# Srinivas Ramanujan Meets PC Mahalanobis

**by**

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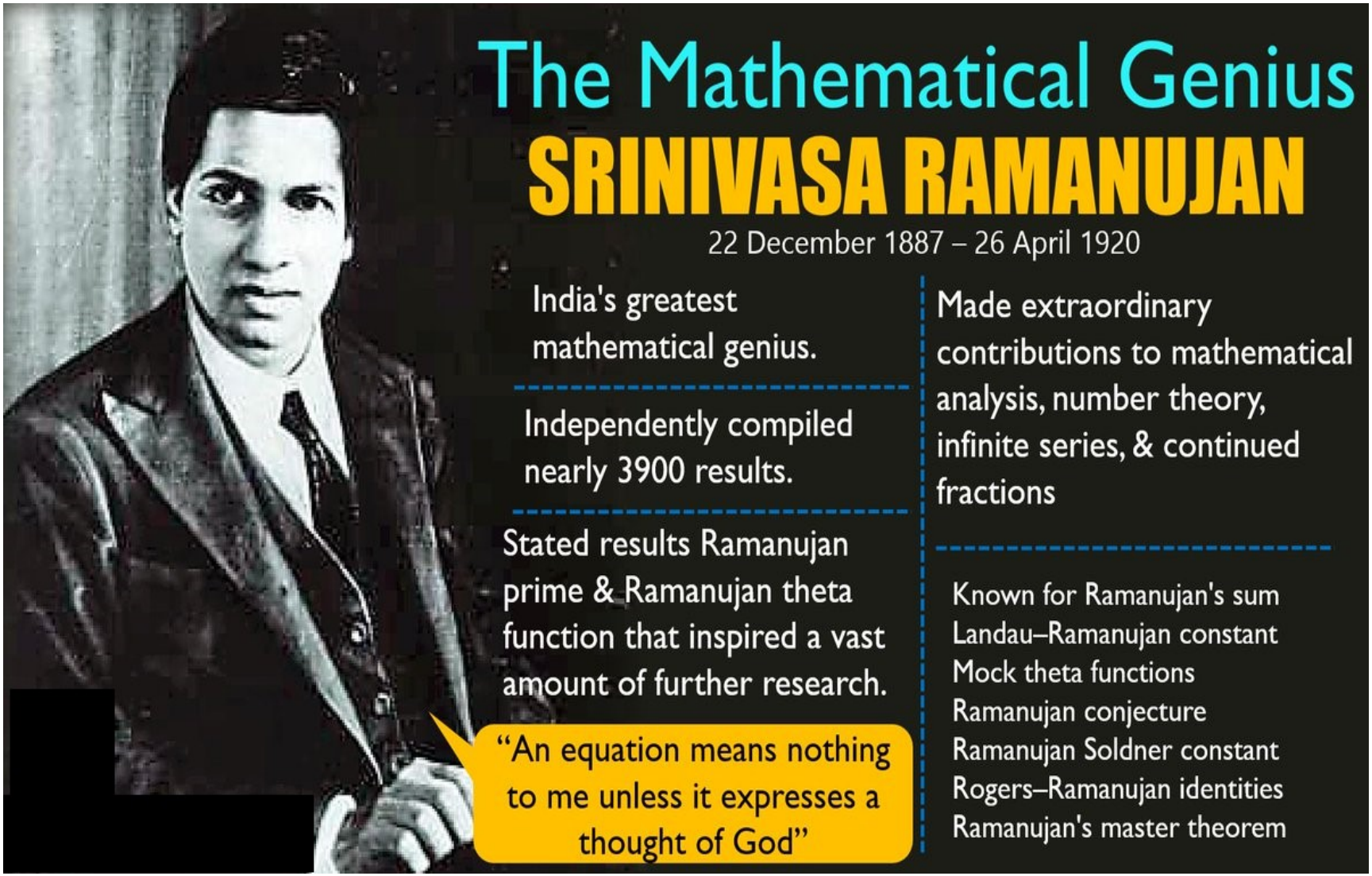
Webinar on National Mathematics Day at Amity University on Dec 22, 2021

# Srinivas Ramanujan (22 Dec 1887 - 26 Apr 1920)

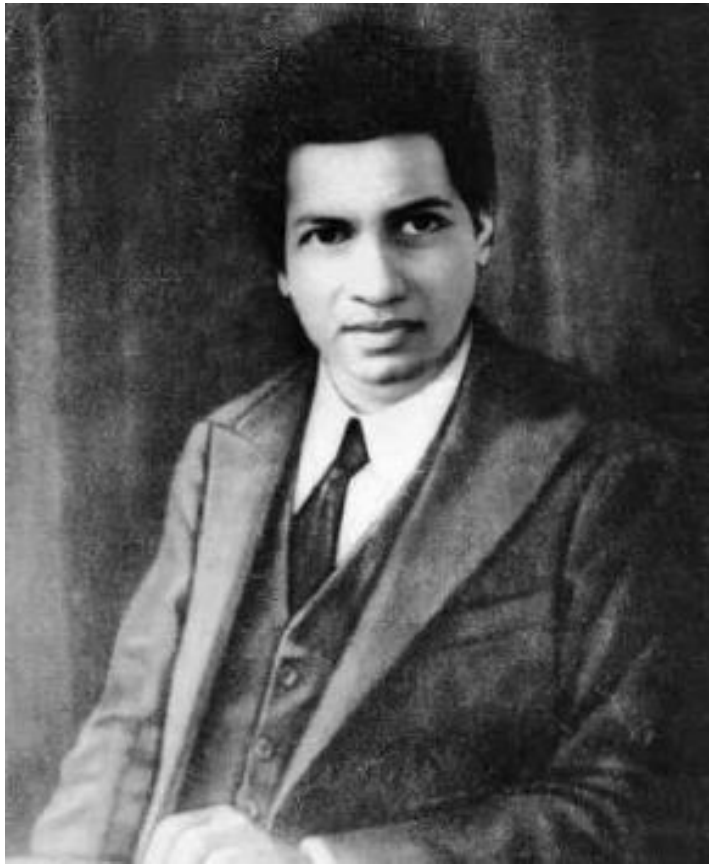


# Basic Information about Ramanujan

- Born on December 22, 1887 in Pallipalayam, Erode and grew up in Kumbakonam, Tamil nadu.
- Son of Srinivas Iyenger and Komalathammal. His mother was a housewife and his father worked as a clerk in a sari shop.
- He did most of his mathematical exploration by his own (In particular he discovered much trigonometry by himself as a 13 years old boy!)
- About his talent,G.H. Hardy, who was known a big mathematician and one of Ramanujan's academic advisors with J.E. Littlewood, said only a few giant mathematicians like Euler,Gauss,Newton had the same talent which Ramanujan had.
- He could not spent a stable childhood because of his poor family and their life standards.

# His Early Life

- By age 13, he mastered an advanced trigonometry book written by S.L. Loney by himself.
- After his graduation from high school,he could not get a degree from both colleges he entered at different times(Government College,Pachaiyappa's College) due to his unwillingness about subjects except mathematics and he could not enter any university.
- He found a Clerical job in Madras port to help his family from poverty. (All other free time were spent for maths)



# Glory & Tragedy

- Ramanujan wrote many letters (with his mathematical findings) to G.H. Hardy.
- Hardy invited Ramanujan to Cambridge. During his visit, Ramanujan wrote many research papers (some on his own, some joint with Hardy)
- Ramanujan had to overcome many difficulties like during World War I, Inability to eat English food.
- Despite these hardships, for his field-changing work he was elected "Fellow of the Royal Society".
- Due to Malnutrition, he felt ill, and he returned to home, where he died one year later in 1920 at the young age of 32.

# The Letter

- Dear Sir,
  I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum. I am now about 23 years of age. I have no University education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at Mathematics. I have not trodden through the conventional regular course followed in a University course, but I am striking out a new path for myself. I have made special investigation of divergent series in general and the results I get are termed by the local mathematicians as startling.

- By the second paragraph of his letter he was insisting he could give meaning to negative values of the gamma function.

- By the third he was disputing an assertion made by Hardy in his work called Orders of Infinity. Ramanujan claimed to have found an approximation for the prime counting function $\pi(x)$ using which the error is negligible. In other words he was challenging the prime number theorem - the best approximation known then.

# Letter Conclusion

- I would request you to go through the enclosed papers. Being poor, if you are convinced that there is anything of value I would like to have my theorems published. I have not given the actual investigations nor the expressions that I get but I have indicated the lines on which I proceed. Being inexperienced I would very highly value any advice you give me. Requesting to be excused for the trouble I give you.

- Yours truly,
  S. Ramanujan (Dated on January 16, 1913).

- The enclosed papers to which Ramanujan referred went on for nine pages. Hardy would rank Ramanujan's letter as "certainly the most remarkable I have ever received, its author a mathematician of the highest quality, a man of altogether exceptional originality and power."

# Ramanujan's Background

- Srinivasa Ramanujan Iyengar came from a poor orthodox Bramhin family in Madras. He was a brilliant student at school. When he passed out, his headmaster introduced him as an outstanding student who deserved scores higher than the maximum possible marks. He received a scholarship for college education.

- But at college he lost interest in all subjects except Mathematics. As a result he failed in most and lost his scholarship. He enrolled in other colleges but again he excelled in maths but failed most others. As a result, he could not get any 'certificate' education beyond school.

# Ramanujan's childhood abode at Kumbakonam, Madras

# Ramanujan's Background

- Day and night he would do nothing but math. He would forget his meals, his sleep in pursuit of math. His mother and wife had to feed him while he carried out his passion.
- Being a married person, his parent's and wife's responsibility fell on him. He had to earn a living to sustain his family.
- But who would give him a job? His work was of no value, least to say incomprehensible to Indian mathematicians. He became so poor that he had no paper to work on and would do all his math on a slate, only write down the results in paper.

# In Search for Patrons

- What was Ramanujan seeking through his appeals? Financial support? World recognition? or something else. This is clear from the conclusion,
- if you are convinced that there is anything of value I would like to have my theorems published...
- Ramanujan just wanted to present his work before the world. He always knew the genius in him and did not care for any world recognition to accept it. Nor did he crave for money or power. He was a very simple man who just wanted the bare necessities of life to preserve his brain.
- Ramanujan's letter was aimed for the world to make use of his extraordinary gifts, so that his work may not be buried unknown with him.

# Discovery of Ramanujan

- Ramanujan's letter Ans so, before midnight, after hours of rummaging through the papers, Hardy and Littlewood began to appreciate that they have been browsing through the works of a genius.
- Hardy sprung into action, he showed the letter to everyone, sent parts of it to experts in particular fields, advised the India Office in London of his interest in Ramanujan and of his desire to bring him to Cambridge.
- Hardy replied back to Ramanujan, acknowledging of his deep interest in the theorems and results and his wish to see their proofs. Ramanujan was notified of Hardy's wish that he came to Cambridge.
- But Hardy soon learned that Ramanujan was not coming. Religious bindings prevented Brahmin and other orthodox Hindus from leaving the borders.

# Discovery of Ramanujan

- But Ramanujan was extremely gratified by Hardy's response. In his second letter, he wrote, "I have found a friend in you who views my labours sympathetically."
- But of proofs, he shot back,
- "...you will not be able to follow my methods of proof if I indicate the lines on which I proceed in a single letter. ...What I tell you is this: Verify the results I give and if they agree with your results, got by treading on the groove in which the present day mathematicians move, you should at least grant that there may be some truths in my fundamental basis."
- He then proceed to pile up more theorems, expanding on the ideas about prime numbers on which Hardy had challenged him, and going on to new work - in all, nine more theorem stuffed pages.

# Ramanujan's Spring

- Many more letters were exchanged after that. In their correspondence, Hardy was trying to wrest rigorous proofs from Ramanujan. He knew what Ramanujan presented was just the tip, many more were hidden under his sleeves. Ramanujan held back, offering excuses.

- Hardy was just "genuinely anxious to see what can be done to give you a better chance of making the best of your obvious mathematical gifts."

- On Hardy's recommendation, Ramanujan was admitted to the Presidence College in Madras and allowed to practice math with single-minded devotion which had been his life long dream. This was an exception made in Ramanujan's case who even lacked a bachelor's degree.

# Ramanujan's Spring

- Ramanujan's only task was to prepare reports every three months detailing his progress which he delivered dutifully on time. In his first report, he offered a theorem - later to be called Ramanujan's Master Theorem - which would provide the means to evaluate many definite integrals whose evaluation was still not possible by known methods.

- Ramanujan presented a powerful generalization of Frullani's 1821 integral theorem that could be made to evaluate formerly unyielding integrals. Hardy had written a paper on the Frullani integral, but he had never seen in it what Ramanujan saw now.

- This time Ramanujan provided proofs for his assertions but they were his by unique methods different from the existing ones. Ramanujan had discovered a new path of mathematics for himself.

# Ramanujan's faith in God

- Ramanujan had strong faith in God. Everything that he discovered, he attributed to the will of God. Goddess Namagiri, whom he worshiped, he said, would come to his dreams and give him the theorems and without Her counsel he could not take any step. He claimed to be just the agent, God the real doer.

- Once on a car, the driver was enjoying himself, was alternating sudden stops with sharp accelerations, jerking the passengers around unmercifully. Said Ramanujan,

- "That man imagines he has the power to go slow or fast at his pleasure. He forgets that he gets the power through the current that flows in the overhead wires. That is the way Maya works in the world."

# Ramanujan's faith in God

- He often said, "An equation for me has no meaning, unless it represents a thought of God." And this quality of Ramanujan inspires me the most.

- Ramanujan was not a dogmatic ritual observer. A deity for him meant just a symbol for God. Hardy cites Ramanujan as remarking that all religions seemed equally true to him.

- On the contrary, Hardy was a confirmed atheist. But even he had to admit that Ramanujan "who for most of his life took counsel from a family goddess, declaring it was she to whom his mathematical insights were owed; whose theorems would at intellectually backbreaking cost, be proved true - yet leave mathematicians baffled that anyone could divine them at the first place."

# About Ramanujan's works

- About his work, Littlewood had written Hardy, "I can believe that he's at least a Jacobi." Hardy would rate him even higher: "I have never met his equal, I can compare him only with Euler or Jacobi."
- On a scale out of 100 of natural mathematical ability, Hardy assigned himself 25, Littlewood 30, and to David Hilbert, the most eminent mathematician of his day, he assigned 80. To Ramanujan he assigned 100.
- In 1918 Hardy and Ramanujan studied the partition function p(n) and gave a non-convergent asymptotic series that permits exact computation of the number of partitions of an integer.

## Ramanujan's Magic Square

| | | | |
|---|---|---|---|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9 | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

This square looks like any other normal magic square. But this is formed by great mathematician of our country – Srinivasa Ramanujan.

What is so great in it?

## Ramanujan's Magic Square

| | | | |
|---|---|---|---|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9 | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

| | | | |
|---|---|---|---|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9 | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

- **Sum of numbers of any row is 139.**
- **Sum of numbers of any Column is 139.**

## RAMANUJAN'S MAGIC SQUARE

| | | | |
|---|---|---|---|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9 | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

| | | | |
|---|---|---|---|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9 | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

**Sum of numbers of any diagonal is also 139.**
**Sum of corner numbers is also 139.**

# Some Interesting Findings of Ramanujan

| | | | |
|---|---|---|---|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9 | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

| | | | |
|---|---|---|---|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9 | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

**RAMANUJAN'S MAGIC SQUARE**

Look at these possibilities. Sum of identical colored boxes is also 139.

*Interesting..?*

| | | | |
|---|---|---|---|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9 | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

## RAMANUJAN'S MAGIC SQUARE

| | | | |
|---|---|---|---|
| 22 | 12 | 18 | 87 |
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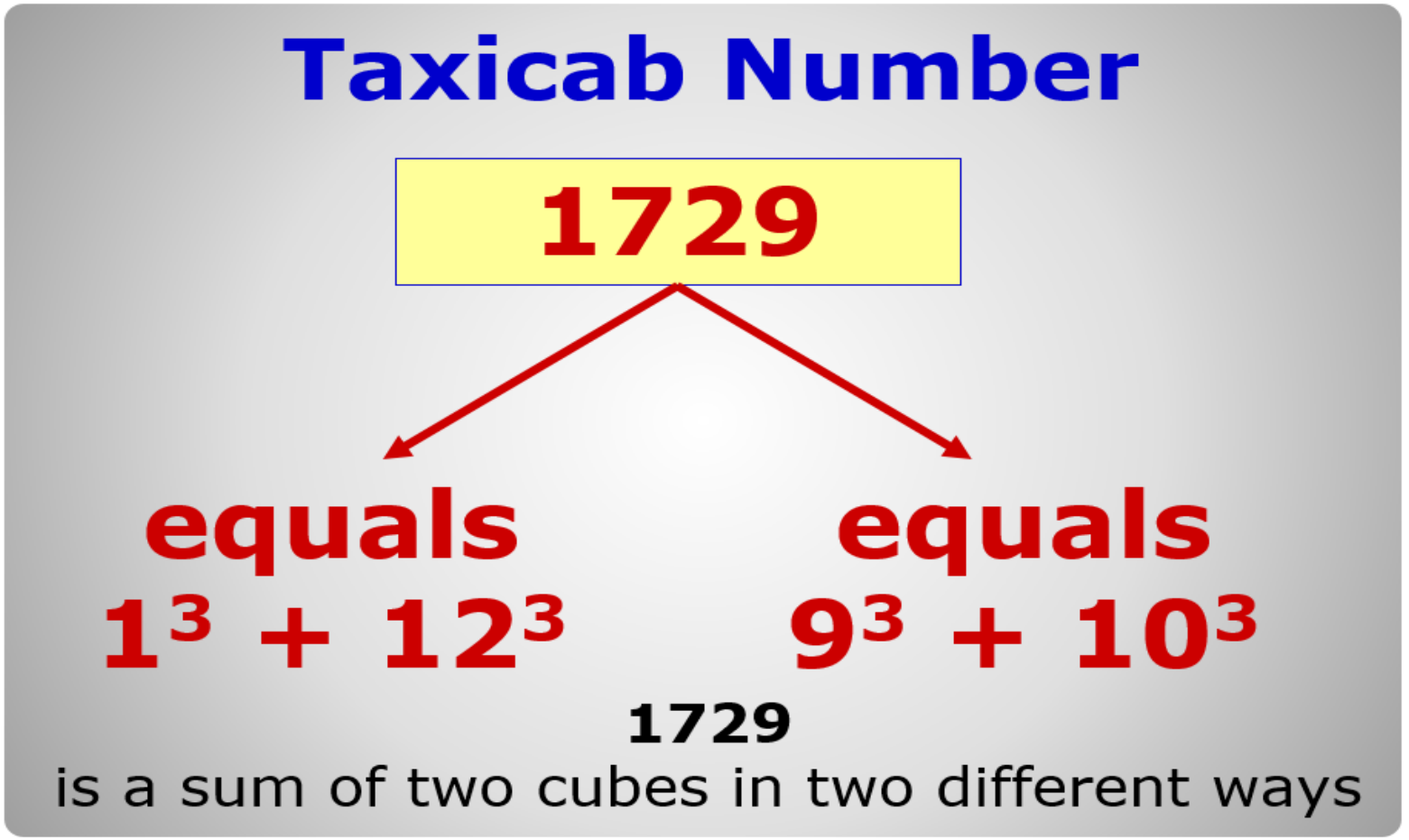
| | | | |
|---|---|---|---|
| 22 | 12 | 18 | 87 |
| 88 | 17 | 9 | 25 |
| 10 | 24 | 89 | 16 |
| 19 | 86 | 23 | 11 |

- **Can you find Ramanujan Birthday from the square?**
- **Yes. It is 22.12.1887**

# Ramanujan-Hardy number

- 1729 is the natural number following 1728 and preceding 1730.
- It is a taxicab number, and is variously known as Ramanujan's number and the Ramanujan-Hardy number.
- Ramanujan found the mystery in the number, 1729, while he was in his bed when he was sick.
- British mathematician G. H. Hardy visited Indian mathematician Srinivasa Ramanujan in hospital. He related their conversation:
- I remember once going to see him when he was ill at Putney. I had ridden in taxi cab number 1729 and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavourable omen. "No," he replied, "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways."

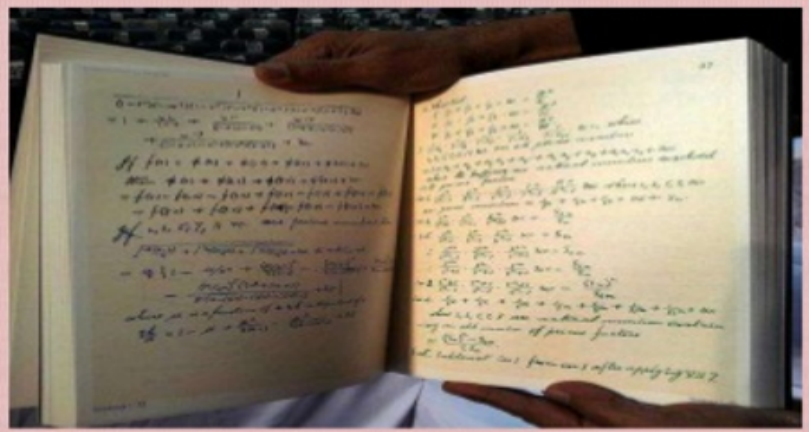

Taxicab Number
1729
equals
$1^3 + 12^3$
equals
$9^3 + 10^3$
1729
is a sum of two cubes in two different ways

# Ramanujan's work

## Ramanujan note books

Ramanujan's note book with his own hand writing

The reprint of Ramanujan's note book



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# Generating conjectures on fundamental constants with the Ramanujan Machine

Gal Raayoni, Shahar Gottlieb, Yahel Manor, George Pisha, Yoav Harris, Uri Mendlovic, Doron Haviv, Yaron Hadad & Ido Kaminer

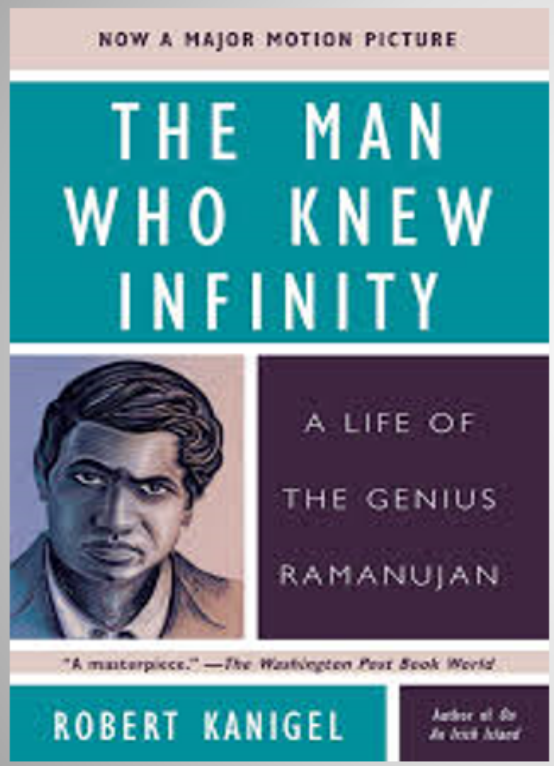
*Nature* **590**, 67–73 (2021) | Cite this article

**12k** Accesses | **7** Citations | **418** Altmetric | Metrics

## Abstract

Fundamental mathematical constants such as e and π are ubiquitous in diverse fields of science, from abstract mathematics and geometry to physics, biology and chemistry[1,2]. Nevertheless, for centuries new mathematical formulas relating fundamental constants have been scarce and usually discovered sporadically[3,4,5,6]. Such discoveries are often considered an act of mathematical ingenuity or profound intuition by great mathematicians such as Gauss and Ramanujan[7]. Here we propose a systematic approach that leverages

# To know more about Ramanujan





# When two geniuses meet

# Mahalanobis and Ramanujan

- Ramanujan found an Indian friend in Prasanta Chandra Mahalanobis. The reputed statistician Mahalanobis who would go on to found the Indian Statistical Institute was then a fellow student of Ramanujan at the King's College, Cambridge.

- Mahalanobis was intrigued by a problem and thought he'd try it out on his friend. He asked, "Imagine that you are on a street with houses marked 1 through n. There is a house in between, say, numbered x, such that the sum of the house numbers to left of it equals the sum of the house numbers to its right. If n is between 50 and 500, what is x?"

- Through trial and error, Mahalanobis had figured it out in a few minutes. Ramanujan figured it out too, but with a twist.

- "Please take down the solution", he said and proceeded to dictate a continued fraction. This wasn't just the solution to the particular problem, it was the solution to a whole class of problems implicit in the puzzle.

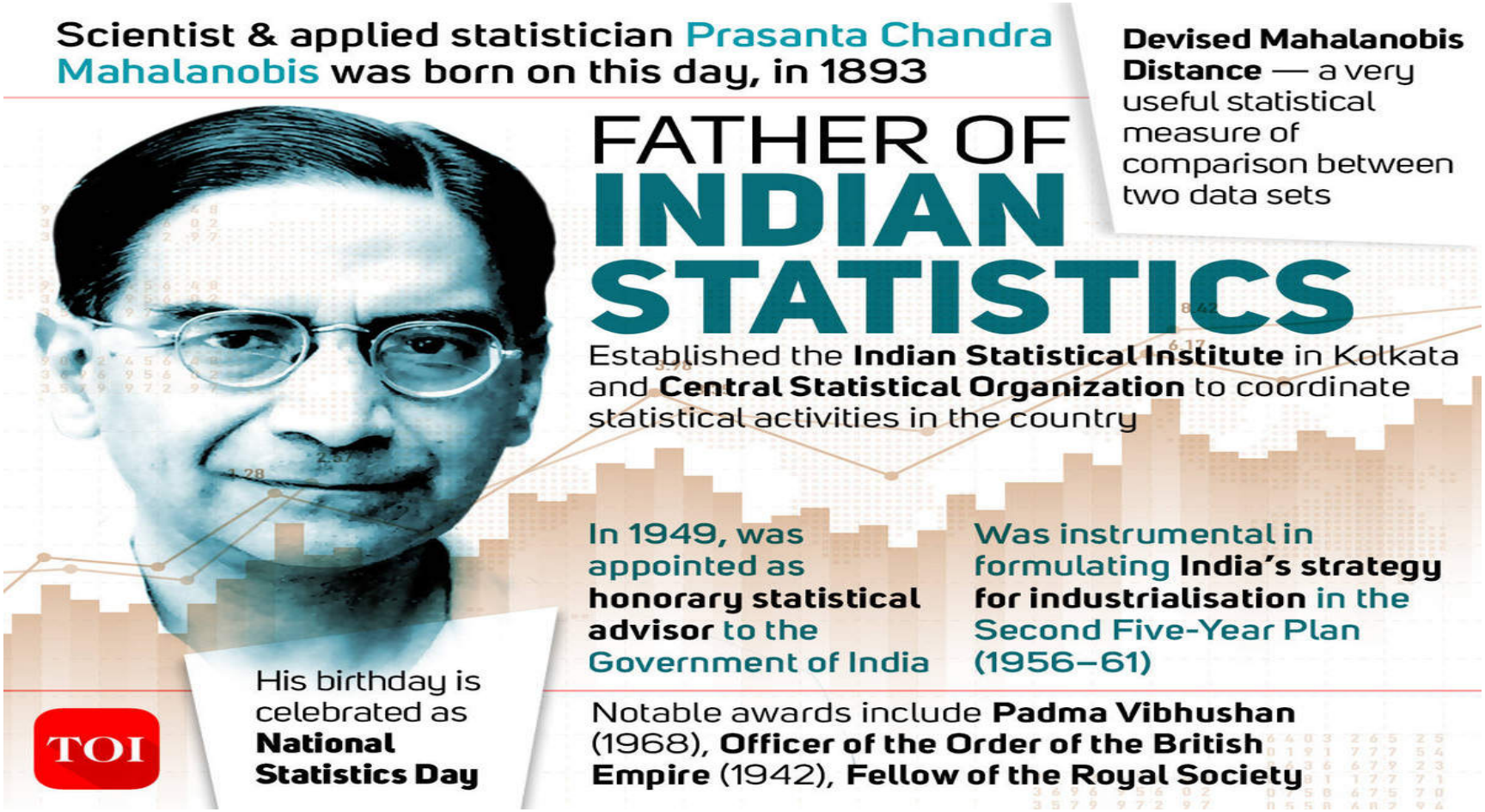
- This problem had a unique solution, 204; $1 + 2 + ... + 203 = 205 + 206 + ... + 288$. But without the 50-500 constraint, there are many more. For example 6 is another. Ramanujan's continued fraction comprised within a single expression all the correct answers.

- Mahalanobis was astounded. How had Ramanujan done it, he asked.

# Mahalanobis and Ramanujan

- "Immediately I heard the problem it was clear that the solution should obviously be a continued fraction; I then thought, which continued fraction? And the answer came to my mind.", replied Ramanujan.

# Prof. Prasanta Chandra Mahalanobis (29 Jun 1893 - 28 Jun 1972)

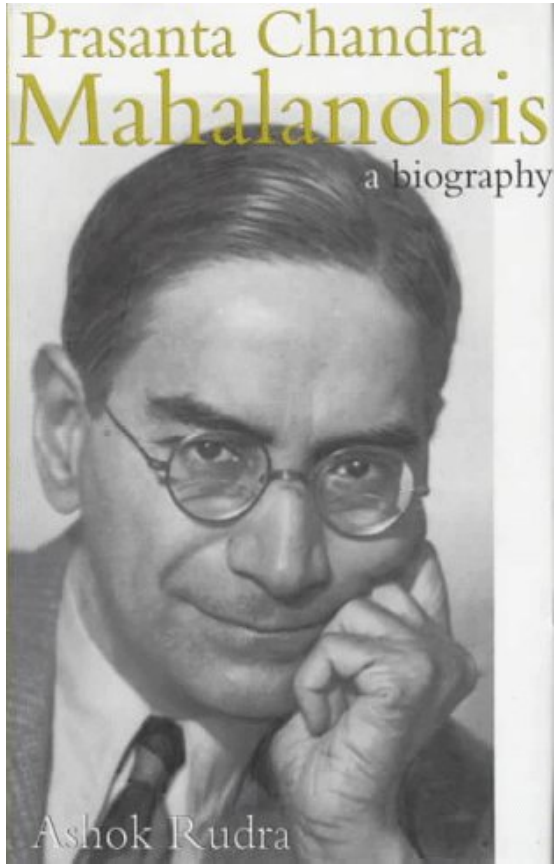


# Historical Facts

- "Prasanta Chandra Mahalanobis (Professor) was a physicist by training, a statistician by instinct and an economist by conviction."
  - Professor C R Rao.

- Mahalanobis met Nelson Annandale (Director of GSI) at the 1920 Indian Science Congress. Annandale asked Mahalanobis to analyze anthropometric measurements of Anglo-Indians in Calcutta. Mahalanobis distance (1922) is a device that compares two different populations.

- His main contributions to statistical theory and applications are multivariate methods in taxonomy (Mahalanobis distance), optimum design of large scale sample surveys, and use of econometric models in planning.



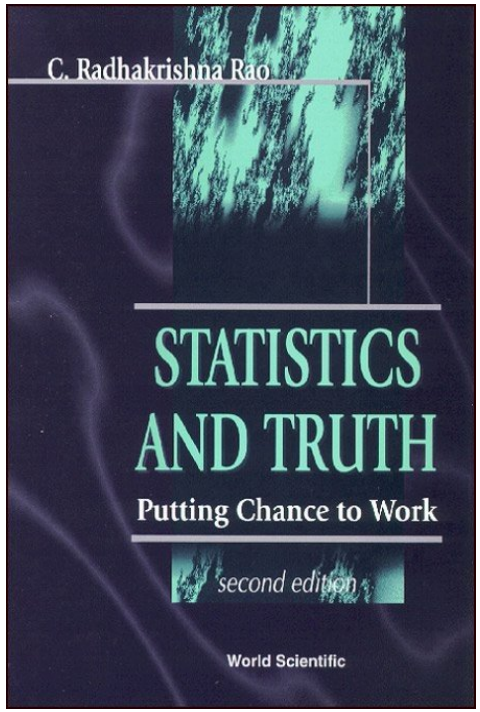
# Reality & Truth

- All knowledge is, in final analysis, history.
  All sciences are, in the abstract, mathematics.
  All judgements are, in their rationale, statistics.

  - Professor C R Rao, Professor Emeritus at Pennsylvania State University, Jan 1987.

- When you're fundraising, it's AI.
  When you're hiring, it's ML.
  When you're implementing, it's Linear Regression.
  When you're debugging, it's *printf()*.

  - Baron Schwartz, Founder and CEO of VividCortex, 2017.

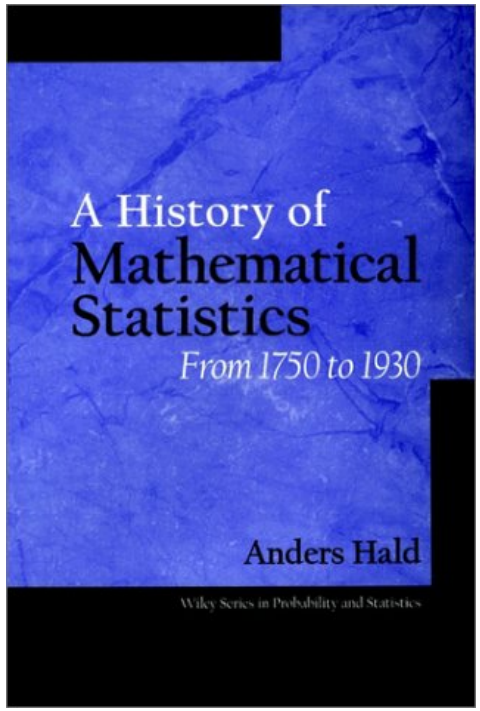


# Statistics: View Points

*"Statistics is the universal tool of inductive inference, research in natural and social sciences, and technological applications.*

*Statistics must have a clearly defined purpose, one aspect of which is scientific advance and the other, human welfare and national develop ment"*

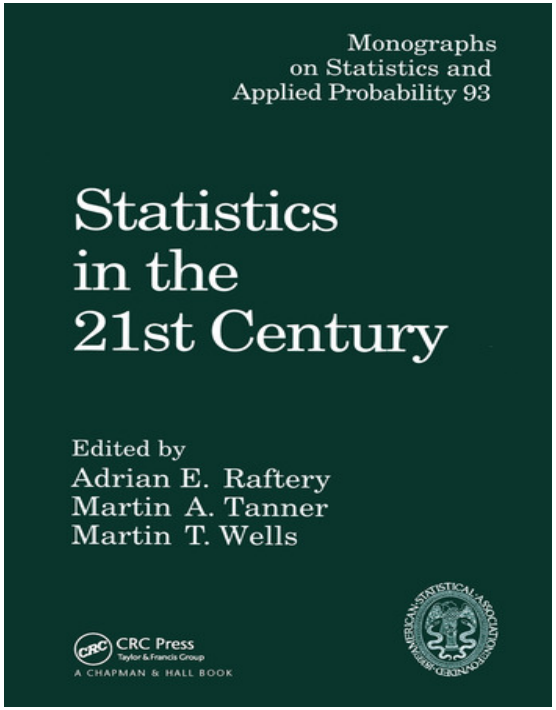
*- Professor P C Mahalanobis.*




- Role of Statistics:
  1. Making inference from samples
  2. Development of new methods for complex data sets
  3. Quantification of uncertainty and variability
- Two Views of Statistics:
  1. Statistics as a Mathematical Science
  2. Statistics as a Data Science

# History of Statistics : Upto 1980

- Parametric Models : One Sample, two sample, linear models, survival data, Estimation, Testing of Hypothesis.
- Probability distributions were believed to generate data (e.g., Gaussian, Logistic, Poisson, Exponential, etc.).
- Semiparametric & Nonparametric Models : Dropping assumptions on population, dependence and errors.
- Emphases on Optimality in various ways : Bayes optimality, Decision theory, minimax and unbiasedness.
- Exact distributional (t, F) approaches and asymptotic methods (samples size $\to \infty$ viewed as approximation).



# Statistics : 1980 - Present

- Data : Large bodies of data with complex data structures are generated from computers, sensors, manufacturing industries, etc.
- Models : Non/Semiparametric models but in complex probability spaces / high-dimensional functional spaces (e.g., deep neural net, reinforcement learning, decision trees, etc.).
- Emphases : Making predictions, causation, algorithmic convergence.
- Data are necessary and at the core of Statistical Learning, Data Science & Machine Learning.

# Statistics : Present (2021)

- Probability : Has moved to the center of Mathematics and having strong interactions with Statistical Physics and Theoretical Computer Science.
- Statistics : Not only has strong interactions with Probability but also other parts of Data Science (Machine Learning, Artificial Intelligence, etc.).
- Computational : Computing skills are essential, construction of fast training algorithms and computation time.
- Applications : Strong interactions with substantive fields in all areas. Applications of statistical methods in almost all the fields are evident. Statistics became a key technology driven by data (**"Data is the new oil"**).

# Role of data: Present

# The World is Data Rich

Astronomy

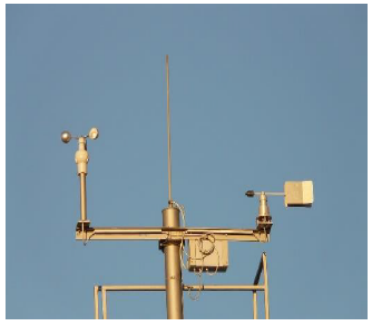
Social Networks


Healthcare

Banking


Genomics

Weather measurements



# Statistics: Past Vs. Present

- Traditional Problems in Applied Statistics:

  - Well formulated question that we would like to answer.
  - Expensive to gathering data and/or expensive to do computation.
  - Create specially designed experiments to collect high quality data.

- Current Situation : Information Revolution

  - Improvements in computers and data storage devices.
  - Powerful data capturing devices.
  - Lots of data with potentially valuable information available.

# What is the Difference?

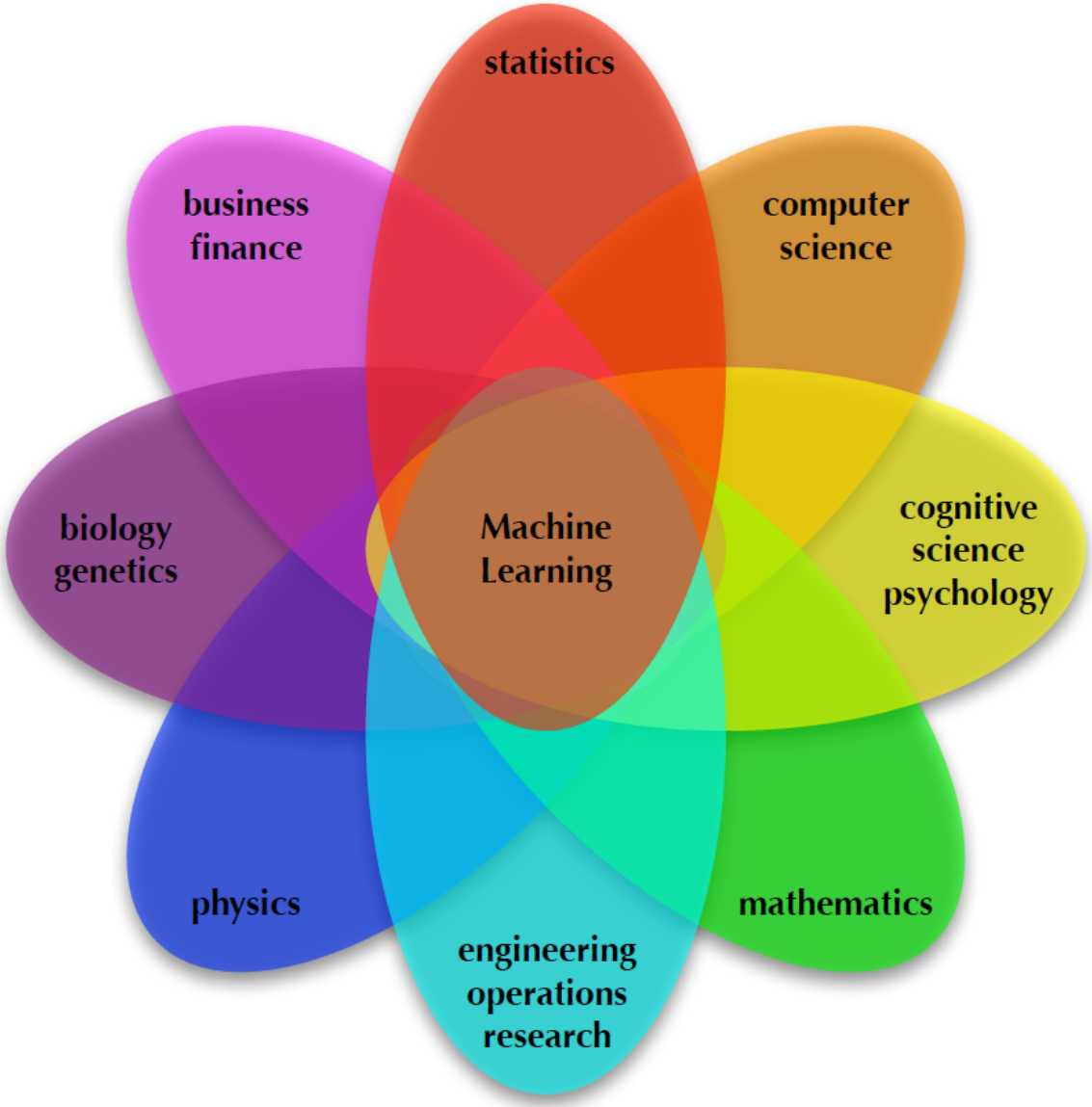
- Data characteristics:
  - Size
  - Dimensionality
  - Complexity
  - Messy
  - Secondary sources
- Focus on generalization performance :
  - Prediction on new data
  - Action in new circumstances
  - Complex models needed for good generalization
- Computational considerations :
  - Large scale and complex systems

# Recent Trends & Buzzwords

- Statistics is the study of the collection, analysis, interpretation, presentation and organization of data.
- Data science is the study of the generalizable extraction of knowledge from data, yet the key word is science.
- Machine learning is the sub-field of computer science that gives computers the ability to learn without being explicitly programmed.
- Artificial Intelligence research is defined as the study of intelligent agents: any device that perceives its environment and takes actions that maximize its chance of success at some goal.
- Big data is an all-encompassing term for any collection of data sets so large and complex that it becomes difficult to process using traditional data processing applications.

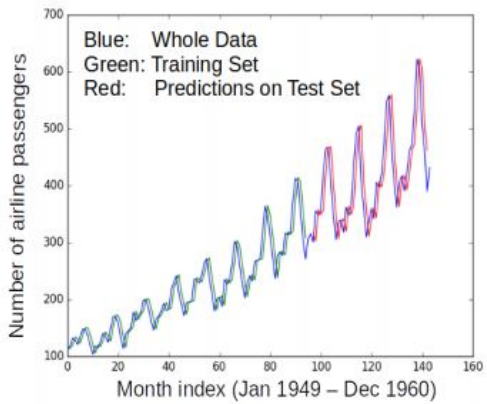
# What is Machine Learning?

Machine learning is the field of study that gives computers the ability to learn without being explicitly programmed.



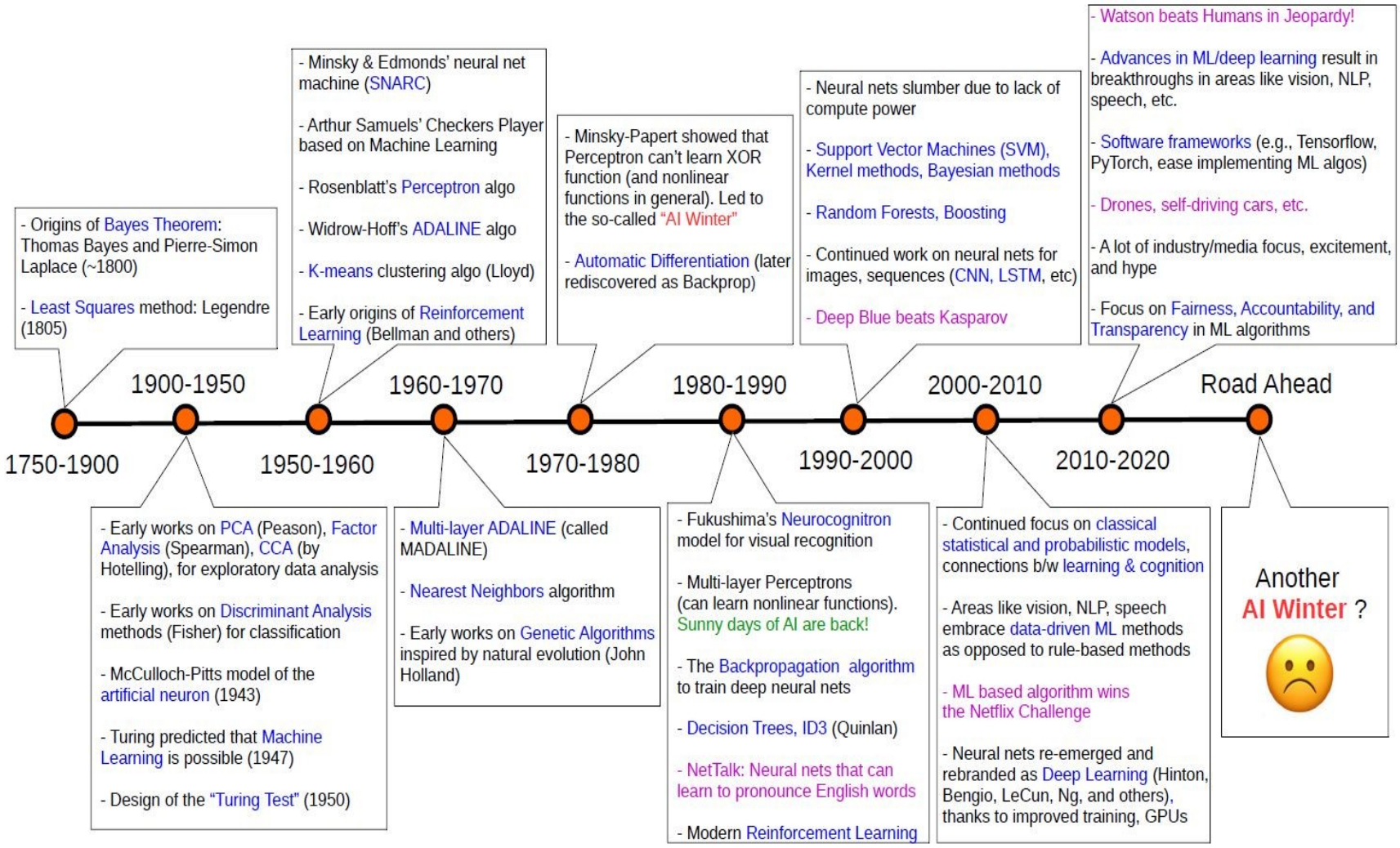
# What is Machine Learning?

- Designing algorithms that ingest data and learn a model of the data.
- The learned model can be used to
  1. Detect patterns/structures/themes/trends etc. in the data
  2. Make predictions about future data and make decisions

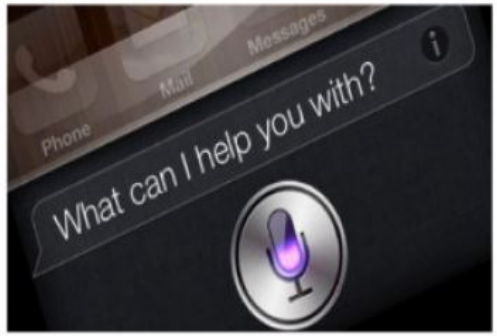

- Modern ML algorithms are heavily "data-driven".
- Optimize a performance criterion using example data or past experience.

# Machine Learning: A Brief Timeline and Some Milestones



# Machine Learning in the real-world

Broadly applicable in many domains (e.g., internet, robotics, healthcare and biology, computer vision, NLP, databases, computer systems, finance, etc.).


Recommendation systems



Predictive Policing

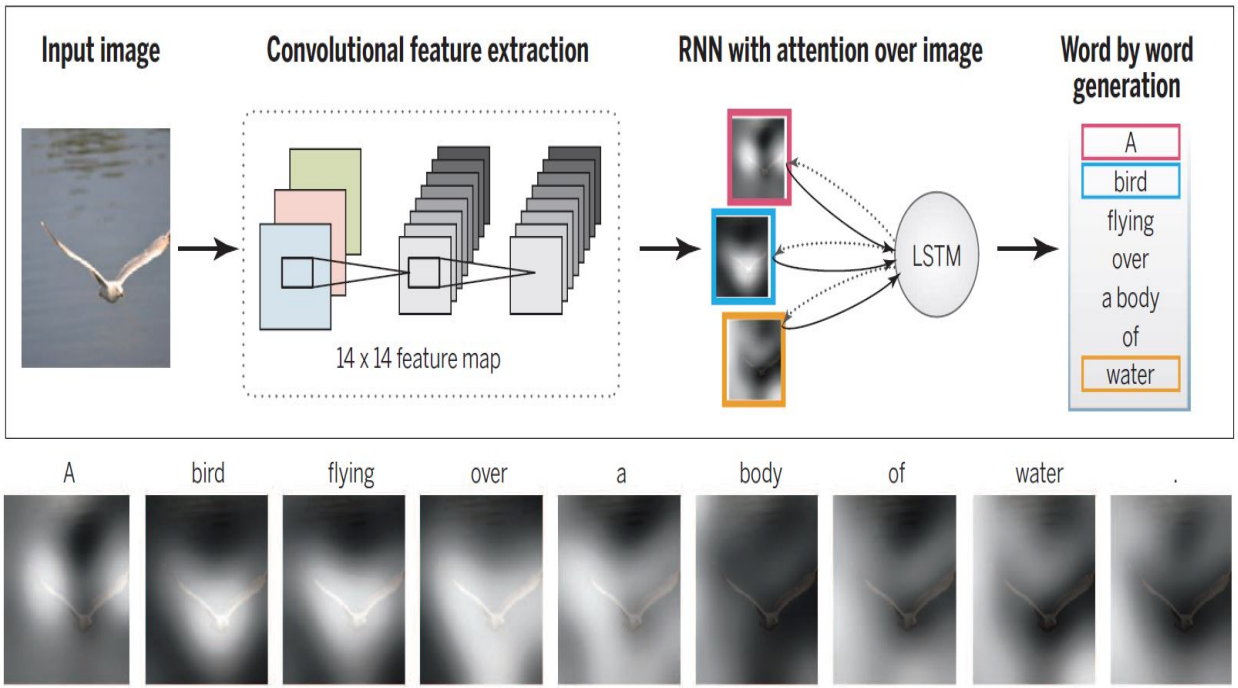

Online Fraud Detection

# Machine Learning helps Natural Language Processing

**ML algorithm can learn to translate text**

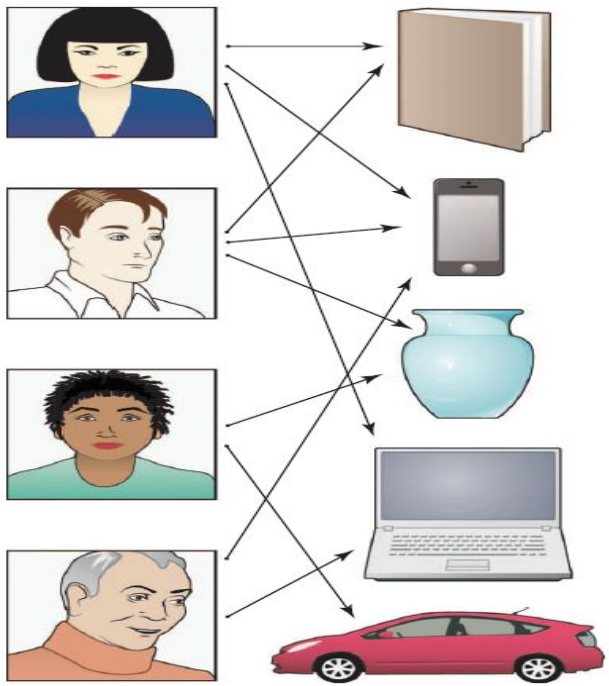
# Machine Learning helps Computer Vision

- Automatic generation of text captions for images:
  **A convolutional neural network** is trained to interpret images, and its output is then used by a recurrent neural network trained to generate a text caption.
- The sequence at the bottom shows the word-by-word focus of the network on different parts of input image while it generates the caption word-by-word.



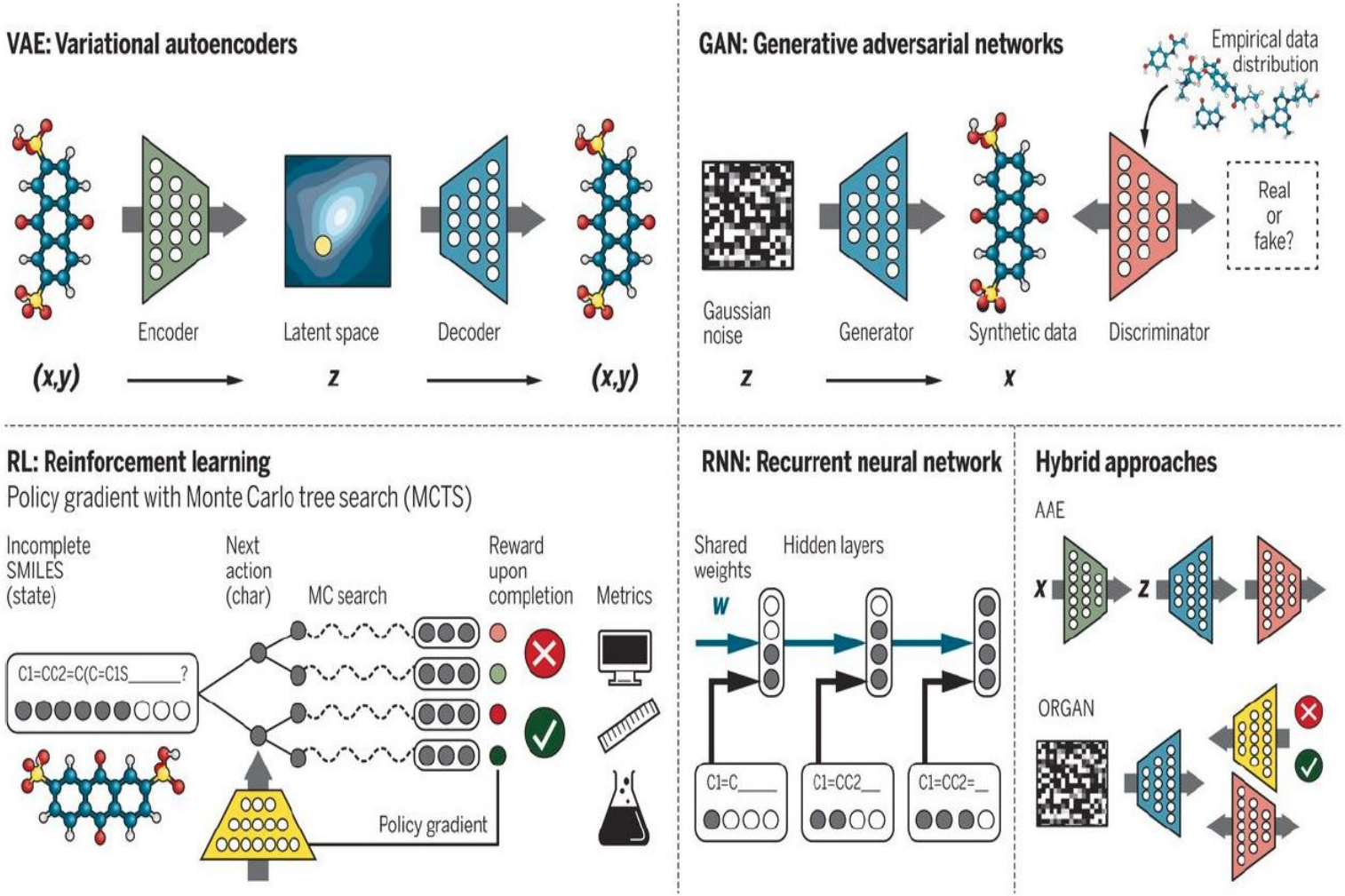
# Machine Learning helps Recommendation systems

- **A recommendation system** is a machine-learning system that is based on data that indicate links between a set of a users (e.g., people) and a set of items (e.g., products).
- A link between a user and a product means that the user has indicated an interest in the product in some fashion (perhaps by purchasing that item in the past).
- The machine-learning problem is to suggest other items to a given user that he or she may also be interested in, based on the data across all users.



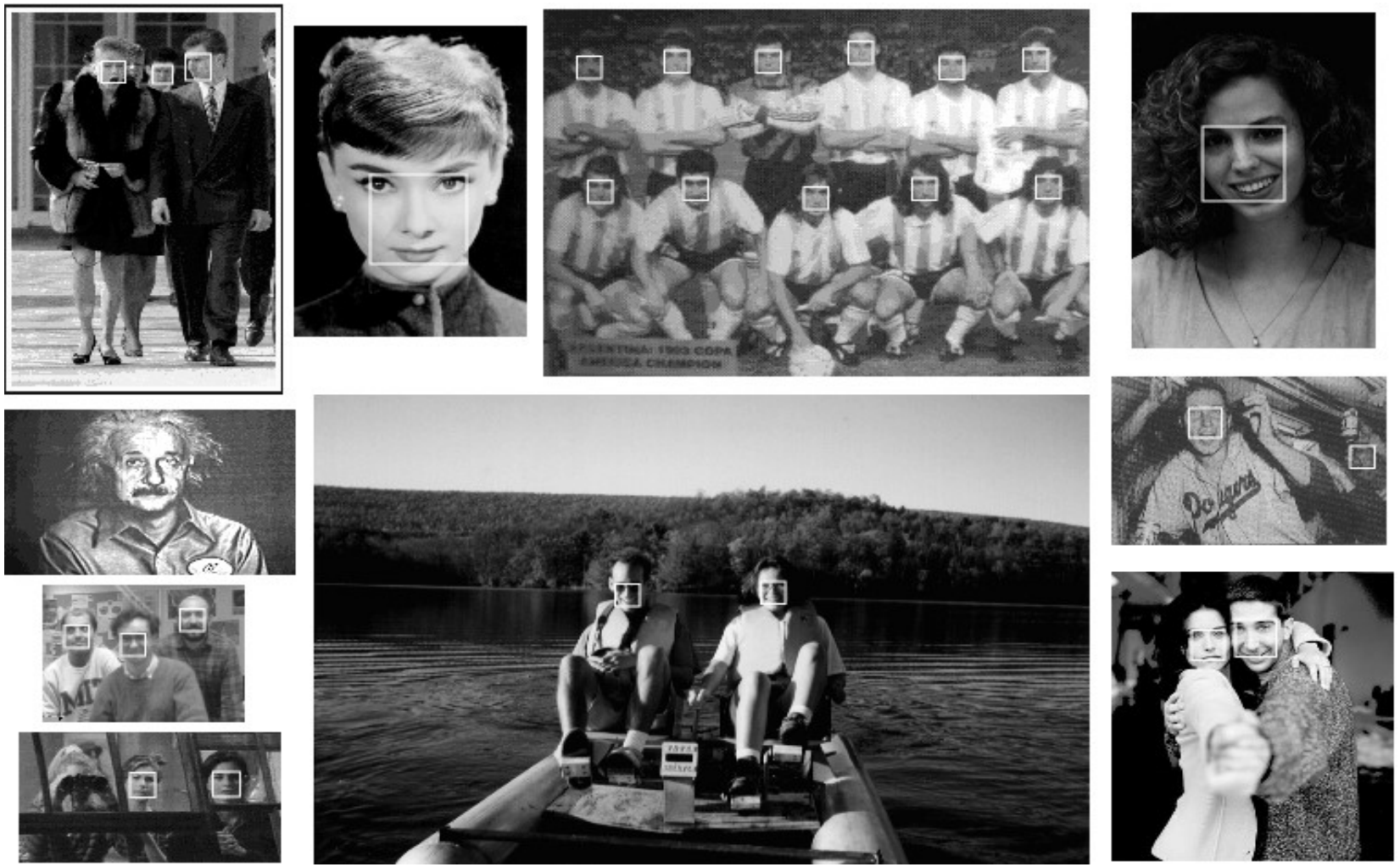
# Machine Learning helps Chemistry

ML algorithms can understand properties of molecules and learn to synthesize new molecules[1].



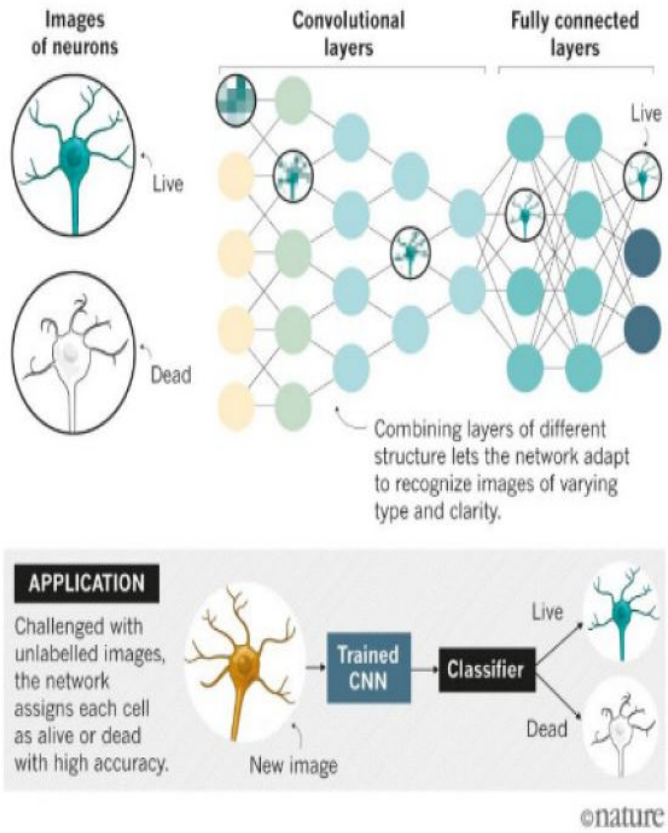

[1] Inverse molecular design using machine learning: Generative models for matter engineering (Science, 2018)

# Machine Learning helps Image Recognition



# Machine Learning helps Many Other Areas...

## Biology




## Finance

# Emerging Research Areas

## Machine Learning:

- Theoretical ML (Explainable & Robust AI)
- Applied ML (Supervised & Unsupervised Problems)
- Deep Learning (Application to CV, IP, IOT, etc.)

## Big Data & Analytics:

- Business Analytics (Quality, Reliability, OR Modelling)
- Data Assimilation
- Methods for Big Data
- High-Dimensional Statistics

## Other Areas:

- Quantum Computing
- 3D Modelling, AR & VR
- Blockchain & Cryptography
- Robotics

# Exploring the beauty of pure mathematics in novel ways

- An Extract from a Post by DeepMind on December 1, 2021 (LINK):

  - More than a century ago, Srinivasa Ramanujan shocked the mathematical world with his extraordinary ability to see remarkable patterns in numbers that no one else could see.
  - The self-taught mathematician from India described his insights as deeply intuitive and spiritual, and patterns often came to him in vivid dreams.
  - These observations captured the tremendous beauty and sheer possibility of the abstract world of pure mathematics.
  - In recent years, we have begun to see AI make breakthroughs in areas involving deep human intuition, and more recently on some of the hardest problems across the sciences.
  - As part of DeepMind's mission to solve intelligence, we explored the potential of machine learning (ML) to recognize mathematical structures and patterns, and help guide mathematicians toward discoveries they may otherwise never have found — demonstrating for the first time that AI can help at the forefront of pure mathematics.

# Exploring the beauty of pure mathematics in novel ways

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Article | Open Access | Published: 01 December 2021

## Advancing mathematics by guiding human intuition with AI

Alex Davies ✉, Petar Veličković, Lars Buesing, Sam Blackwell, Daniel Zheng, Nenad Tomašev, Richard Tanburn, Peter Battaglia, Charles Blundell, András Juhász, Marc Lackenby, Geordie Williamson, Demis Hassabis & Pushmeet Kohli ✉

*Nature* **600**, 70–74 (2021) | Cite this article

**122k** Accesses | **1** Citations | **1553** Altmetric | Metrics

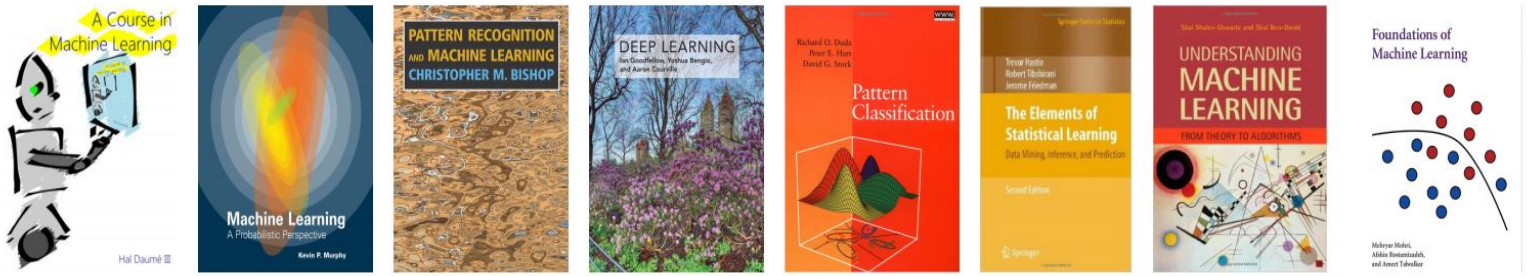
### Abstract

The practice of mathematics involves discovering patterns and using these to formulate and prove conjectures, resulting in theorems. Since the 1960s, mathematicians have used computers to assist in the discovery of patterns and formulation of conjectures[1], most

# Textbook and References

Start with these three books


Then you can start reading these books

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# Textbook and References (Continued...)


thank you