Multivariate Control Chart:- Used where simultaneous monitoring or control of two or more related quality characteristics is necessary.

$P(\bar{x}_1 > 3\sigma) = P(\bar{x}_2 > 3\sigma) = 0.0027$

$P(\bar{x}_1 > 3\sigma, \bar{x}_2 > 3\sigma) = 0.0027 \times 0.0027 < 0.0027$

So, the use of two independent $\bar{x}$ charts has distroted the simultaneous monitoring of $\bar{x}_1$, and $\bar{x}_2$.

Normal distn :- $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$

Multivariate Normal:- $f(\underset{\sim}{x}) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(\underset{\sim}{x}-\underset{\sim}{\mu})'\Sigma^{-1}(\underset{\sim}{x}-\underset{\sim}{\mu})}$;

$-\infty < x_j < \infty$

$j = 1(1)p.$

$\underset{\sim}{x} = (x_1, \ldots, x_p)$.

$\underset{\sim}{\mu} = (\mu_1, \ldots, \mu_p)$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2 & \cdots \\ & \sigma_2^2 & \cdots \\ & & \ddots \\ & & \sigma_p^2 \end{bmatrix}$$

$(x-\mu)(\sigma^2)^{-1}(x-\mu)$

$= (\underset{\sim}{x}-\underset{\sim}{\mu})'\Sigma^{-1}(\underset{\sim}{x}-\underset{\sim}{\mu})$

Σ : covariance matrix

The most familiar multivariate SPC procedure is the Hotelling T^2 control chart for monitoring the mean vector of the process.

Suppose two quality characteristic x_1, x_2 are jointly distd. according to the Bivariate Normal Distn.

$E(X_1) = \mu_1$, $E(X_2) = \mu_2$, $V(X_1) = \sigma_1^2$, $V(X_2) = \sigma_2^2$; $\text{cov}(x_1, x_2) = \sigma_{12}$

Assuming that $\sigma_1, \sigma_2, \sigma_{12}$ are known.

If $\bar{x}_1, \bar{x}_2$ are sample averages of the two quality characteristic computed from a sample of size n, then the statistic

$$\chi_0^2 = \frac{n}{\sigma_1^2\sigma_2^2 - \sigma_{12}^2}\left[\sigma_2^2(\bar{x}_1-\mu_1)^2 + \sigma_1^2(\bar{x}_2-\mu_2)^2 - 2\sigma_{12}(\bar{x}_1-\mu_1)(\bar{x}_2-\mu_2)\right]$$

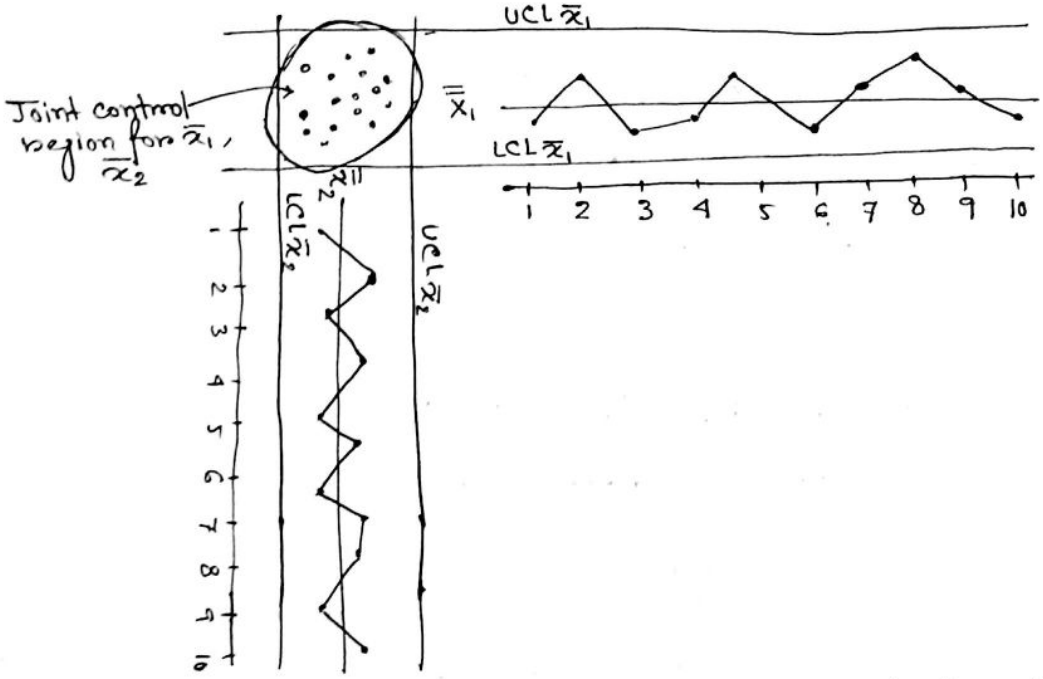
$\sim \chi_2^2$.

$UCL = \chi^2_{\alpha,2}$ = upper α percentage point of χ_2^2.

$LCL = 0$

Example:- x_1 and x_2 are independent, i.e., $\rho_{12} = 0$.

If corresponding value of χ_0^2 plots outside the ellipse, the process is out of control.



Modified Control Charts ($\bar{x}$ Chart):— It is used when the natural variability or spread of the process is considerably smaller than the spread in the specification limits; i.e. C_p or C_{pk} is much > 1.

Usually $C_{pk} \geq 2$.

process output is normally distributed.

$$\mu_L = LSL + Z_\delta \sigma$$

$$\mu_U = USL - Z_\delta \sigma$$

where, Z_δ is the upper $100(1-\delta)$ percentage point of $N(0,1)$.

If we specify type I error of α, then

$$UCL = \mu_U + \frac{3\sigma}{\sqrt{n}}$$

$$= USL - \left(Z_\delta - \frac{3}{\sqrt{n}}\right)\sigma$$

$$LCL = \mu_L - \frac{3\sigma}{\sqrt{n}}$$

$$= LSL + \left(Z_\delta - \frac{3}{\sqrt{n}}\right)\sigma$$

Note that the modified control chart is equivalent to testing the hypothesis that the process mean lies in $\mu_L \leq \mu \leq \mu_U$

Ex. Consider a normally distd process with a target value $\mu = 20$, $\sigma = 2$. LSL = 8, USL = 32, $C_p = C_{pk} = 2$.
In this six-sigma process it is assumed that the mean may drift as much as 1.5 s.d.s off target without causing serious problems. Set up a control chart for monitoring the mean of this 6σ process with $n = 4$.

Solution:-

$$Z_\delta = 3\delta = 3 \times 1.5 = 4.5$$

$$UCL = USL - \left(4.5 - \frac{3}{\sqrt{4}}\right)\sigma = 32 - (4.5 - 1.5)2 = 26$$

and

$$LCL = LSL + \left(4.5 - \frac{3}{\sqrt{4}}\right)\sigma = 8 + (4.5 - 1.5)2 = 14.$$

$\bar{x}$ and R Charts for Short Production Runs:-

- Deviation from Normal (DNOM) Control Chart

$T_A = 50$ mm, $T_B = 25$ mm

M_i : i^{th} actual sample measurement in mm.

$x_i = M_i - T_A$ could be the deviation from Nominal

Sample No	Part No	Measurement M_1	M_2	M_3	DNOM x_1	x_2	x_3	$\bar{x}$	R
1	A	50	51	52	0	1	2	1	2
2	A								
3	A								
4	A								
5	A								
6	B	25	27	24	0	2	−1	0.33	3
7	B								
8	B								
9	B								
10	B								

$\bar{\bar{x}} = 0.17$ $\bar{R} = 2.7$

1. An assumption is process s.d. is approx. same for all parts. If this assumption is invalid, then used standardized $\bar{x}$ & R chart.
2. This procedure works best when the sample size is constant for all part numbers.

▨ Acceptance Sampling :— Online Quality control tool : SPC
Offline Quality control tool : Acceptance sampling

— 100% inspection (either if the process is stable/capable)
— Sampling (the process should be stable and capable)
— No inspection ($C_{PK} > 2$)

- Part criticality
- Capability of the process
- Type of inspection (destructive or non-destructive)
- Cost of inspection
- Availability of Resources.

Variable Inspection:- Measurable, part dimension measured by an instrument. eg. length, power.

Attribute Inspection:- When check by visual inspections.

Two stategy :-
- Acceptance Rejection: The quality can be improved (customer)
- Acceptance Rectifying: The quality can be made better (manufacturer)

When lot quality is good → 100% inspection

risk : good lot quality product getting rejected (Producer's risk)
bad lot quality product getting accepted (Customer's risk)

When we can do sampling?

— When the lot is homogeneous (all the parts in the lot is similar; i.e., from same batch, same machine)
— When the process is stable and capable.

Online Quality Control:- When we can take action back to the process.

Skip-lot Sampling Plan

One step ahead of Chain sampling plan.

→ When quality by vendor is very good.

→ and he demonstrated it for very long time.

→ lot by lot inspection plan supplied

→ if the manufacturing parts are very good, we can skip inspecting few lots.

→ an extension of CSP from part to lot.

Start a reference sampling plan

↓

Start checking every lot (normal inspection)

↓

i consecutive lots are accepted under normal inspection

↓

switch to skip lot inspection ($0 < f < 1$)

↓

moment a lot is rejected go back to normal inspection

$$P_a(f,i) = \frac{f P_a + (1-f) P_a^i}{f + (1-f) P_a^i}$$; where P_a = Prob. of occurrence of reference plan

Case I let $f_2 < f_1$ for fixed i

$P_a(f_1, i) \leq P(f_2, i)$

Case II When $i > j$ for a fixed f.

$P_a(f, j) \leq P_a(f, i)$

$$ASN(SkSP) = ASN(R) \times K = ASN(R) \times \frac{f}{(1-f)P_a^i + f}$$

Skip-lot Sampling plan ← Reference Sampling plan

$\Rightarrow ASN(SkSP) < ASN(R)$.

Sequential Sampling Plan

Checking one item at a time and counting the no. of defective pieces we get.

Item by item sequential sampling plan by Wald (1947)

Acceptance line, $X_A = -h_1 + sn$

Rejection line, $X_R = h_2 + sn$

where $h_1 = \left(\log \frac{1-\alpha}{\beta}\right) / K$

$h_2 = \left(\log \frac{1-\beta}{\alpha}\right) / K$

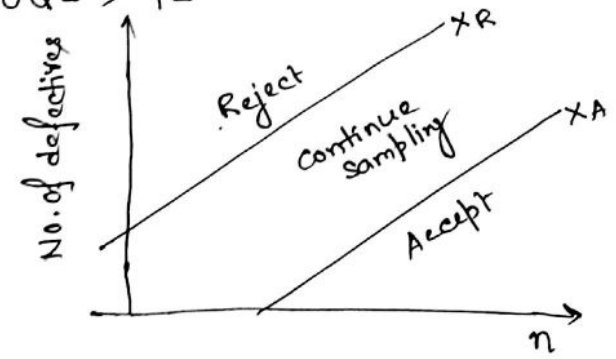
$K = \log \frac{p_2(1-p_1)}{p_1(1-p_2)}$

$s = \log\left[(1-p_1)/(1-p_2)\right] / K$

OC curve with prob. α, β;

α = producer's risk, β = consumer's risk;

p_1 = AOQL, p_2 = LTPD.



$ASN = P_a\left(\frac{A}{c}\right) + (1-P_a)\frac{B}{c}$

$A = \log \frac{\beta}{1-\alpha}$

$B = \log \frac{1-\beta}{\alpha}$

$c = p \log\left(\frac{p_2}{p_1}\right) + (1-p)\log\left(\frac{1-p_2}{1-p_1}\right)$, $p = s$.

Rectifying inspection:-

$ATI = P_a\left(\frac{A}{c}\right) + (1-P_a)N$

$AOQ = P_a \cdot P$.

Chain Sampling Plan

Condition:- 1. You take small sample size
2. May be test is destructive/lot quality is very good & consistent.

Draw a sample size n.

(i) $c = 0$ — accept the lot.
(ii) $c = 1$ — accept the lot if i preceding lots were accepted.
(iii) $c > 1$ — Reject the lot.

The points on the OC curve of a chain sampling plan are given by

$$P_a = P(0,n) + P(1,n)\,[P(0,n)]^i$$

where, $P(0,n)$ and $P(1,n)$ are the probabilities of obtaining 0 and 1 defectives, respectively, out of a random sample of size n.

Example:- Chain Sampling plan with $n = 5$, $c = 0$, and $i = 3$.

For $p = 0.10$, we have

$$P(0,n) = \frac{n!}{d!\,(n-d)!}\,p^d(1-p)^{n-d}$$

$$= \frac{5!}{0!\,5!}\,(0.10)^0(0.9)^5 = 0.590$$

$$P(1,n) = \frac{5!}{1!\,5!}\,(0.10)^1(0.90)^4 = 0.328$$

$$P_a = P(0,n) + P(1,n)\,[P(0,n)]^i$$

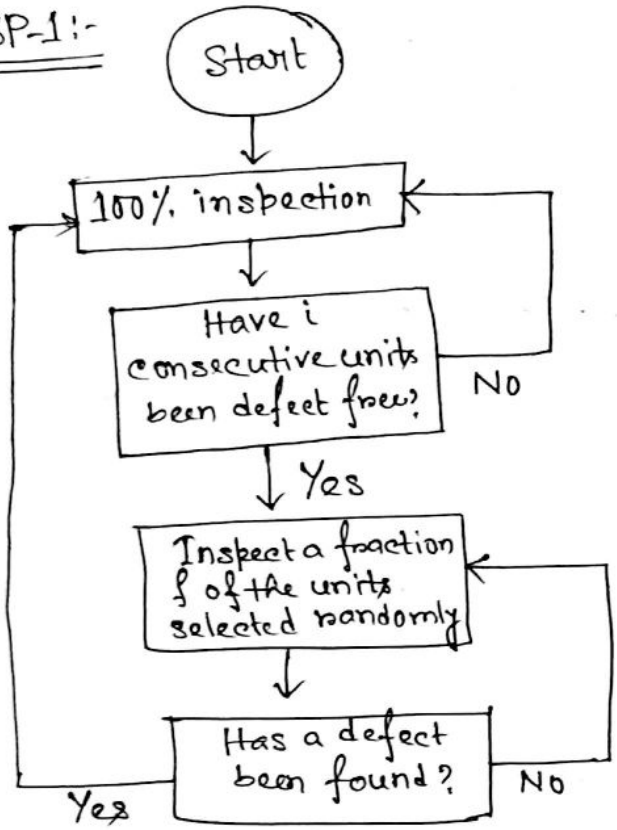
$$= 0.590 + (0.328)(0.590)^3$$

$$= 0.657.$$

Continuous Sampling Plan (CSP)

In this sampling plan first do 100% inspection. After a definite length if all items are good then go for sampling. If in sampling any bad item is detected then go for 100% inspection.

CSP-1:-



i = clearance number

f = fraction of inspection

AOQL = 0.143%.

if $f = \frac{1}{25}$, $i = 1147$

$f = \frac{1}{7}$ if $i = 623$

So, if we decide to do sampling once in every 7 items, we should have 623 units defect free.

The average number of units inspected in a 100% screening sequence following occurrence of a defect is equal to

$$u = \frac{1 - q^i}{pq^i}$$

where, $q = 1 - p$, and p is the fraction defective produced when the process is operating in control.

The average number of units passed under the sampling inspection procedure before a defective unit is found is

$$v = \frac{1}{fp}$$

The average fraction of total manufactured units inspected in the long run is

$$AFI = \frac{u + fv}{u + v}$$

The average fraction of manufactured units passed under the sampling procedure is

$$P_a = \frac{v}{u + v}.$$

β-correction technique

When process suffer from adjustment problem then we apply β-correction technique.

Data(x_i)	add cons 2	add cons 8	add next 2	Total sum
75	145			
70		285		
70	140		587	
70				
75	150			
75		302		1198
77	152			
75				
75	147			
72		300		
78	153		611	
75				
75	153			
78		311		
78	158			
80				

So, $\sum_{i=1}^{16} x_i = 1198$, $\sum_{i=1}^{16} x_i^2 = 89840$

Total sum of square $= \sum x_i^2 - CF$; $CF = \frac{(\sum x_i)^2}{16} = \frac{(89840)^2}{16}$

$= 89700.25$

$= 139.75$

$SS_{\text{between 8 obsn.}} = \frac{587^2 + 611^2}{8} - CF = 36$

$SS_{\text{between 4 obsn}} = \frac{285^2 + 302^2 + 300^2 + 311^2}{4} - CF - SS_8 = 51.25$

$SS_{\text{between 2 obsn.}} = \frac{145^2 + 140^2 + 150^2 + 152^2 + \cdots + 158^2}{2} - CF - SS_8 - SS_4$

$= 22.5$

ANOVA Table:-

Source of Variation	df	SS	MS	Fcal	F-tab
among 8 obs.	1	36	36		
among 4 obs.	2	51.25	25.625	5.857 >	$F_{0.05,2,12} = 3.89$
among 2 obs.	4	22.5	5.625	1.5 <	$F_{0.05,4,8} = 3.84$
Error	8	30	3.75		Not significant
Total	15				

Now, $MSE^* = \frac{SSE + SS_2}{df_E + df_2} = \frac{30+22.5}{4+8} = 4.375$

So, $MS_4 = 25.625$

So, $Fcal = \frac{MS_4}{MSE^*} = 5.857$

So, MSE^* = Variance = $4.375 = \sigma^2$

Now, m = target

$\hat{\mu}$ = Estimator = population mean.

$D = \hat{\mu} - m$ = Off target

Adjustment = $-\beta D$

Define β as $\beta = \begin{cases} 0 & \text{if } \frac{D^2}{\sigma^2} < 1 \\ 1 - \frac{1}{F} & \text{OW} \end{cases}$

where $F = \frac{D^2}{\sigma^2} =$

Method:- Data on $\hat{\mu}$ & σ

Check $(\hat{\mu} - m) > \sigma$

Calculate $F = \frac{(\hat{\mu} - m)^2}{\sigma^2}$

then find $\beta = 1 - \frac{1}{F}$.

:- Taguchi Loss Function:-

Loss function is defined as deviation as the quantity proportional to the squared deviation from the target quantity characteristic. At zero deviation, the performance is at target and the loss is zero.

Y = Deviation from target

Y_0 = Target

$$L(Y) = K(Y - Y_0)^2$$

Derivation:- $L(Y) = L(Y_0) + L'(Y_0)(Y-Y_0) + \frac{1}{2!}L''(Y_0)(Y-Y_0)^2$

$= \frac{1}{2!}L''(Y_0)(Y-Y_0)^2$

$= K(Y-Y_0)^2$

$L(Y_0) = K(Y_0 - Y_0)^2 = 0$

$L'(Y_0) = 2K(Y-Y_0) = 0$

L_0 = Loss at $Y_0 + \Delta$

$= K(Y_0 + \Delta - Y_0)^2$

$= K\Delta^2$

$\therefore K = \frac{L_0}{\Delta^2}$

So, $L(y) = \frac{L_0}{\Delta^2}(y - y_0)^2$.

$= K\left[\frac{(y_1 - y_0)^2 + \cdots + (y_n - y_0)^2}{n}\right]$

$= K.MSD$.

Ex.1. Target = 12

Tolerance = ± 0.35

L_0 = Rs. 20/-

Data: 11.80 12.30 12.20 12.40 12.10
12.20 11.90 11.80 11.85 12.15

Estimate $L(y) = ?$

Solution:- $\Delta = 0.35$

$$K = \frac{L_0}{\Delta^2} = \frac{20}{(0.35)^2} = 163.265$$

$L(Y) = KMSD$

$= 7.10$

Ex.2. The target value of a quality characteristic is 100. The loss to customer beyond 115 is Rs. 40. The internal loss is Rs. 15 for the same value. What should be the mfg. tolerance for this characteristic?

Sol.

$$L(y) = k(y - y_0)^2$$

$$40 = k(115 - 100)^2$$

$$\therefore k = \frac{40}{15^2} = 0.178$$

$$L_0 = k\Delta^2$$

$$15 = \frac{40}{15^2}\Delta^2$$

$$\Rightarrow \Delta^2 = \frac{15^3}{40} = 9.185$$

Note:- Taguchi said that external loss is much more higher than internal loss.

Quality talks mainly performance feature.

Performance characteristic

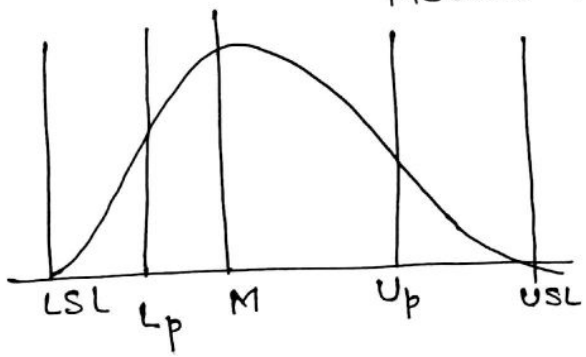
- Warranty
- Inhouse testing
- Accelerated life testing

Process Capability for Non-Normal Distributions

Process capability index = $\frac{\text{Allowed Variation}}{\text{Actual Variation}} = \frac{T}{6s}$

$$C_p = \frac{USL - Median}{Maximum - Median} \quad \text{(lower the better)}$$

$$= \frac{Median - LSL}{Median - Min} \quad \text{(higher the better)}$$



$$C_p = \frac{USL - LSL}{U_p - L_p}$$

$$C_{pU} = \frac{USL - M}{U_p - M}$$

$$C_{pL} = \frac{M - LSL}{M - L_p}$$

$$C_{pK} = \min\{C_{pU}, C_{pL}\}$$

$$L_p = \bar{X} - s.L_p'$$

$$U_p = \bar{X} + s.U_p'$$

$$M = \bar{X} + s.M'$$

L_p', U_p' & M' value is in table.

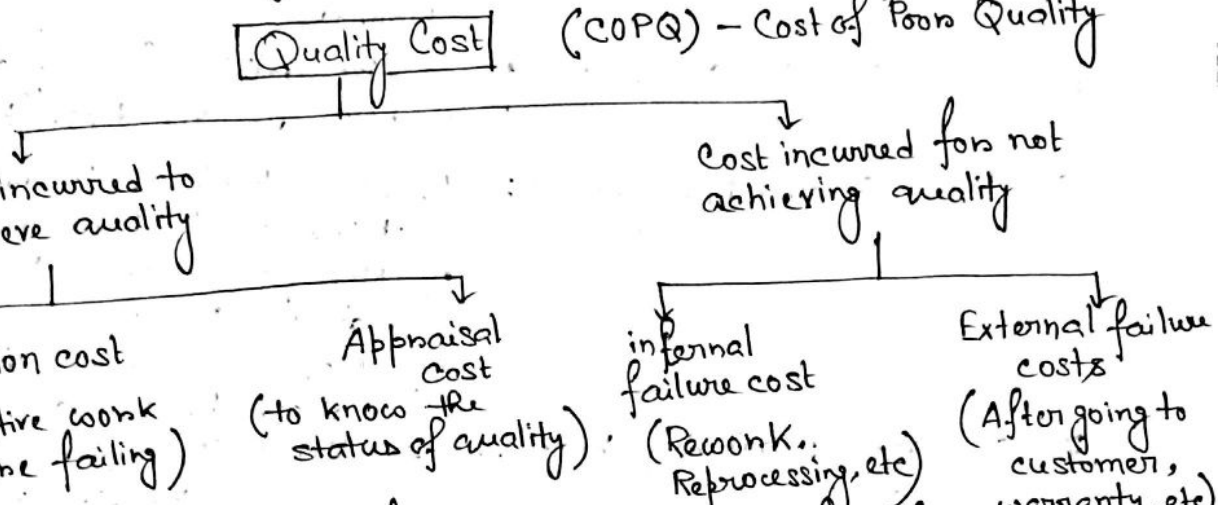
[Process Capability Calculations for Non-normal distn. (Quality Progress) by John. A. Clements]

Trouble Shoting & Problem Solving for Quality Improvement

CTQ : Critical to Quality characteristics

COPQ : Cost of Poor Quality

Bias :- 1. Instant Bias : how much on an average deviated from target.
2. Linearity : What happened to the bias factor when measuring through its range.
3. Stability : Over a period of time bias should be stable.



Invitro :- (within glass) refers to the techniaue of performing a given procedure in a control environment outside of a living organism (cellular biology environment) – fail to replicate the precise cellular condition of an organism. So, this may lead to results that don't correspond to the circumtances occuring around a living organism.

Invivo :- (within the living) refers to experimentation using a whole, living organism as opposed to a particular on dead organism. Animal studies & clinical trials are two forms of invivo research. This is suited for observing the overall effects of an experiment on a living subject.

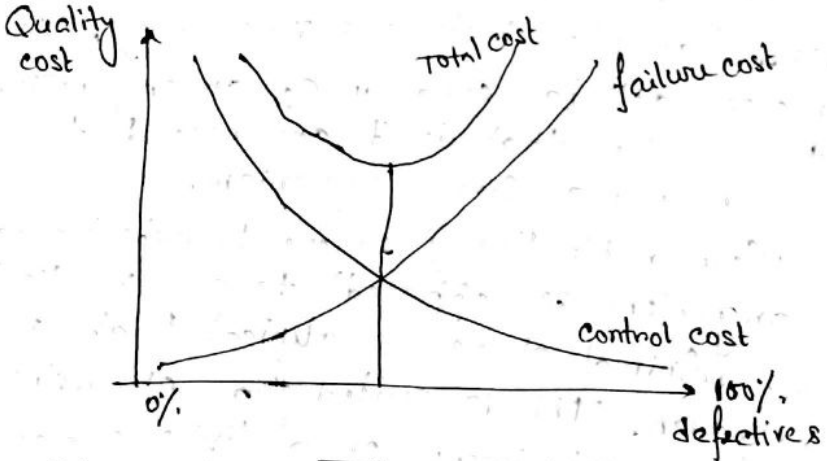
Invitro is better than Invivo :-

- reduce cost
- more directly assess product performance.
- offer benefits in terms of ethical consideration.
- Invivo is costly, tested on living, so error is high.

Note :- For world class Company, COPQ is less than 10%, for average company 10 – 30%, poor company > 30%.

Terms:-

1. Craftman: people develop some trades (individual come with some idea)
2. Inspection: An activity to segregate good from bad.
3. Quality Control:- prevents defects from occuring (in the process)
4. Quality Assurance: Set of all activity which ensure every activity associated is working as planned.
5. Total Quality Control: Associative activity (manufacturing) control throughout various dipt. (production, design, etc)
6. Total Quality Management:
   1. Don't think of product, think about the process producing it.
   2. Never think about profit, think about customers.
   3. Never think about the task, think about people doing the task.
7. Six Sigma: achiving business excellence (balancing product & profit margin)
8. Lean Six Sigma: There are 8 waste due to manufacturing, how to minimize these waste.

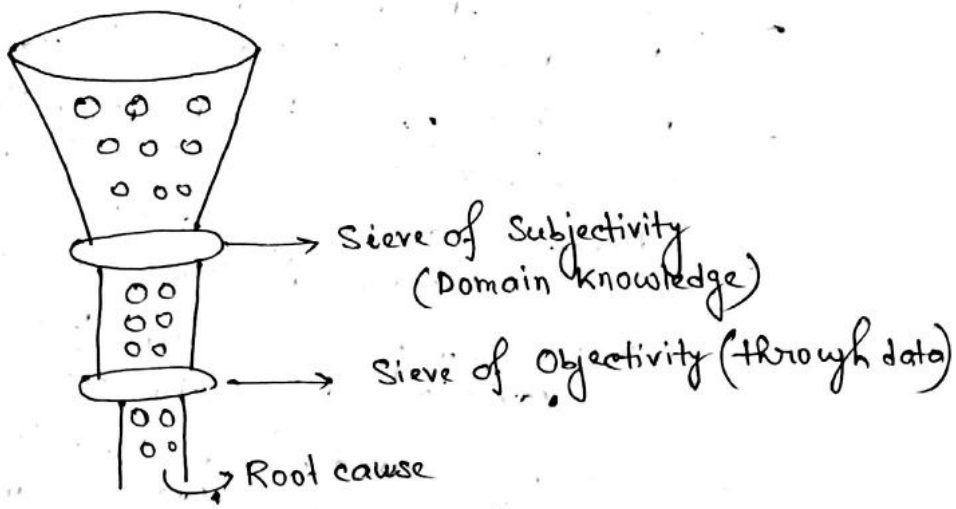


C_p: $\frac{USL - LSL}{6\sigma}$, $\hat{\sigma} = \frac{\overline{MR}}{d_2} = \frac{\bar{R}}{d_2}$

P_p: $\frac{USL - LSL}{6\sigma}$, $\hat{\sigma} = \frac{1}{n-1}\sum (x_i - \bar{x})^2$.

House of Quality:-
- Product Planning
- Parts Deployment
- Process Deployment
- Design Process
- Product deployment

Root-cause Identification:- Funelling approach



Best model conditions:-
- VIF < 5
- R^2-adjusted > 0.6
- Residual Plot (Normal)
- Residual vs fitted value
- Histogram
- Residual vs. order

Seven QC Tools:-
1. Flow Diagram
2. Stratification and Check Sheet
3. Pareto Analysis
4. Graphs, Charts & Plots
5. Cause & Effect diagram
6. Histogram
7. Scatter Diagram

New 7M tools:-
1. Relation Diagram } Planning stage, identifying problems
2. Affinity Diagram (KJ) }
3. Systematic Diagram } matching goals with the means
4. Matrix Diagram }
5. Matrix Data Analysis (PCA)
6. Process Decision Program chart (PDPC) }
7. Arrow Diagram (PERT/CPM) } Implementation stage.

Note:- Cause & Effect Diagram can't interconnect the various causes, but Relation diagram does.

Principal Component Analysis:-

- Describes the variation in a set of correlated variables (x's) by a set of uncorrelated variables.
- Each principal component is a linear combination of the x's.
- The new variables are derived in decreasing order of importance
- Hence y_1 account for maximum possible variation in x among all linear combination of x.
- y_2 account for maximum possible of the remaining variation subject to being uncorrelated to y_1 & so on.
- Helps to understand the variability in large data sets with intercorrelated variables using a smaller number of uncorrelated factors.
- Explaining variability of a set of n variables using m factors, $m<n$

Objective:- 1. Reduces the complexity of a large set of variables by summarizing them in a smaller set of components / factors.

2. Tries to improve the interpretation of complex data through logical factors.

Relation Diagram:- When something achieved by intution in past, depending upon the past experience some logic is made.

Affinity Diagram:- This techniaue clarifies important but unresolved problems by collecting verbal data. One way to understand VOC.

Systematic Diagram:- This techniaue searches for the most appropriate & effective means of accomplishing given objectives.

Matrix Diagram:- When we have multiple solutions, finding the best. A techniaues that clarifies problematic spots through multidimensional thinking.

☒ Rate of Improvement (RI) = $\frac{\text{Stage what you want to achieve}}{\text{Current stage}}$

PDPC :- This techniaue helps to determine which process to use to obtain desired result by evaluating the progress of events and the variety of conceivable outcomes.

Looks difficulties in the process (diffence between flow chart)

FMECA → static

PDPC → Dynamic

Six-Sigma for Business Excellence

DMAIC: Define, Measure, Analyse, Improve, control.

Statistical Thinking:- 1. Variation is inevitable
2. Everything is executed as a process.
3. Understanding & reducing variation is key to success.

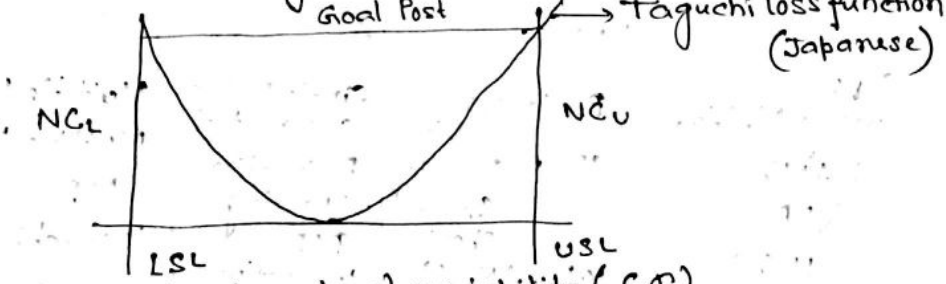
Six-sigma try to achieve as much as less variation possible.
- Cause & Effect relationship; $E = f(C)$.

Variation

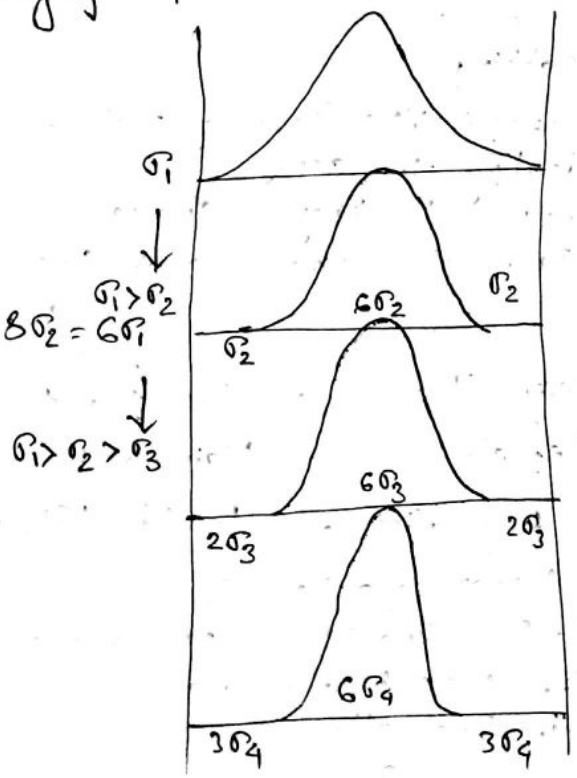
Assignable
- small in number
- contribution is large
- Easily identifiable
- can be eliminated / should be eliminated.

Chance/Random
- large in number
- Each small contribution
- Very difficult to identify individual.
- Difficult to eliminate for the individual.

American Approach to Quality (Goal post syndrome)



Capability of a process is its natural variability (6σ).



$C_p = 1$
$C_{pk} = 1$

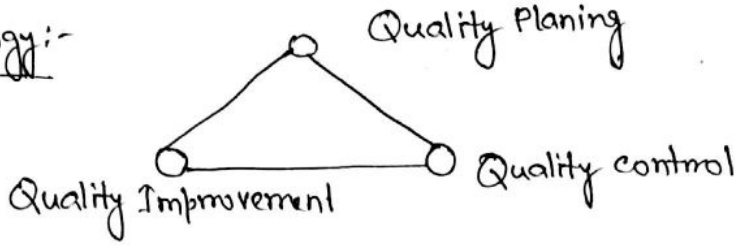
$8\sigma_2 = 6\sigma_1$
$C_p = 1.33$ $= \frac{8}{6} = 1.33$

$10\sigma_3 = 6\sigma_1$
$C_p = 1.67$

$12\sigma_4 = 6\sigma_1$
$C_p = 2$
if moving by $1.5\sigma_4$, $C_{pk} = 1.5$

Stability Analysis:-
- Run chart
- Histogram (if # observation > 70)
- Control Chart (I-MR)
- N.P.P.
- Box Plot. — Six-pack

Juran Triology:-



PDCA Cycle:- Plan - Do - Check - Act.

R D M A I C S I → Integrate

Recognize, Define, Measure, Analyse, Improve, control, Standardize

Modelling :- DOE, Multiple Regression.

D	M	A	I	C
VOC	Basic stat	Simple Problem solving tool	Multiple Reg.	MSA
Survey Method	SPC ← Stability, Capability, Control Chart	Estimation	Multiple Method	SPC
KANO	$\bar{X}$-R, ANOVA	Testing	DOE	Sign off
QFD	Gauge R&R	DOE	Taguchi	
SIPOC	Kappa	Correlation		
Project Charter		Regression		

VSM: Value Stream Mapping

Define
1. Capture the VOC (Voice of customer)
2. Identify the CTQ
3. Prepare the project charter
4. Draw the process map (SIPOC)

VOC	CTQ	Solution
Slow room service	— Delivery Time	— Deliver food to room in 20min
Quality is not good	— Defects	— Maintain it
Picture Quality is bad	— Resolution	— Provide 16M colour pics.

Measure
1. Performance Variable
2. Establish the performance variable
3. MSA performance
4. Evaluation

Kappa Analysis:-

	1	2	Total
1	a	b	a+b
2	c	d	c+d
Total	a+c	b+d	N

Expected prob. for cell (1,1)

$$= \frac{a+b}{N} \times \frac{a+c}{N} \times N$$

$$= \frac{(a+b)(a+c)}{N} \quad \text{(Marginal prob.)}$$

$Pr(0)$ = Prob. of obs. agreement $= \frac{a+d}{N}$

$Pr(e)$ = Prob. of expected agreement $= \frac{(a+b)(a+c)+(b+d)(c+d)}{N^2}$

$$K = \frac{Pr(0) - Pr(e)}{\underbrace{1 - Pr(e)}_{\text{disagreement}}}$$

Kappa ranges from -1 to 1.

$0 \rightarrow$ agreeing by chance

$-1 \rightarrow$ complete disagreement

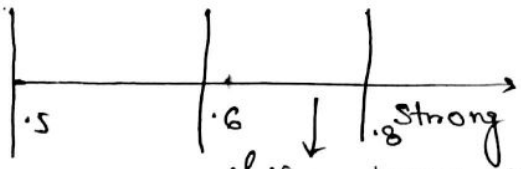
$1 \rightarrow$ Complete agreement

Sigma level → is Z-level (so it can be -ve) ← point in Z-scale corresponding to area meeting our requirements

- SIPOC: Supplier + Input + Process + Output + Customer

DFSS: Design for Six-sigma

Thumb Rule for R^2-adjusted:-



if there is any other variable, try to add, or can go further.

EXAM:- 1. Six sigma Case Study

2. Need to identify a concept/objective, what's the six sigma stage on which we will be using what tool or techniaue.

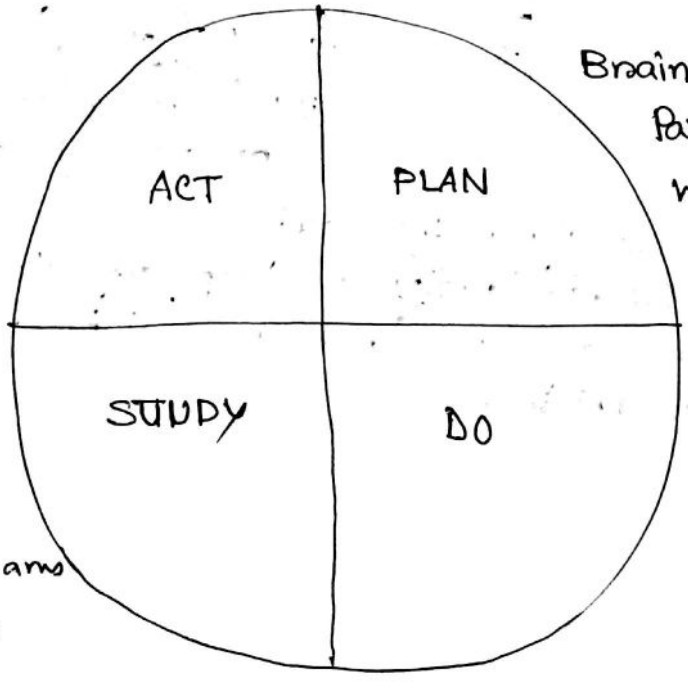
7 STEP PROBLEM SOLVING METHODS:-

1. Select a Theme 2. Collect Data 3. Analyse Causes	PLAN	}	Check Sheet, Graph, Histogram, Scatter diagram, Pareto, C&E diagram, Flowcharts
4. Plan and implement solution	DO	}	Flowcharts
5. Evaluate effects	CHECK	}	Check sheets, graphs, Histogram, scatter diagram, Pareto, C&E diagram, Run chart/control chart
6. Standardize 7. Reflect on Process	ACT	}	Flowchart

" Problem Solving, the isolation and analysis of a problem and the development of a permanent solution, is an integral part of the quality improvement process.

Problem Solving Process:-

Symptom Recognition → Fact finding → Problem Identification → Idea generation → Solution Development → Plan Implementation → Follow up



Brainstroming
Pareto Analysis
Why-Why diagram

Run charts
Control Charts
Histograms
Check sheets
Scatter diagrams
Pareto charts

C & E diagram
Scatter diagram
Control Charts
Chick Sheets
Run Charts

8D PROBLEM SOLVING TECHNIQUES:-

1. Define the Team
2. Define Problem/Failure
3. Choose and Verify Interim Containment Action (ICA)
4. Define and Verify root causes
5. Choose & Verify Permanent Corrective Action (PCA)
6. Implement & Validate PCA
7. System Prevent Actions to Prevent Recurrence
8. Team Recognition/ Celebration

Deming Quality Principles:- (14 Point Management Philosophy)

1. Create constancy of purpose for continual improvement of products.
2. Adopt a commitment to seek continual i
3. Switch from defect detection to defect prevention
4. In dealing with suppliers one should end the practice of awarding business on price. Move towards quality of product, reliability of delivery and willingness to cooperate and improve. Build partnerships.
5. Improvement is not confined to products and their direct processes but to all supporting services and activities.
6. Train a modern way
7. Supervision must change from chasing, to coaching and support.
8. Drive out fear and encourage two way communication.
9. Remove barriers between departments.
10. Do not have unrealistic targets.
11. Eliminate quotas and numerical targets.
12. Remove barriers that prevent employees having pride in the work that they perform.
13. Encourage education and self-improvement for everyone.
14. Publish top management's permanent commitment to continuous improvement of quality & productivity.