INDUSTRIAL EXPERIMENTATION

BY

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Industrial Experimentation (IE)

Experiment:- Something doing intensely to see the effect.

Generally experiments are used to study the performance of process and systems. It can be modelled.

Process is defined as a combination of operations, machines, methods, people and other resources that transform some input into an output that has one or more observable response variables. Some of the process variables are controllable where as others are uncontrollable.

Design of Experiment Objectives:-

(i) Determine or identify the critical or most influential parameters which influences the output or response (Y) of process.

(ii) Identify the optimal level of the critical variable.

(iii) Determine where to set the influential x's so that the response Y is almost always near the desired nominal value.

(iv) Determine where to set the influential x's so that the variability in Y is small.

(v) Determine where to set the influential x's so that the effects of the uncontrollable variables, say, z_i's are minimized.

For performing experiments which is affected by different factors, We need to draw a plan for conducting this which is called Strategy of experiment.

Any output or response of an experiment is affected by the factors and its level of the impact.

In any experiment the correct way to deal with several factors is to conduct factorial experiment. eg:- If there are two factors and two levels then number of experiment performed is 2^2. It is also called as 2^2 factorial design.

Applications of Experimental design (in Process development) can result in

- Improved process yield
- Reduced variability and closer conformance to nominal or target requirement.
- Reduced the time (development time)
- Reduced overall cost.

There are two aspects of any experimental problem

- Design of the experiment
- Statistical analysis of the data

So, for any Industrial Experiment basic process involves

(i) Identifying the need for experiment.
(ii) Identifying the response or output.
(iii) factors and levels affecting the experiment.
(iv) Design of Experiment.
(v) Conducting the experiment and collecting data.
(vi) Analysis and recommendation.

Basic Principles of Experimental Design are :-

- Randomization (through this process systematic pattern can be removed/recognized).
- Replication (to determine Experimental error)
- Local control/blocking (Non-source of variability is detected).

CRD

This design provides a one-way classified data according to levels of a single factor, treatment, for its analysis the following model is taken:

$$y_{ij} = \mu + \tau_i + e_{ij} \quad (i = 1(1)k,\ j = 1(1)n)$$

y_{ij} : Observation from the i^{th} treatment and j^{th} replication.

μ : overall mean

τ_i : Effect of the i^{th} treatment.

$e_{ij} \overset{iid}{\sim} N(0, \sigma_e^2)$.

Assumption:- $\sum_{i=1}^{k} n\tau_i = 0$.

ANOVA Table:-

Source of Variation	DF	SS	MS	Fcalculate
Treatment	$k-1$	$SS_{Treatment}$	$MS_{Treat} = \frac{SS_{Treat}}{k-1}$	$\frac{MS_{Treat}}{MS_E} \sim F_{k-1,\,k(n-1)}$
Error	$k(n-1)$	SS_E	$MS_E = \frac{SS_E}{k(n-1)}$	
Total	$kn-1$	SS_T		

The hypothesis that treatments have equal effects is tested by the F-test.

We reject $H_0: \tau_1 = \tau_2 = \cdots\cdots = \tau_k = 0$ Vs. $H_1: \tau_i \neq 0$ for at least one i at level α if $F = \frac{MST}{MSE} > F_\alpha$.

where, $SS_{Total} = \sum_i \sum_j y_{ij}^2 - CF$ (Correction Factor)

$$= \sum_i \sum_j y_{ij}^2 - \frac{T^2}{kn}, \quad T = \sum_i \sum_j y_{ij}$$

$$SS_{Treatment} = \frac{\sum_i y_{io}^2}{n} - CF$$

$$SS_{Error} = SS_T - SS_{Treat}$$

Conclusion:- If $F_{cal} > F_\alpha$ then the treatment is significant.

In MINITAB, if p-value $< \alpha$ then it has significant effect.

RBD

The data collected from experiments with RBD from a two-way classification, i.e., classified to the levels of two factors, viz., blocks and treatments. We take the model:

$$y_{ij} = \mu + \tau_i + B_j + e_{ij}$$

y_{ij} : Observation from ith treatment in the j^{th} block.

μ : general mean

τ_i : Effect of the ith treatment ; $i = 1(1)k$,

B_j : Effect of the j^{th} block. ; $j = 1(1)n$.

e_{ij} : error component

Assumption :- $e_{ij} \overset{iid}{\sim} N(0, \sigma_e^2)$ $\forall$ i,j and $\sum_i \tau_i = \sum_j B_j = 0$.

Test :- i) $H_0 : \tau_1 = \tau_2 = \cdots = \tau_k = 0$ Vs $H_1 : \tau_i \neq 0$ for at least one i.

ii) $H_0 : \beta_1 = \beta_2 = \cdots = \beta_n = 0$ Vs. $H_1 : \beta_j \neq 0$ for at least one j.

ANOVA Table :-

Source	DF	SS	MS $\left(\frac{SS}{DF}\right)$	F
Treatment	$k-1$	$\frac{\sum y_{i0}^2}{n} - CF = SS_T$	$\frac{SS_T}{k-1}$	$\frac{MS_T}{MS_E} \sim F_{\alpha,(k-1),(k-1)(n-1)}$
Block	$n-1$	$\frac{\sum y_{0j}^2}{k} - CF = SS_B$	$\frac{SS_B}{n-1}$	$\frac{MS_B}{MS_E} \sim F_{\alpha,(n-1),(k-1)(n-1)}$
Error	$(k-1)(n-1)$	By subtraction $= SS_E$	$\frac{SSE}{(k-1)(n-1)}$	
Total	$kn-1$	$\sum y_{ij}^2 - CF$		

$$CF = \frac{y_{00}^2}{k \times n}.$$

H_0 is rejected at level α if $F = \frac{MST}{MSE} > F_{\alpha,(k-1),(k-1)(n-1)}$

→ Unknown source of variations are included, generally called blocks.

→ If we don't consider block variations then the SS_E will be more which will lead us in wrong calculation of F and finally allowing us to conclude wrong significance of the source, which will be of no use.

LSD

The data collected from this design has 3 factors, therefore, analysed as a three-way classified data.

LSD is an incomplete 3-way layout, where all the 3 factors (row, column, treatment) are at same no. of level a.

So let us take the model:

$$y_{ijk} = \mu + \alpha_i + B_j + \tau_k + e_{ijk}, \quad i, j, k = 1(1)a.$$

y_{ijk}: obsⁿ in the i^{th} row, j^{th} column and under K^{th} treatment.

μ: general mean

α_i: row effect

B_j: column effect

τ_k: treatment effect.

Assumption:- $e_{ijk} \overset{iid}{\sim} N(0, \sigma_e^2)$, $\sum_i \alpha_i = \sum_j B_j = \sum_k \tau_k = 0$.

Tests:- (a) $H_0: \alpha_i = 0$ Vs. $H_1:$ at least one $\alpha_i \neq 0$

(b) $H_0: B_j = 0$ Vs. $H_1:$ at least one $B_j \neq 0$

(c) $H_0: \tau_k = 0$ Vs. $H_1:$ at least one $\tau_k \neq 0$

ANOVA Table:-

Source	DF	SS	MS	F
Treatment	$a-1$	$\frac{\sum_k y_{00k}^2}{a} - CF$	$SS_{Treat}/a-1$	$\frac{MS_T}{MS_E}$
Row	$a-1$	$\frac{\sum_i y_{i00}^2}{a} - CF$	$SS_{Row}/a-1$	$\frac{MS_R}{MS_E}$
Column	$a-1$	$\frac{\sum_j y_{0j0}^2}{a} - CF$	$SS_{col}/a-1$	$\frac{MS_C}{MS_E}$
Error	$(a-1)(a-2)$	By substitution	$SS_{Err}/(a-1)(a-2)$	
Total	a^2-1	$\sum_i \sum_j y_{ij}^2 - CF$		

$$CF = \frac{y_{00}^2}{a^2}$$

H_0 is rejected at level α if $F_{cal} > F_\alpha$.

Balanced Design:- When all factors level can be replicated equal no. of times in the design.

Nested ANOVA

Data (x_i)	add cons 2	add cons 4	add 2	Total
75	145			
70		285		
70	140			
70			587	
75	150	302		
75				1198
77	152	300		
75	147		611	
75	153			
72		311		
78	153			
75				
75	158			
78				
78				
80				

$$TSS = \sum_{i=1}^{16} x_i^2 - \frac{(\sum x_i)^2}{16} = 139.75$$

$$SS_8 = \frac{587^2 + 611^2}{8} - CF = 36$$

$$SS_4 = \frac{285^2 + 302^2 + 300^2 + 311^2}{4} - CF - SS_8 = 51.25$$

$$SS_2 = \frac{145^2 + 140^2 + \cdots + 158^2}{2} - CF - SS_8 - SS_4 = 22.5$$

ANOVA Table:-

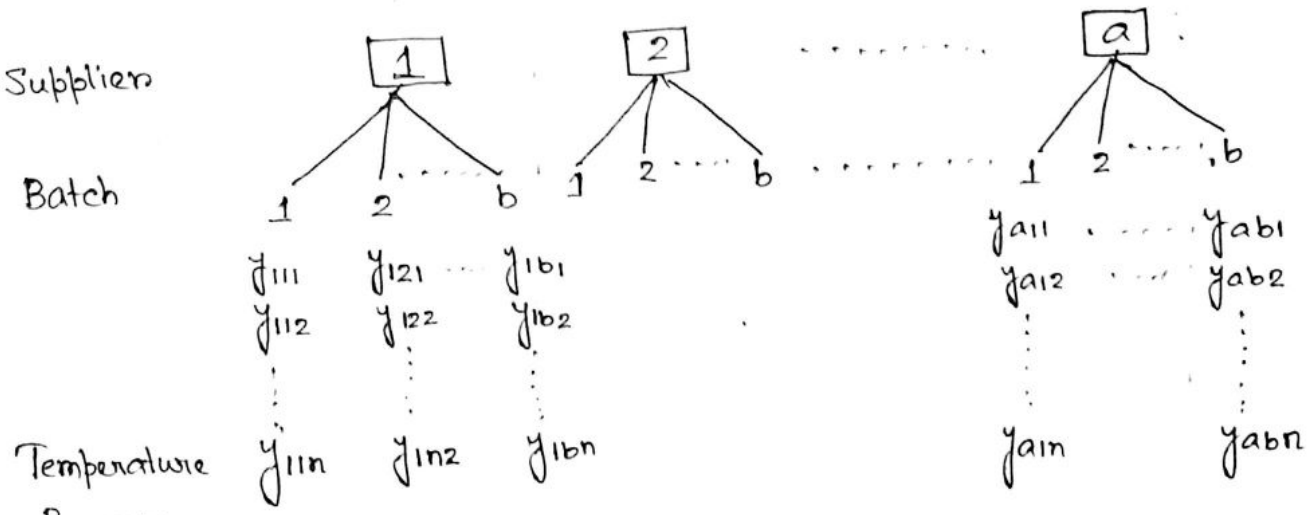
Source of Variation	DF	SS	MS	Fcal	Ftab
among 8	1	36	36		
among 4	2	51.25	25.625	5.857 >	$F_{0.05,2,12} = 3.89$
among 2	4	22.5	5.625	1.5 <	$F_{0.05,4,8} = 3.84$
Error	8	30	3.75		→ not significant
Total	15	—			

$$MSE^* = \frac{SSE + SS_2}{df_E + df_2} = \frac{30 + 22.5}{4 + 8} = 4.375$$

$$MS_4 = 25.625$$

$$F_{cal} = \frac{MS_4}{MSE^*} = 5.857$$

Nested Design



Degree of freedom for suppliers = $a-1$

" " " Batches = $a(b-1)$

Total d.f. = $abn-1$.

Model:- $y_{ijk} = \mu + \tau_i + \beta_{j(i)} + e_{(ij)k}$; $e_{(ij)k} \sim NID(0, \sigma^2)$

$i = 1, 2, \ldots, a$
$j = 1, 2, \ldots, b$
$k = 1, 2, \ldots, n$

Total sum of square:-

$$SS_T = SS_A + SS_{B(A)} + SS_E$$

d.f. $abn-1$ $a-1$ $a(b-1)$ $ab(n-1)$

Test:- $H_0: \tau_i = 0$

$H_1: \tau_i \neq 0$ for atleast one i.

$\rightarrow$ $H_0: \beta_{j(i)} = 0$

$H_1: \beta_{j(i)} \neq 0$ for at least one i.

$$SS_T = \sum_i \sum_j \sum_k (y_{ijk} - \bar{y})^2 = \sum_{ijk} y_{ijk}^2 - CF$$

$$SS_A = \frac{1}{bn} \sum_i y_{i00}^2 - \frac{y_{000}^2}{abn} \; (= CF)$$

$$SS_{B \text{ within } A} = SS_{B(A)} = \frac{1}{n} \sum_{ij} y_{ij0}^2 - \frac{1}{bn} \sum_i y_{i00}^2 .$$

$$SS_E = SS_T - SS_A - SS_{B(A)}$$

Source	DF	SS	MS	F
A	$a-1$	SS_A	MS_A	$F = \frac{MS_A}{MS_{B(A)}}$
B(A)	$a(b-1)$	$SS_{B(A)}$	$MS_{B(A)}$	$F = \frac{MS_{B(A)}}{MSE}$
Error	$ab(n-1)$	SS_E	MS_E	
Total	$abn-1$	SS_T		

Problem:-

Batch	Process-1				Process 2				Process 3			
	1	2	3	4	1	2	3	4	1	2	3	4
	25	19	15	15	19	23	18	35	14	35	38	25
	30	28	17	16	17	24	21	27	15	21	54	29
	26	20	14	13	14	21	17	25	20	24	50	33
Total	81	67	46	44	50	68	56	87	49	80	142	89
			238				261					360

Total = 859 = T

$CF = \frac{T^2}{36} = 20496.69$

$SS_{Total} = 23745 - CF = 3248.306$

$= \sum_{i,j,k=1}^{12} (y_{ijk})^2 - CF$

$SS_A = \frac{(238)^2 + (261)^2 + (360)^2}{12} - CF = 700.3889$

$SS_{B(A)} = \frac{(81)^2 + (67)^2 + \cdots + (89)^2}{3} - SS_A - CF = 2675.25$

$SS_E = 472.667$

Source	DF	SS	MS	F	$F_{0.05,n_1,n_2}$
A	2	700.39	350.2	1.52	< 4.25
B(A)	9	2075.25	230.58	11.70	> 2.30
Error	24	472.67	19.69		
Total	35	3248.31			

∴ B(A) is significant but Process A is not significant

Industrial Experimentation

Assignment -1

Last Date for Submission: 10 March 2015

All questions carry equal marks. Max. Marks : 50

1 Four different designs for a digital computer circuit are being studied to compare the amount of noise present. The following data have been obtained:

Circuit Design	Noise Observed				
1	19	20	19	30	8
2	80	61	73	56	80
3	47	26	25	35	50
4	95	46	83	78	97

(a) Is the amount of noise present the same for all four designs? Use $\alpha = 0.05$.
(b) Analyze the residuals from this experiment. Are the analysis of variance assumptions satisfied?
(c) Which circuit design would you select for use? Low noise is best.

2 Three different washing solutions are being compared to study their effectiveness in retarding bacteria growth in 5-gallon milk containers. The analysis is done in a laboratory, and only three trials can be run on any day. Because days could represent a potential source of variability, the experimenter decides to use a randomized block design. Observations are taken for four days, and the data are shown here. Analyze the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

	Days			
Solution	1	2	3	4
1	13	22	18	39
2	16	24	17	44
3	5	4	1	22

3 The effect of five different ingredients (A, B, C, D, E) on the reaction time of a chemical process is being studied. Each batch of new material is only large enough to permit five runs to be made. Furthermore, each run requires approximately $1\frac{1}{2}$ hours, so only five runs can be made in one day. The experimenter decides to run the experiment as a Latin square so that day and batch effects may be systematically controlled. She obtains the data that follow. Analyze the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

	Day				
Batch	1	2	3	4	5
1	$A = 8$	$B = 7$	$D = 1$	$C = 7$	$E = 3$
2	$C = 11$	$E = 2$	$A = 7$	$D = 3$	$B = 8$
3	$B = 4$	$A = 9$	$C = 10$	$E = 1$	$D = 5$
4	$D = 6$	$C = 8$	$E = 6$	$B = 6$	$A = 10$
5	$E = 4$	$D = 2$	$B = 3$	$A = 8$	$C = 8$

4. A bacteriologist is interested in the effects of two different culture media and two different times on the growth of a particular virus. She performs six replicates of a 2^2 design, making the runs in random order. Analyse the bacterial growth data that follows and draws appropriate conclusions. Analyse the residuals and comment on the model's adequacy.

Time, h	Culture Medium 1		Culture Medium 2	
12	21	22	25	26
	23	28	24	25
	20	26	29	27
18	37	39	31	34
	38	38	29	33
	35	36	30	35

5. The yield of a chemical process is being studied. The two most important variables are thought to be the pressure and the temperature. Three levels of each factors are selected, and a factorial experiment with two replicates is performed. Data: (yield)

Temperature (°C)	Pressure (psig) 200	215	230
150	90.4	90.7	90.2
	90.2	90.6	90.4
160	90.1	90.5	89.9
	90.3	90.6	90.1
170	90.5	90.8	90.4
	90.7	90.9	90.1

(a) Analyze the data and draws conclusions. Use $\alpha = 0.05$.

(b) Prepare appropriate residual plots and comment on model's adequacy.

(c) Under what conditions would you operate this process?

Assignment-1 (Tanujit Chakraborty)

1.

(a)

Circuit Design	Noise observed					Treatment sum (y_{i0})	$\bar{y}_{i0}$
1	19	20	19	30	8	96	19.2
2	80	61	73	56	80	350	70
3	47	26	25	35	50	183	36.6
4	95	46	83	78	97	399	79.8
						$\Sigma y_{ij} = 1028$	

Model is:- $y_{ij} = \mu + \tau_i + e_{ij}$, $i = 1(1)k$, $j = 1(1)n$

No. of level $= k = 4$

No. of replicates $= n = 5$

To test $H_0: \tau_i = 0$ vs. $H_1: \tau_i \neq 0$ for at least one i.

$$SS_{Treatment} = \frac{\sum_i y_{i0}^2}{n} - CF$$

$$= \frac{96^2 + 350^2 + 183^2 + 399^2}{5} - 52839.2$$

$$= 12042.$$

$$SS_{Total} = \sum\sum y_{ij}^2 - CF$$

$$= 67830 - 52839.2 = 14990.8$$

$$\left[CF = \frac{(\sum y_{ij})^2}{k \times n} = \frac{(1028)^2}{4 \times 5} = 52839.2 \right]$$

$SS_{Total} = SS_{Treatment} + SS_{Error}$ $\Rightarrow$ $SS_{error} = 2948.2$

ANOVA Table:-

Source	DF	SS	MS $\left(\frac{SS}{DF}\right)$	F_{cal}
Treatment	3	12042	4014	$\frac{MS_{Treat}}{MS_{Error}} = 21.784$
Error	$(19-3)=16$	2948.2	184.2625	
Total	19	14990.8		

For $\alpha = 0.05$, $F_{\alpha, 3, 16} = 3.24$

So, $F_{cal} > F_\alpha$, then the treatment is significant.

No, The amount of noise are different for all four design.

(b) Residuals are: $y_{ij} - \bar{y}_{i0}$

Circuit design	Residuals Noise observed				
1	−0.2	0.8	−0.2	10.8	−11.2
2	10	−9	0.3	−14	10
3	10.4	−10.6	−11.6	−1.6	13.4
4	15.2	−33.8	3.2	−1.8	17.2

We are to check whether the assumption is true or not.

Assumption: $e_{ij} \sim NID(0, \sigma^2)$

(c) Circuit design-1 is best. (minimum $\bar{y}_{i0}$)

Circuit design

To check normality, we can plot a histogram.

Minimum value → −33.8
Maximum value → 17.2
Total observation → 20

$$\text{Class width} = \frac{(17.2) - (-33.8)}{5} = \frac{51}{5} \approx 11$$

class interval	Frequency
−33.85 to −22.85	1
−22.85 to −11.85	1
−11.85 to −0.85	6
−0.85 to 10.15	7
10.15 to 21.85	5

[Plot is done is graph sheet]

(c)

Circuit design	Average noise
1	19.2 (lowest)
2	70
3	36.6
4	79.8

Circuit design '1' is best design to use.

2.

$k = 3$
$n = 4$

Solution	Days 1	2	3	4	y_{io}
1	13	22	18	39	92
2	16	24	17	44	101
3	5	4	1	22	32
y_{oj}	34	50	36	105	$y_{oo} = 225$

Model: $y_{ij} = \mu + \tau_i + \beta_j + e_{ij}$

$i = 1(1)3,$
$j = 1(1)4$

$$CF = \frac{y_{oo}^2}{k \times n} = \frac{225^2}{12} = 4218.75$$

$$SS_{Treatment} = \frac{\sum y_{io}^2}{n} - CF$$

$$= \frac{(92)^2 + (101)^2 + (32)^2}{4} - 4218.35$$

$$= 703.9$$

$$SS_{Block} = \frac{\sum y_{oj}^2}{k} - CF$$

$$= \frac{(34)^2 + (50)^2 + (36)^2 + (105)^2}{3} - 4218.35$$

$$= 1106.92$$

$$SS_{Total} = \frac{\sum\sum y_{ij}^2}{1} - CF = 6081 - 4218.35 = 1862.25$$

$$SS_E = SS_T - SS_{Treat} - SS_{Block} = 51.83$$

ANOVA Table:-

Source	DF	SS	MS $\left(\frac{SS}{DF}\right)$	Fcal
Treatment	2	703.9	351.75	40.721
Block	3	1106.92	368.97	42.719
Error	6	51.83	8.638	
Total	11	1862.25		

$F_{0.05,2,6} = 5.14$, $F_{0.05,3,6} = 4.76$

So, we reject the hypothesis.

$F_{cal} > F_\alpha$, implying both treatment & block have significant effect.

3.

Batch	Day 1	2	3	4	5	Row sum
1	A=8	B=7	D=1	C=7	E=3	26
2	C=11	E=2	A=7	D=3	B=8	31
3	B=4	A=9	C=10	E=1	D=5	29
4	D=6	C=8	E=6	B=6	A=10	36
5	E=4	D=2	B=3	A=8	C=8	25
Column sum	33	28	27	25	34	147

$$CF = \frac{y_{00}^2}{a^2} = \frac{(147)^2}{25} = 864.36$$

$$SS_{Total} = \sum y_{ij}^2 - CF = 1071 - 864.36 = 206.64$$

$$SS_{Row} = \frac{\sum y_{i0}^2}{a} - CF = \frac{(26)^2 + 31^2 + 29^2 + 36^2 + 25^2}{5} - 864.36$$

$$= 15.44$$

$$SS_{Col} = \frac{\sum y_{0j}^2}{a} - CF = \frac{(33)^2 + 28^2 + 27^2 + 25^2 + 34^2}{5} - 864.36$$

$$= 12.24$$

$$SS_{Treatment} = \frac{(\text{Sum of A's})^2 + (\text{Sum of B's})^2 + \cdots + (\text{Sum of E's})^2}{5} - CF$$

$$= 1005.8 - 864.36$$

$$= 141.44$$

$$SS_E = SS_{Total} - SS_R - SS_C - SS_{Tr} = 37.52$$

Treat	A	B	C	D	E
Average	8.4	5.6	8.8	3.4	3.2

∵ Reaction time is lower the better. So, ingradient E is better.

ANOVA Table:-

Source	DF	SS	MS	Fcal	$F_{0.05,4,12}$
Treatment	4	141.44	35.36	11.31 >	3.26
Row	4	15.44	3.86	1.233 <	3.26
Column	4	12.24	3.06	0.977 <	3.26
Error	12	37.52	3.13		
Total	24	206.64			

So, $F_{cal} > F_\alpha$ for treatment showing five different ingridients (A, B, C, D, E) on reaction time has significant effects. Others have insignificant effect.

4.

A	B		Replication I	II	III	IV	V	VI	Total
−	−	(1)	21	22	23	28	20	26	140
−	+	(b)	37	39	38	38	35	36	223
+	−	(a)	25	26	24	25	29	27	156
+	+	(ab)	31	34	29	33	30	35	192
									711

$n = 6$

$$SS_A = \frac{[(a) + (ab) - (b) - (1)]^2}{4n} = \frac{18.75}{2} = 9.375$$

$$SS_B = \frac{[(b) + (ab) - (a) - (1)]^2}{24} = \frac{1180.083}{2} = 590.04$$

$$SS_{AB} = \frac{[(1) + (ab) - (a) - (b)]^2}{24} = \frac{184.083}{2} = 92.04$$

$$CF = \frac{505521}{24} = 21063.37$$

$$SS_T = \sum y_{ijk}^2 - \frac{(y_{000})^2}{24}$$

$$= 21857 - 21063.37 = 793.63$$

$$SS_E = 102.165$$

ANOVA Table :-

Source	DF	SS	MS	F_{cal}	$F_{\alpha,1,20}$
A	1	9.375	9.395	1.835 <	4.35
B	1	590.04	590.04	115.51 >	4.35
AB	1	92.04	92.04	18.02 >	4.35
Error	20	102.165	5.10835		
Total	23	793.63			

So, Time & interaction have significant effect.

So, with the change of time, growth of virus changes and also there is a relationship between culture media and time.

5. (a)

Temp. \ Process	B_1	B_2	B_3	y_{i00}
A_1	90.1, 180.6, 90.2	90.7, 181.3, 90.6	90.2, 180.6, 90.4	542.5
A_2	90.1, 180.4, 90.3	90.5, 181.1, 90.6	89.9, 180, 90.1	541.5
A_3	90.5, 181.2, 90.7	90.8, 181.7, 90.9	90.4, 180.5, 90.1	543.1
y_{0j0}	542.2	544.1	541.1	$y_{000} = 1627.4$

$a = 3,\ b = 3,\ n = 2$

$$CF = \frac{y_{000}^2}{abn} = 147135.04$$

$$SS_{Total} = \sum_i \sum_j \sum_k y_{ijk}^2 - CF = 147136.34 - 147135.04 = 1.3$$

$$SS_A = \frac{1}{bn} \sum_{i=1}^{3} y_{i00}^2 - CF = 147135.34 - 147135.04 = 0.30$$

$$SS_B = \frac{1}{an} \sum_{i=1}^{3} y_{0j0}^2 - CF = 0.77$$

$$SS_{AB} = \frac{1}{n} \sum\sum y_{0j0}^2 - \frac{y_{000}^2}{abn} - SS_A - SS_B$$

$$= 0.07$$

$$SS_{Error} = SS_T - SS_A - SS_B - SS_{AB} = 0.16$$

ANOVA Table:-

Source	DF	SS	MS	Fcal	F_{α, n_1, n_2}	Conclusion
A	2	0.30	0.15	8.427	4.26	significant
B	2	0.77	0.385	21.629	4.26	significant
AB	4	0.07	0.0175	0.983	3.63	not significant
Error	9	0.16	0.0178			
Total	17	1.3				

Temperature & process both have significant effect.

(b) Residual plot.

Residuals :- $e_{ijk} = y_{ijk} - \bar{y}_{ij}$

A	B	I	II
−	−	0.1	−0.1
−	0	0.05	−0.05
−	+	−0.1	0.1
0	−	−0.1	0.1
0	0	−0.05	0.05
0	+	−0.1	0.1
+	−	−0.1	0.1
+	0	−0.05	0.05
+	+	0.15	−0.15

Please [See next page for Normal Probability Plot]

(c)

Main effect table :-

	1	2	3
A	90.42	90.25	90.57
B	90.37	90.68	90.18

Now, for yield, higher is the better, so for A level 3, i.e. 170°C and for B level 2, i.e. 215 psig is optimum level. Under this condition we will operate the process.

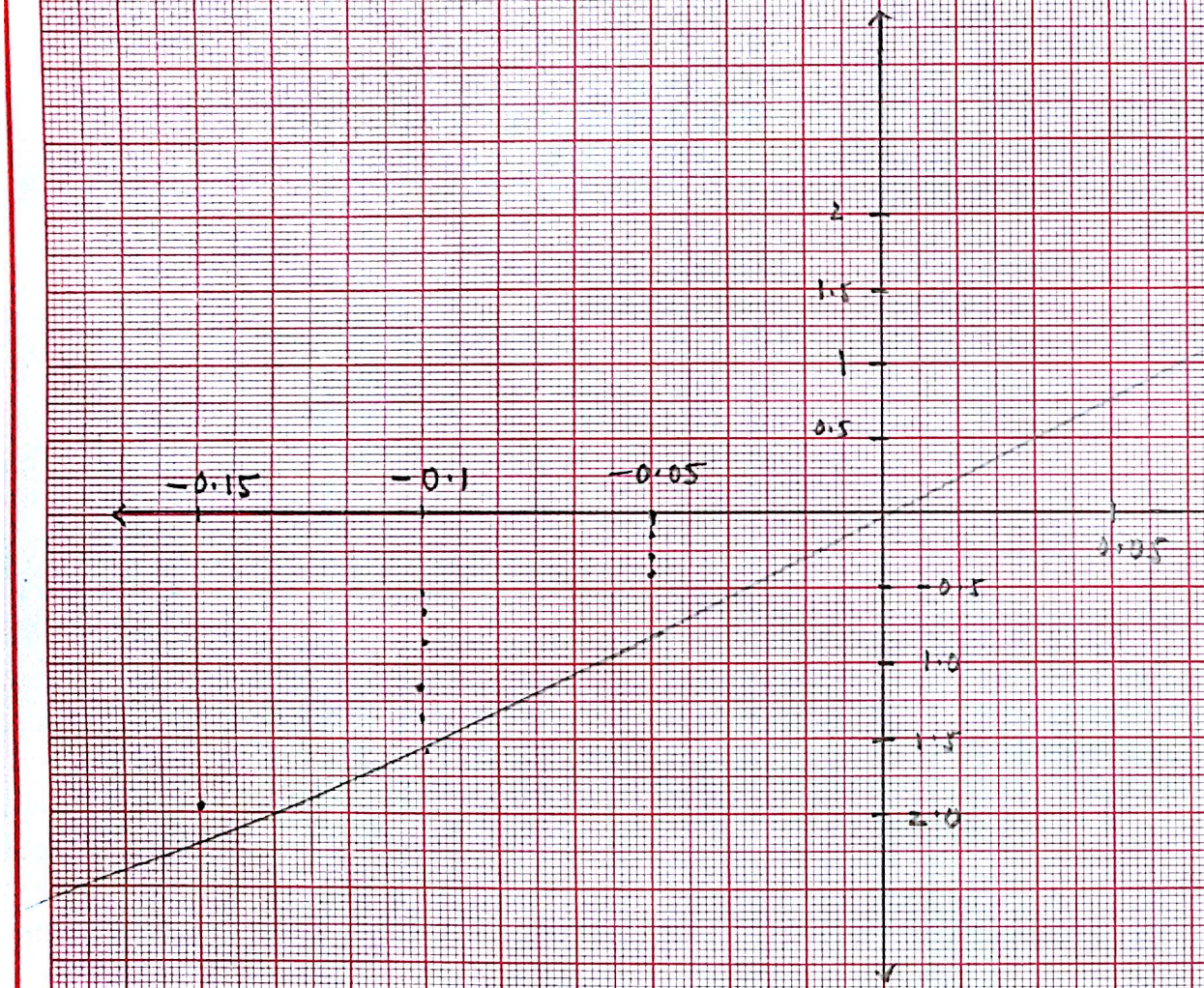
5. (b)

i	z_i	$m_i = \frac{i-0.5}{n}$ = mean rank	$y_i = \Phi^{-1}(m_i)$
1	−0.15	0.0278	−1.915
2	−0.1	0.0833	−1.383
3	−0.1	0.1389	−1.085
4	−0.1	0.1944	−0.862
5	−0.1	0.24	−0.694
6	−0.1	0.3055	−0.508
7	−0.05	0.3611	−0.355
8	−0.05	0.4167	−0.210
9	−0.05	0.4722	−0.07
10	0.05	0.5278	0.07
11	0.05	0.5833	0.21
12	0.05	0.6389	0.355
13	0.1	0.6944	0.508
14	0.1	0.75	0.694
15	0.1	0.8065	0.862
16	0.1	0.8611	1.085
17	0.1	0.9167	1.383
18	0.15	0.9722	1.915

Scale:-

Xaxis: 3 cms = 0.05 units

Yaxis:- 10 cm = 0.5 unit



The points can be approximated by a straight line, it is valid to assume that residual data are following normal distribution.

CONFOUNDING IN FACTORIAL EXPERIMENTS

Introduction

When the number of factors and/or levels of the factors increase, the number of treatment combinations increase very rapidly and it is not possible to accommodate all these treatment combinations in a single homogeneous block. For example, a 2^5 factorial would have 32 treatment combinations and blocks of 32 plots are quite big to ensure homogeneity within them. A new technique is therefore necessary for designing experiments with a large number of treatments. One such device is to take blocks of size less than the number of treatments and have more than one block per replication. The treatment combinations are then divided into as many groups as the number of blocks per replication. The different groups of treatments are allocated to the blocks.

There are many ways of grouping the treatments into as many groups as the number of blocks per replication. It is known that for obtaining the interaction contrast in a factorial experiment where each factor is at two levels, the treatment combinations are divided into two groups. Such two groups representing a suitable interaction can be taken to form the contrasts of two blocks each containing half the total number of treatments. In such case the contrast of the interaction and the contrast between the two block totals are given by the same function. They are, therefore, mixed up and can not be separated. In other words, the interaction has been confounded with the blocks. Evidently the interaction confounded has been lost but the other interactions and main effects can now be estimated with better precision because of reduced block size. This device of reducing the block size by taking one or more interaction contrasts identical with block contrasts is known as confounding. Preferably only higher order interactions, that is, interactions with three or more factors are confounded, because their loss is immaterial. As an experimenter is generally interested in main effects and two factor interactions, these should not be confounded as far as possible.

When there are two or more replications, if the same set of interactions are confounded in all the replications, confounding is called complete and if different sets of interaction are confounded in different replications, confounding is called partial. In complete confounding all the information on confounded interactions are lost. But in partial Confounding in Factorial Experiments and Fractional Factorials confounding, the confounded interactions can be recovered from those replications in which they are not confounded.

Confounding is used to indicate that the value of a main effect estimate comes from both the main effect itself and also bias from higher order interaction.

Advantages of Confounding

- It reduces the experimental error considerably by stratifying the experimental material into homogeneous subsets or subgroups.
- The removal of the variation among incomplete blocks (freed from treatments) within replicates results in smaller error mean square as compared with a RBD, thus making the comparisons among some treatment effects more precise.

Disadvantages of Confounding

- In the confounding scheme, the increased precision is obtained at the cost of sacrifice of information (partial or complete) on certain relatively unimportant interactions.
- The confounded contrasts are replicated fewer times than are the other contrasts and as such there is loss of information on them and they can be estimated with a lower degree of precision as the number of replications for them is reduced.
- An indiscriminate use of confounding may result is complete or partial loss of information on the contrasts or comparisons of greatest importance. As such the experimenter should confound only those treatment combinations or contrasts which are of relatively less or of importance at all.
- The algebraic calculations are usually more difficult and the statistical analysis is complex, especially when some of the units (observations) are missing.
- A number of problems arise if the treatments interact with blocks.

Confounding in 2^3 Experiment

Although 23 is a factorial with small number of treatment combinations but for illustration purpose, this example has been considered. Let the three factors be A, B, C each at two levels.

Factorial Effects → Treat. Combinations ↓	A	B	C	AB	AC	BC	ABC
(1)	-	-	-	+	+	+	-
(a)	+	-	-	-	-	+	+
(b)	-	+	-	-	+	-	+
(ab)	+	+	-	+	-	-	-
(c)	-	-	+	+	-	-	+
(ac)	+	-	+	-	+	-	-
(bc)	-	+	+	-	-	+	-
(abc)	+	+	+	+	+	+	+

The various factorial effects are as follows:

A = (abc) + (ac) + (ab) + (a) - (bc) - (c) - (b) - (1)
B = (abc) + (bc) + (ab) + (b) - (ac) - (c) - (a) - (1)
C = (abc) + (bc) + (ac) + (c) - (ab) - (b) - (a) - (1)
AB = (abc) + (c) + (ab) + (1) - (bc) - (ac) - (b) - (a)
AC = (abc) + (ac) + (b) + (1) - (bc) - (c) - (ab) - (a)
BC = (abc) + (bc) + (a) + (1) - (ac) - (c) - (ab) - (b)
ABC = (abc) + (c) + (b) + (a) - (bc) - (ac) - (ab) - (1)

Let the highest order interaction ABC be confounded and we decide to use two blocks of 4 units (plots) each per replicate.

Thus in order to confound the interaction ABC with blocks all the treatment combinations with positive sign are allocated at random in one block and those with negative signs in the other block. Thus the following arrangement gives ABC confounded with blocks and hence we loose information on ABC.

Replication I

Block 1: (1) (ab) (ac) (bc)
Block 2 : (a) (b) (c) (abc)

It can be observed that the contrast estimating ABC is identical to the contrast estimating block effects.

The other six factorial effects viz. A, B, C, AB, AC, BC each contain two treatments in block 1 (or 2) with the positive signs and two with negative sign so that they are orthogonal with block totals and hence these differences are not influenced among blocks and can thus be estimated and tested as usual without any difficulty. Whereas for confounded interaction, all the treatments in one group are with positive sign and in the other with negative signs.

Similarly if AB is to be confounded, then the two blocks will consists of

Block 1 (abc) (c) (ab) (1)
Block 2 (bc) (ac) (b) (a)

Here AB is confounded with block effects and cannot be estimated independently whereas all other effects A, B, C, AC, BC and ABC can be estimated independently.

Resolution III Design: No main effect is confounded with other main effects but main effects are confounded with 2-factor interactions.

Resolution IV Design: No main effect is confounded with other main effects and 2-factor interactions but 2-factor interactions are confounded with each other.

Resolution IV Design: No main effect is confounded with other main effects but main effects are confounded with 2-factor interactions.

Orthogonal array:-

L_4 → L: OA, 4: No. of experiments.

$L_4(2^3)$ → 3: no. of columns/factors; 2: no. of levels

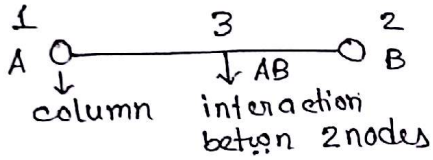
$L_8(2^7)$, $L_{16}(2^{15})$.

In 3^K factorial experiments, $L_9(3^4)$

$L_{27}(3^{13})$ $L_{81}(3^{40})$

Consider $L_{16}(2^{15})$ if we do full factorial experiment, we have to do 2^{15} experiments instead of we are doing only 16 experiments.

In orthogonal array, we can estimate
→ all main effects
→ Selected interactions (only 2-factor interaction)



4 factors each at 2 levels:- AB, AC.

1. Total degree of freedom (TDF) = 6
2. Minimum # expts. = TDF+1 = 7
3. Look at the nearest highest node OA = $L_8(2^7)$.

Method (Steps):—

1. Calculate the TDF for main effect and selected interaction
2. Minimum Number of Experiments = TDF + 1
3. Look for the nearest OA.
4. Required Linear graph (RLG)
5. Identify appropriate standard Linear graph (SLG)
6. Prepare the modified linear graph (MLG)
7. Make allocation of factors and interactions as per as MLG.
8. Prepare master plan (design layout) for experimentation.

Note:- Without error column, we can't do ANOVA.
In L_8; If we need all 7 df in main & interaction effect, then we can't do ANOVA. Then we have to replicate so that error column comes in ANOVA.

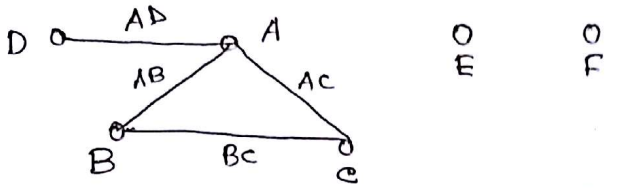
Ex.1. A, B, C, D, E, F, AB, AC, AD, BC all are at 2-levels.

Sol. 1. Total degree of freedom = TDF = 10

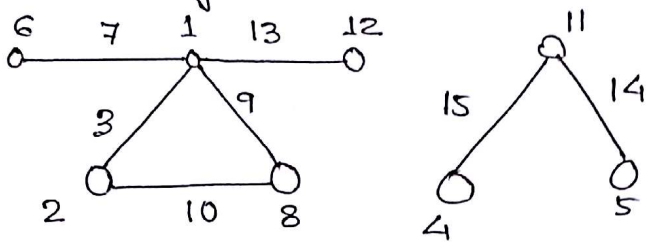
2. Minimum Number of Experiments = MNE = TDF + 1 = 11.

3. Nearest orthogonal array = L_{16}

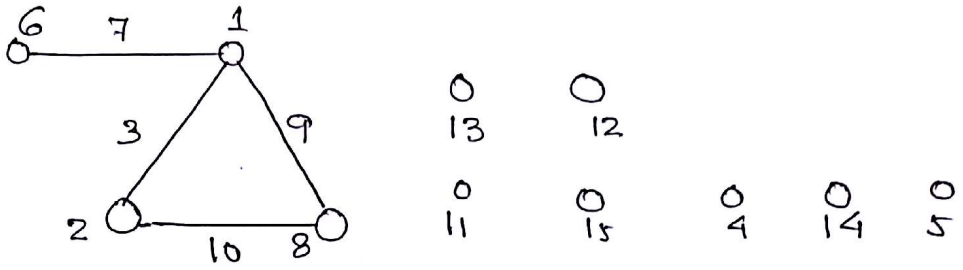
4. Required Linear graph = RLG :-



5. Standard Linear graph (SLG) :- [See $L_{16}(2^{15})$ (3)]



6. Modified Linear Graph (MLG) :-



7. Allocation :-

A → 1, B → 2, C → 8, D → 6, AB → 3, BC → 10, AC → 9, AD → 7, E → 11, F → 12 ; Errors → 4, 5, 13, 14, 15

8. Master plan :-

Exp No.	A 1	B 2	D 6	C 8	E 11	F 12
1	1	1	1	1	1	1
2	1	1	1	2	2	2
3	1	1	2	1	1	2
⋮	⋮	⋮	⋮	⋮	⋮	⋮
16						

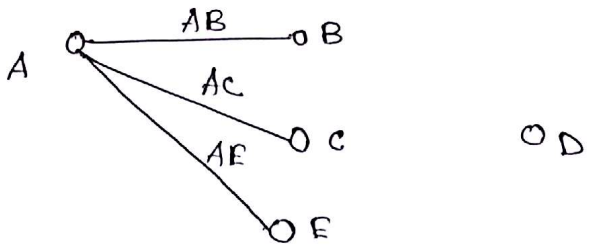
Ex.2. A, B, C, D, E, AB, AC, AE, all are at 3 levels.

Sol. 1. TDF = 2+2+2+2+2+4+4+4 = 22

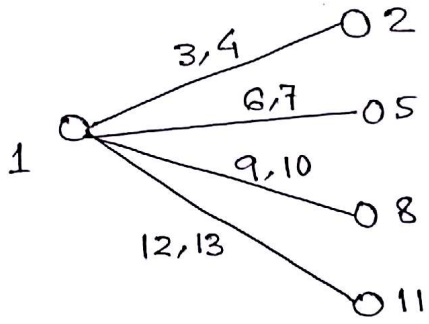
2. MNE = TDF + 1 = 23

3. Nearest OA: L_{27}

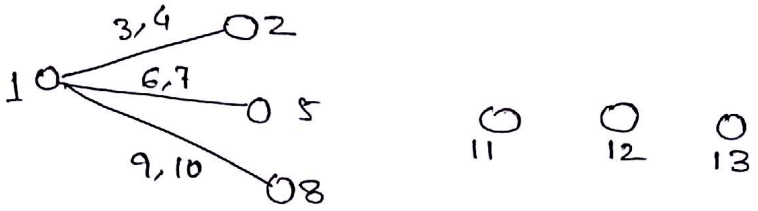
4. RLG:



5. SLG :- $L_{27}(3^{13})$ (2)(a)



6. MLG :-



7. Allocation:- A → 1, B → 2, C → 5, E → 8, A;B → 3, 4
A,C → 6, 7, A,E → 9, 10, D → 11

8.

Exp No	A 1	B 2	AB 3 4	C 5	AC 6 7	E 8	AE 9 10	D 11	En 12	En 13	En 14	En 15
1												
2												
:												
27												

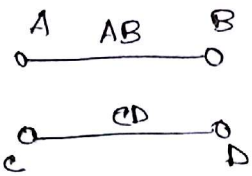
Master Plan:-

Exp No	A 1	B 2	C 5	E 8	D 11
1					
2					
:					
27					

Ex.3. A, B, C, D all at 2-levels, AB, CD

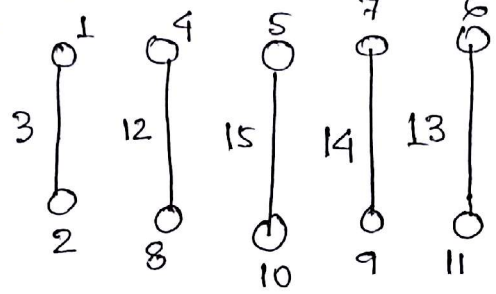
Sol.

1. TDF = 1+1+1+1+1+1 = 6
2. MNE = 6+1 = 7
3. OA: L_{16}
4. RLG :-



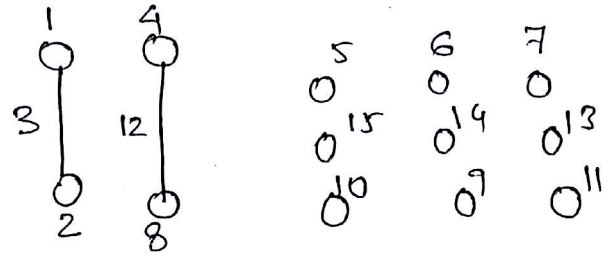
[L_8 not possible since this RLG does not match with any Linear graph in L_8]

5. SLG:-



Here L_{16} is 2^4 equal to full factorial. In this case, OA can't help in reducing expts.

6. MLG:-



7. Allocation:- A → 1, C → 4, B → 2, D → 8, AB → 3, CD → 12

8.

Exp. No.	A 1	B 2	AB 3	C 4	Er 5 6 7	D 8	Er 9 10 11	CD 12	Er 13 14 15
1									
⋮									
16									

Master plan:-

Exp	A 1	B 2	C 4	D 8
1				
2				
⋮				
16				

Dummy Level Technique:- Allocation of a lower level factor to higher level Orthogonal array.

A is at 2 level and B C D at 3 level.

Sol. TDF = 7, MNE = 8

A_1, A_2, A_3 → dummy level (to put in L9)

Exp No.	A 1	B 2	C 3	D 4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	1'	1	3	2
8	1'	2	1	3
9	1'	3	2	1

→ lower dummy level

Collapsing method:- A → 4 levels, B, C → 2 levels

TDF = 3 + 2 = 5

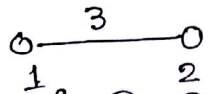
MNE = 6 L8

Each column have 1 df. combine 3 columns to get 3 df.

Hoco to combine?

→ A/c Taguchi, take any two column and their interaction, if 1 and 2 then 3 is their interaction. This method is called collapsing

	A			Er 4	Er 5	B 6	C 7
Exp No	1	2	3				
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2



Combination of ①, ② mapping:-

$(1,1) \rightarrow 1$
$(1,2) \rightarrow 2$
$(2,1) \rightarrow 3$
$(2,2) \rightarrow 4$

Exp. No	A (1 2 3)	4	5	B 6	7
1	1				
2	1				
3	2				
4	2				
5	3				
6	3				
7	4				
8	4				

Ex. A → 3 levels and B, C, D, E → 2 levels
Combination of collapsing and dummy technique

L_8

Exp	A (1 2 3)	B 4	C 5	D 6	F 7
1	1				
2	1				
3	2				
4	2				
5	3				
6	3				
7	1′				
8	1′				

Drawback of Taguchi Method:-

1. There is no mathematical explanation of using logarithmic function.
2. No statistical justification has been given there.

SIGNAL TO NOISE RATIO:- (S/N Ratio):-

σ is variation (harmful to system): so it is noise

μ is signal

Smaller the better:- $\eta = -10\log\left\{\frac{1}{n}\sum_{i=1}^{n} y_i^2\right\}$; y_{ij}, $j=1,2,\ldots,n$ is the response of ith expt.

Larger the better:- $\eta = -10\log\left\{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right\}$

Nominal the best:- $\eta = 10\log_{10}\left(\frac{\mu^2}{\sigma^2}\right)$

For Fraction defective:- (Discrete case) $\eta = -10\log_{10}\left(\frac{p}{1-p}\right)$

According to taguchi S/N Ratio reduces variation.

For discrete case:- Let for expt. No.1; we check 10 items; let we find out $x < 10$ defective pieces, then $p = \frac{x}{10}$ and SN Ratio be $-10\log\left(\frac{p}{1-p}\right)$

Note:- Factors which affect the variability and factor which effect in the average may be different. In ANOVA, we want to see the average & S.N. Ratio talks about variation. So, results in ANOVA & S.N. Ratio analysis may be different.

Ex.1. Higher the better case: $S/N = \eta = -10\log\left\{\frac{1}{n}\sum\frac{1}{y_i^2}\right\}$ → not ln

Exp No.	A	B	C	D	I	II	S/N
1	1	1	1	1	4.5	4.8	13.3354
2	1	1	2	2	5	4.7	13.7023
3	1	2	1	2	5.7	6	15.3345
4	1	2	2	1	4.6	4.4	13.0574
5	2	1	1	2	3.3	3.2	10.2345
6	2	1	2	1	2.5	2.2	7.3682
7	2	2	1	1	3.1	3.1	9.8272
8	2	2	2	2	3.9	4.2	12.1312

Total = Sum of all S/N values = 94.99

$$CF = \frac{(94.99)^2}{8} = 1127.9136$$

$$SS_{Total} = \sum (S.N.values)^2 - CF = 1174.0142 - CF = 46.1006$$

$$SS_A = \frac{A_1^2 + A_2^2}{4} - CF$$

$$= \frac{(13.3354 + 13.7023 + 15.3345 + 13.0578)^2 + (\ldots + \ldots)^2}{4} - CF$$

$$= 31.4777$$

$SS_B = 4.0759$

$SS_C = 0.7639$

$SS_D = 7.6321$

$SS_{Error} = SS_T - SS_A - SS_B - SS_C - SS_D = 1.521$

ANOVA Table:-

Source	DF	SS	MS	F_{cal}	f_{tab} for $\alpha = 0.05$
A	1	31.4777	31.4777	43.9019	> 10.13
B	1	4.0759	4.0759	5.6846	< 10.13
C	1	0.7639	0.7639	1.0654	< 10.13
D	1	7.6321	7.6321	10.6445	> 10.13
Error	3	2.151	0.717		
Total	7	46.1006			

A, D are significant.
B, C are not significant.

In SN Ratio, for higher the better or lower the better case, we always have to see maximum average b coz we maximize here in the ratio.

Main effect table:-

	1	2
A	13.85	9.8902
B	13.3354	12.5878
C		
D	10.8471	12.8506

Resolution III Design:- No main effect is aliased with any other main effect but main effects are aliased with 2-factor interaction. 2^{3-1} ; ABC

Resolution IV design:- No main effect is aliased with any other main effect or 2-factor interaction but 2-factor interactions may be aliased with each other. 2^{4-1} ; ABCD

Resolution V design:- No main effect or 2-factor interaction is aliased with any other main effect or 2-factor interaction but 2-factor interaction is aliased with 3-factor interaction.

2^{5-1} ; ABCDE.

Response Surface Methodology:- Useful for modelling & analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize the response. For example, suppose a chemical engineer wishes to find the levels of temperature (x_1) and pressure (x_2) that maximize the yield of a process.

$$y = f(x_1, x_2) + \epsilon$$

ϵ: noise or error observed in the response y.

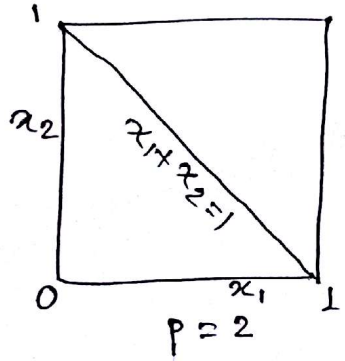
$E(y) = f(x_1, x_2) = \eta$ then surface represented by $\eta = f(x_1, x_2)$ is called response surface. And designs for fitting response surfaces is called response surface design.

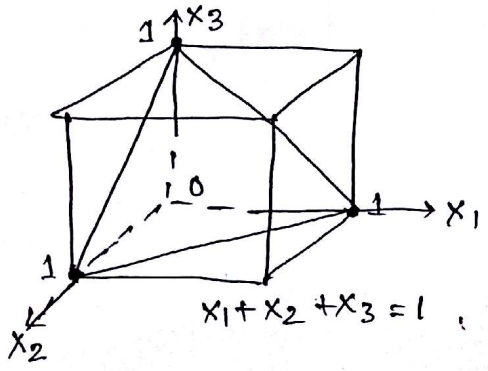
Mixture Design:- In mixture experiments, the factors are the components or ingredients of a mixture, and consequently, their levels are not independent. For example, if $x_1, x_2, \ldots, x_p$ denote the proportions of p-components of a mixture, then

$$0 \le x_i \le 1 \quad , i = 1, 2, \ldots, p$$

and

$$x_1 + x_2 + \cdots + x_p = 1.$$





Orthogonal Arrays : 2 Series

L_4 Array

Trial no.	Column no. 1	2	3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

L4 Triangular Table (Interactions)

Column no.	2	3
1	3	2
2	—	1

L_8 Array

Trial no.	Column no. 1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

L8 Triangular Table (Interactions)

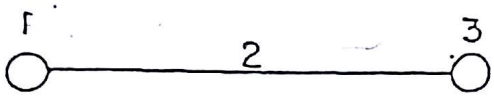
Column no.	2	3	4	5	6	7
1	3	2	5	4	7	6
2	—	1	6	7	4	5
3	—	—	7	6	5	4
4	—	—	—	1	2	3
5	—	—	—	—	3	2
6	—	—	—	—	—	1

Orthogonal Arrays : 2^n Series

LINEAR GRAPH

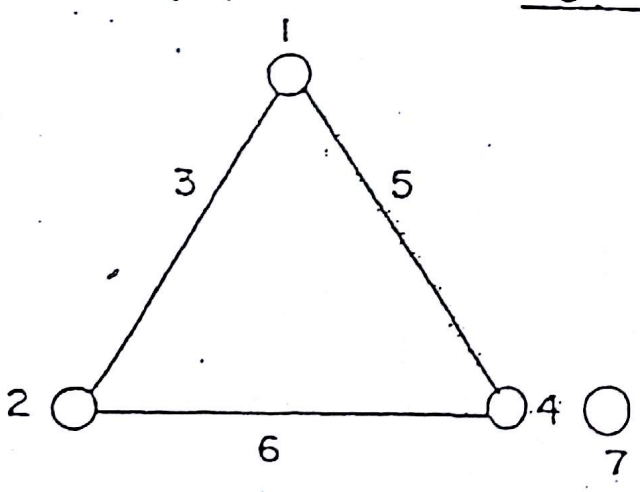
$L_4(2^3)$

(1)

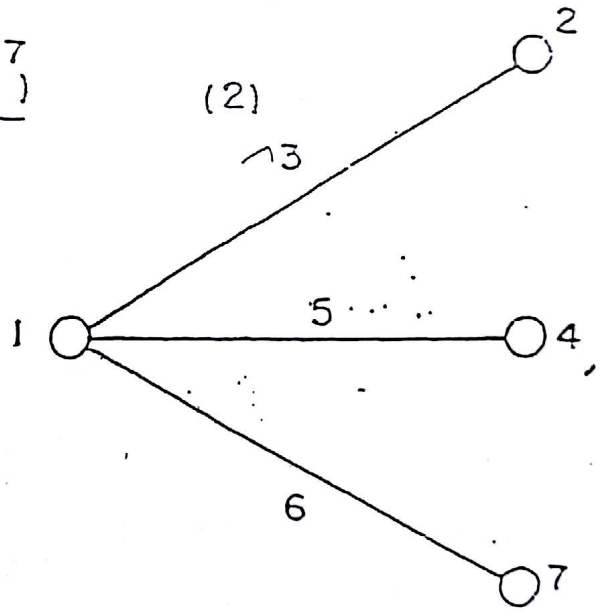


$L_8(2^7)$

(1)



(2)



Orthogonal Arrays : 2^n Series

L_{16} Array

Trial no.	Column no. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	1	1	2	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

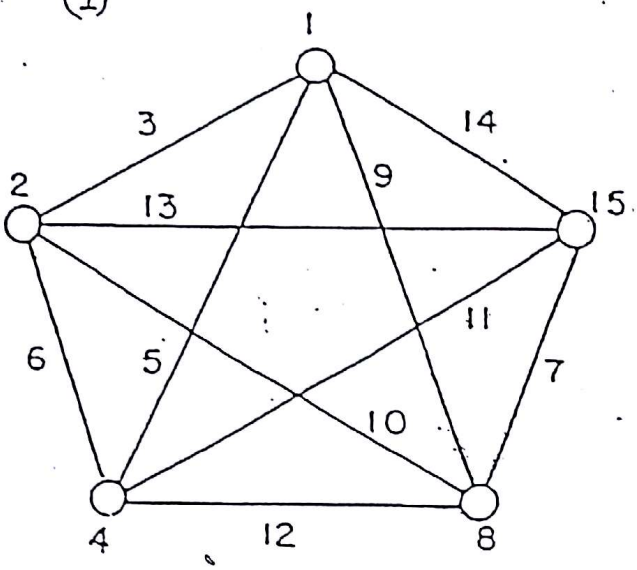
L16 Triangular Table (Interactions)

Column no.	Column no. 2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	3	2	5	4	7	6	9	8	11	10	13	12	15	14
2	—	1	6	7	4	5	10	11	8	9	14	15	12	13
3	—	—	7	6	5	4	11	10	9	8	15	14	13	12
4	—	—	—	1	2	3	12	13	14	15	8	9	10	11
5	—	—	—	—	3	2	13	12	15	14	9	8	11	10
6	—	—	—	—	—	1	14	15	12	13	10	11	8	9
7	—	—	—	—	—	—	15	14	13	12	11	10	9	8
8	—	—	—	—	—	—	—	1	2	3	4	5	6	7
9	—	—	—	—	—	—	—	—	3	2	5	4	7	6
10	—	—	—	—	—	—	—	—	—	1	6	7	4	5
11	—	—	—	—	—	—	—	—	—	—	7	6	5	4
12	—	—	—	—	—	—	—	—	—	—	—	1	2	3
13	—	—	—	—	—	—	—	—	—	—	—	—	3	2
14	—	—	—	—	—	—	—	—	—	—	—	—	—	1

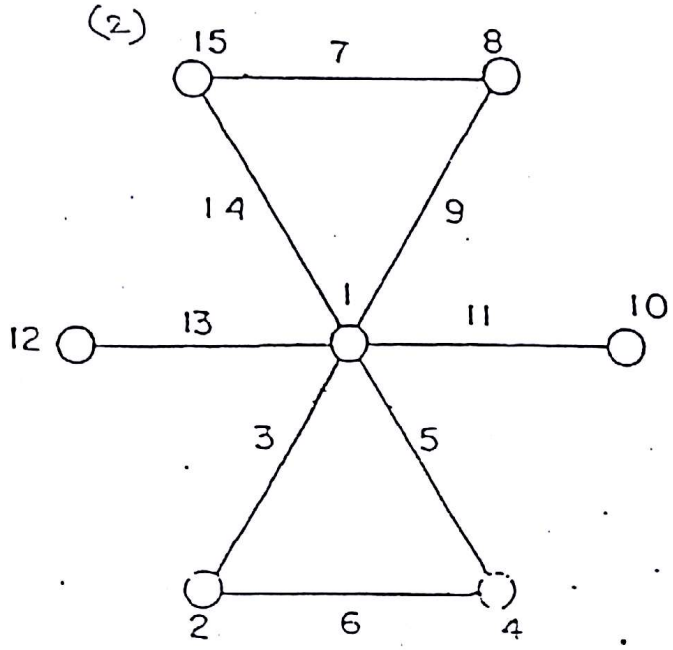
Orthogonal Arrays : 2^n Series

$L_{16}(2^{15})$

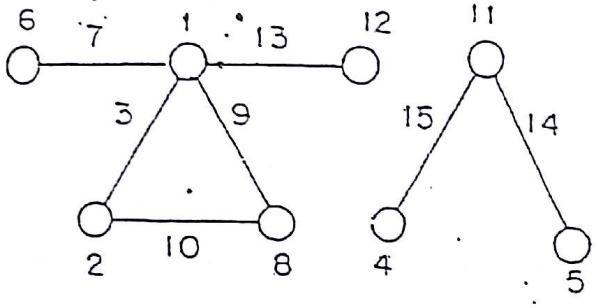
(1)



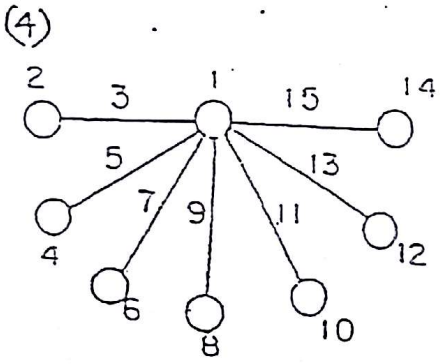
(2)



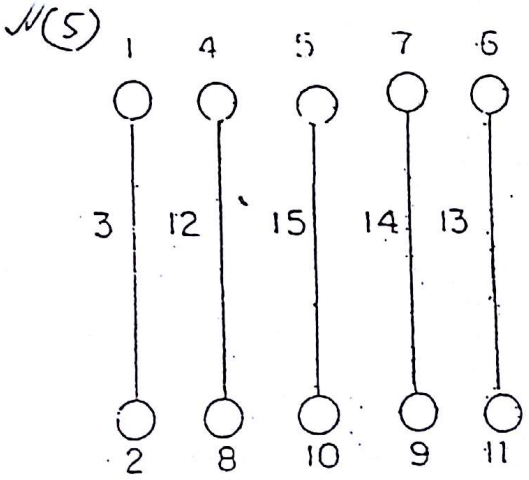
(3)



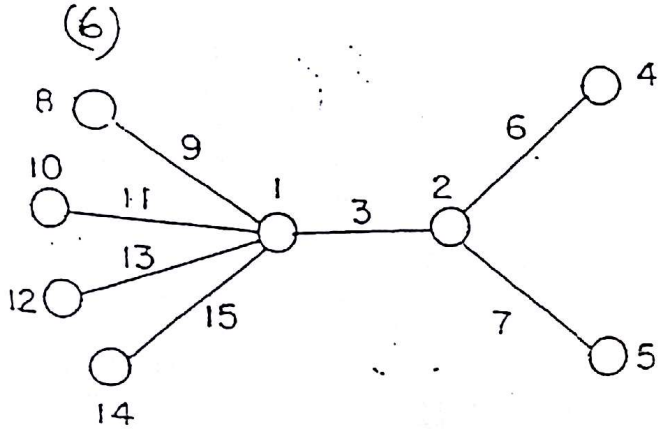
(4)



(5)



(6)

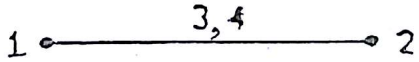


Orthogonal Arrays : 3^n Series

L_9 Array

Trial no.	Column no. 1	2	3	4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

L9 Linear Graph



Orthogonal Arrays : 3^n Series

L 27 Array

Trial no	Column no.												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3	3	3	3	3
4	1	2	2	2	1	1	1	2	2	2	3	3	3
5	1	2	2	2	2	2	2	3	3	3	1	1	1
6	1	2	2	2	3	3	3	1	1	1	2	2	2
7	1	3	3	3	1	1	1	3	3	3	2	2	2
8	1	3	3	3	2	2	2	1	1	1	3	3	3
9	1	3	3	3	3	3	3	2	2	2	1	1	1
10	2	1	2	3	1	2	3	1	2	3	1	2	3
11	2	1	2	3	2	3	1	2	3	1	2	3	1
12	2	1	2	3	3	1	2	3	1	2	3	1	2
13	2	2	3	1	1	2	3	2	3	1	3	1	2
14	2	2	3	1	2	3	1	3	1	2	1	2	3
15	2	2	3	1	3	1	2	1	2	3	2	3	1
16	2	3	1	2	1	2	3	3	1	2	2	3	1
17	2	3	1	2	2	3	1	1	2	3	3	1	2
18	2	3	1	2	3	1	2	2	3	1	1	2	3
19	3	1	3	2	1	3	2	1	3	2	1	3	2
20	3	1	3	2	2	1	3	2	1	3	2	1	3
21	3	1	3	2	3	2	1	3	2	1	3	2	1
22	3	2	1	3	1	3	2	2	1	3	3	2	1
23	3	2	1	3	2	1	3	3	2	1	1	3	2
24	3	2	1	3	3	2	1	1	3	2	2	1	3
25	3	3	2	1	1	3	2	3	2	1	2	1	3
26	3	3	2	1	2	1	3	1	3	2	3	2	1
27	3	3	2	1	3	2	1	2	1	3	1	3	2

Orthogonal Arrays : 3^n Series

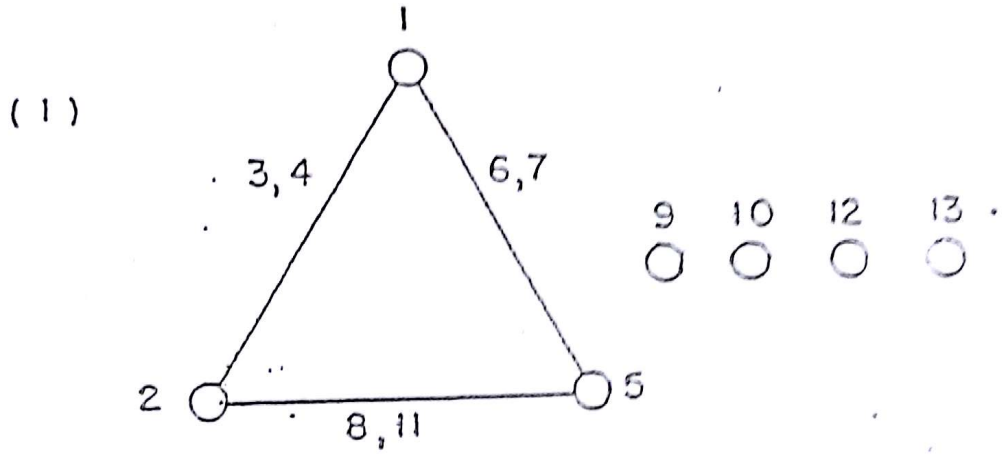
L27 Triangular Table (Interactions)

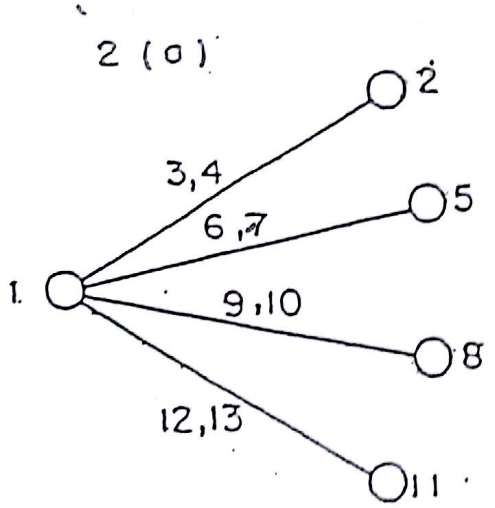
Column no.	Column no. 2	3	4	5	6	7	8	9	10	11	12	13
1	3*	2	2	6	5	5	9	8	8	12	11	11
	4*	4	3	7	7	6	10	10	9	13	13	12
2	—	1	1	8	9	10	5	6	7	5	6	7
	—	4	3	11	12	13	11	12	13	8	9	10
3	—	—	1	9	10	8	7	5	6	6	7	5
	—	—	2	13	11	12	12	13	11	10	8	9
4	—	—	—	10	8	9	6	7	5	7	5	6
	—	—	—	12	13	11	13	11	12	9	10	8
5	—	—	—	—	1	1	2	3	4	2	4	3
	—	—	—	—	7	6	11	13	12	8	10	9
6	—	—	—	—	—	1	4	2	3	3	2	4
	—	—	—	—	—	5	3	12	11	10	9	8
7	—	—	—	—	—	—	3	4	2	4	3	2
	—	—	—	—	—	—	12	11	13	9	8	10
8	—	—	—	—	—	—	—	1	1	2	3	4
	—	—	—	—	—	—	—	10	9	5	7	6
9	—	—	—	—	—	—	—	—	1	4	2	3
	—	—	—	—	—	—	—	—	8	7	6	5
10	—	—	—	—	—	—	—	—	—	3	4	2
	—	—	—	—	—	—	—	—	—	6	7	7
11	—	—	—	—	—	—	—	—	—	—	1	1
	—	—	—	—	—	—	—	—	—	—	13	12
12	—	—	—	—	—	—	—	—	—	—	—	1
	—	—	—	—	—	—	—	—	—	—	—	11

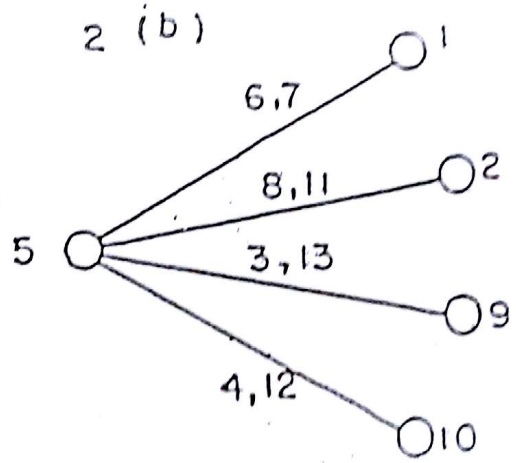
*Pairs of numbers indicate columns that contain the total interaction.

Orthogonal Arrays : 3 Series

$L_{27}(3^{13})$





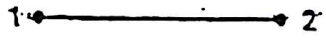


Orthogonal Arrays : Mixed Series

L_{18} Array

	Column no.							
Trial no.	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	2	3	1	2	3	2	1
16	2	3	1	3	2	3	1	2
17	2	3	2	1	3	1	2	3
18	2	3	3	2	1	2	3	1

L18 Linear Graph



Two-way ANOVA of columns 1 and 2 for *only* 1 interaction which has 2 d.f. associated with it.

Orthogonal Arrays : Mixed Series

$L_{36}(2^{11} \times 3^{12})$

No. \ Col. no.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1′	2′	3′	4
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	3	3	3	3	3	3	3	3	3	3	3	3	1	1	1	1
4	1	1	1	1	1	2	2	2	2	2	2	1	1	1	1	2	2	2	2	3	3	3	3	1	2	2	1
5	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	3	3	3	3	1	1	1	1	1	2	2	1
6	1	1	1	1	1	2	2	2	2	2	2	3	3	3	3	1	1	1	1	2	2	2	2	1	2	2	1
7	1	1	2	2	2	1	1	1	2	2	2	1	1	2	3	1	2	3	3	1	2	2	3	2	1	2	1
8	1	1	2	2	2	1	1	1	2	2	2	2	2	3	1	2	3	1	1	2	3	3	1	2	1	2	1
9	1	1	2	2	2	1	1	1	2	2	2	3	3	1	2	3	1	2	2	3	1	1	2	2	1	2	1
10	1	2	1	2	2	1	2	2	1	1	2	1	1	3	2	1	3	2	3	2	1	3	2	2	2	1	1
11	1	2	1	2	2	1	2	2	1	1	2	2	2	1	3	2	1	3	1	3	2	1	3	2	2	1	1
12	1	2	1	2	2	1	2	2	1	1	2	3	3	2	1	3	2	1	2	1	3	2	1	2	2	1	1
13	1	2	2	1	2	2	1	2	1	2	1	1	2	3	1	3	2	1	3	3	2	1	2	1	1	1	2
14	1	2	2	1	2	2	1	2	1	2	1	2	3	1	2	1	3	2	1	1	3	2	3	1	1	1	2
15	1	2	2	1	2	2	1	2	1	2	1	3	1	2	3	2	1	3	2	2	1	3	1	1	1	1	2
16	1	2	2	2	1	2	2	1	2	1	1	1	2	3	2	1	1	3	2	3	3	2	1	1	2	2	2
17	1	2	2	2	1	2	2	1	2	1	1	2	3	1	3	2	2	1	3	1	1	3	2	1	2	2	2
18	1	2	2	2	1	2	2	1	2	1	1	3	1	2	1	3	3	2	1	2	2	1	3	1	2	2	2
19	2	1	2	2	1	1	2	2	1	2	1	1	2	1	3	3	3	1	2	2	1	2	3	2	1	2	2
20	2	1	2	2	1	1	2	2	1	2	1	2	3	2	1	1	1	2	3	3	2	3	1	2	1	2	2
21	2	1	2	2	1	1	2	2	1	2	1	3	1	3	2	2	2	3	1	1	3	1	2	2	1	2	2
22	2	1	2	1	2	2	2	1	1	1	2	1	2	2	3	3	1	2	1	1	3	3	2	2	2	1	2
23	2	1	2	1	2	2	2	1	1	1	2	2	3	3	1	1	2	3	2	2	1	1	3	2	2	1	2
24	2	1	2	1	2	2	2	1	1	1	2	3	1	1	2	2	3	1	3	3	2	2	1	2	2	1	2
25	2	1	1	2	2	2	1	2	2	1	1	1	3	2	1	2	3	3	1	3	1	2	2	1	1	1	3
26	2	1	1	2	2	2	1	2	2	1	1	2	1	3	2	3	1	1	2	1	2	3	3	1	1	1	3
27	2	1	1	2	2	2	1	2	2	1	1	3	2	1	3	1	2	2	3	2	3	1	1	1	1	1	3
28	2	2	2	1	1	1	1	2	2	1	2	1	3	2	2	2	1	1	3	2	3	1	3	1	2	2	3
29	2	2	2	1	1	1	1	2	2	1	2	2	1	3	3	3	2	2	1	3	1	2	1	1	2	2	3
30	2	2	2	1	1	1	1	2	2	1	2	3	2	1	1	1	3	3	2	1	2	3	2	1	2	2	3
31	2	2	1	2	1	2	1	1	1	2	2	1	3	3	3	2	3	2	2	1	2	1	1	2	1	2	3
32	2	2	1	2	1	2	1	1	1	2	2	2	1	1	1	3	1	3	3	2	3	2	2	2	1	2	3
33	2	2	1	2	1	2	1	1	1	2	2	3	2	2	2	1	2	1	1	3	1	3	3	2	1	2	3
34	2	2	1	1	2	1	2	1	2	2	1	1	3	1	2	3	2	3	1	2	2	3	1	2	2	1	3
35	2	2	1	1	2	1	2	1	2	2	1	2	1	2	3	1	3	1	2	3	3	1	2	2	2	1	3
36	2	2	1	1	2	1	2	1	2	2	1	3	2	3	1	2	1	2	3	1	1	2	3	2	2	1	3
	Columns for response analysis, regression analysis												1	2	3	4	5	6	7	8	9	10	11	12			13
	Gp I		Group 2										Group 3														

(Notes)

i. By introducing columns 1′, 2′, 3′, and 4′ in place of columns 1, 2, . . . , 11, one obtains $L_{36}(2^3 \times 3^{13})$.

ii. Since interactions are not orthogonal to other columns in the case of $L_{36}(2^{11} \times 3^{12})$, it is best to avoid assignments to find such interactions.

iii. The assignment type is shown here only for $L_{36}(2^3 \times 3^{13})$.

iv. In Chapter 15 and Chapter 16, the column numbers of the bottom row (column 1–column 13) are used.