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ISI B.STAT/B.MATH OBJECTIVE QUESTIONS & SOLUTIONS

SET – 1

1. How many zeros are at the end of 1000! ?

(a) 240 (b) 248 (c) 249 (d) None

Ans:- (c) The number of two's is enough to match each 5 to get a 10.

So,

$5^1 \longrightarrow 200$

$5^2 \longrightarrow 40$ ∴Thus, 1000! Ends with 249 zeros

$5^3 \longrightarrow 8$

$5^4 \longrightarrow 1$

[Theorem: (de Polinac's formula)
Statement: Let p be a prime and e be the largest exponent of p such that p^e divides n! , then $e=\sum [n/p^i]$, where i is running from 1 to infinity.]

So, [1000/5]+[1000/25]+[1000/125]+[1000/625]=249 .
Thus, 1000! ends with 249 zeros.

2. The product of the first 100 positive integers ends with

(a) 21 zeros (b) 22 zeros (c) 23 zeros (d) 24 zeros.

Ans:- $5^1 \longrightarrow 20$ (d) 24 zeros .

$5^2 \longrightarrow 4$

Alternatively, put p=5,n=100,thus from above theorem we have [100/5]+[100/25]=24 zeros as the answer.

3. Let P (x) be a polynomial of degree 11 such that P (x) = $\frac{1}{x+1}$ $for\ x = 0\ (1) 11$.

Then P (12) = ?

(a) 0 (b) 1 (c) $\frac{1}{13}$ (d) none of these

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Ans:- (a) P (x)= $\frac{1}{x+1}$

⇒ (x+1)[P (x)]-1 = c (x-0)(x-1)….(x-11)

Putting x= -1, 0- 1= c (-1)(-2)….(-12)

⇒ c = - $\frac{1}{12!}$

∴ [P (x)](x+1)-1= - $\frac{1}{12!}$(x-0)(x-1)….(x-11)

⇒ P (12) 13-1 = - $\frac{1}{12!}$ 12 .11. ….2.1

⇒ P (12) 13-1 = -1

⇒P (12) = 0.

4. Let s= {$(x_1, x_2\ , x_3)$| 0≤ $x_i \leq 9$ *and* $x_1 + x_2 + x_3$ is divisible by 3}.

Then the number of elements in s is

(a) 334 (b) 333 (c) 327 (d) 336

Ans:- (a) with each $(x_1, x_2\ , x_3)$ identify a three digit code, where reading zeros are allowed. We have a bijection between s and the set of all non-negative integers less than or equal to 999 divisible by 3. The no. of numbers between 1 and 999, inclusive, divisible by 3 is $\left(\frac{999}{3}\right) = 333$

Also, '0' is divisible by 3. Hence, the number of elements in s is = 333 + 1= 334.

5. Let x and y be positive real number with x< y. Also 0 < b< a < 1.

Define E = $\log_a\left(\frac{y}{x}\right) + \log_b\left(\frac{x}{y}\right)$. Then E can't take the value

(a) -2 (b) -1 (c) -$\sqrt{2}$ (d) 2

Ans :- (d) E =$\log_a\left(\frac{y}{x}\right) + \log_b(\frac{x}{y})$ = $\frac{\log\frac{y}{x}}{\log_a} - \frac{\log\frac{y}{x}}{\log_b}$

$= \log\left(\frac{y}{x}\right)\{\frac{1}{\log_a} - \frac{1}{\log_b}\}$ $= \log\left(\frac{y}{x}\right)\{\frac{\log_b - \log_a}{(\log_a)(\log_b)}\}$

$= \log\left(\frac{y}{x}\right).\frac{\log(\frac{b}{a})}{(\log_a)(\log_b)}$ $= -\log\left(\frac{y}{x}\right).\frac{\log(\frac{a}{b})}{(\log_a)(\log_b)}$

Log 0< a < 1, 0< b <1 ∴ $\log_a$ and $\log_b$ are both negative.

Also $\frac{y}{x} > 1$ and $\frac{a}{b} > 1$. Thus $\log\left(\frac{y}{x}\right)$ and $\log\left(\frac{a}{b}\right)$ are both positive. Finally E turns out to be a negative value. So, E can't take the value '2'.

6. Let S be the set of all 3- digits numbers. Such that

(i) The digits in each number are all from the set {1, 2, 3, …., 9}

(ii) Exactly one digit in each number is even

The sum of all number in S is

(a) 96100 (b) 133200 (c) 66600 (d) 99800

Ans:- (b) The sum of the digits in unit place of all the numbers in s will be same as the sum in tens or hundreds place. The only even digit can have any of the three positions,

i.e. ${}^{3}c_1$ ways.

And the digit itself has 4 choices (2, 4, 6 or 8). The other two digits can be filled in 5× 4 = 20 ways.

Then the number of numbers in S = 240.

Number of numbers containing the even digits in units place = $4 \times 5 \ \times 4$ = 80

The other 160 numbers have digits 1, 3, 5, 7 or 9 in unit place, with each digit appearing

$\frac{160}{5}$ = 32 times. Sum in units place = 32 (1+ 3+ 5+ 7+ 9) + 20 (2+ 4+ 6+ 8)

= $32.5^2 + \ 20 \ \times 2 \times \frac{4\times5}{2}$ = $32\times 25 + 20 \times 20$ = 1200

∴ The sum of all numbers= 1200 (1+ 10 +10^2) = 1200×111 = 133200.

7. Let y = $\frac{x}{x^2+1}$, Then $y^4(1)$is equals

(a) 4 (b) -3 (c) 3 (d) -4

Ans:- (b) Simply differentiating would be tedious,

So we take advantage of 'i' the square root of '-1'

y = $\frac{x}{x^2+1} = \frac{1}{2}\left\{\frac{1}{(x-i)} + \frac{1}{(x+i)}\right\}$

$\frac{d^4y}{dx^4} = \frac{1}{2}\left\{\frac{4!}{(x-i)^5} + \frac{4!}{(x-i)^5}\right\}$

Note that, $\frac{d^n}{dx^n}\left\{\frac{1}{x+a}\right\} = \frac{(-1)^n\, n!}{(x+a)^{n+1}}$

So, $y^4(x) = \frac{4!}{2}\{\frac{1!}{(x-i)^5} + \frac{1!}{(x-i)^5}\}$ Then

$y^4(1) = 12\,\{\frac{1!}{(x-i)^5} + \frac{1!}{(x-i)^5}\}$ = 12 $\{\frac{1-i}{(-2i)^3} + \frac{1-i}{(2i)^3}\}$ = 12 $\{\frac{1-i}{8i} + \frac{1-i}{8i}\}$ = 12 $(-\frac{1}{8} - \frac{1}{8})$ = -3.

8. A real 2× 2 matrix. M such that $M^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1-\epsilon \end{pmatrix}$

(a) exists for all ∊ > 0 **(b) does not exist for any ∊> 0**

(c) exists for same ∊> 0 **(d) none of the above**

Ans:- (b) since M^2 is an diagonal matrix, so M= $\begin{bmatrix} i & 0 \\ 0 & \sqrt{1-\epsilon} \end{bmatrix}$,

So, M is not a real matrix, for any values of ∊

M is a non –real matrix.

9. The value of $\left(\frac{1+i\sqrt{3}}{2}\right)^{2008}$ is

(a) $\frac{1+i\sqrt{3}}{2}$ **(b) $\frac{1-i\sqrt{3}}{2}$** **(c) $\frac{-1-i\sqrt{3}}{2}$** **(d) $\frac{-1+i\sqrt{3}}{2}$**

Ans:- (c) A = $(\frac{1+i\sqrt{3}}{2})$, $A^2 = \frac{-1+i\sqrt{3}}{2}, A^4 = \frac{-1-i\sqrt{3}}{2} = -A$

$\therefore A^{2008} = (A^4)^{502} = A^4 = \frac{-1-i\sqrt{3}}{2}$.

10. Let f(x) be the function f(x)= $\begin{cases} \frac{x^P}{(sinx)^q} & if\ x > 0 \\ 0 & if\ x = 0 \end{cases}$

Then f(x) is continuous at x= 0 if

(a) p > q **(b) p > 0** **(c) q > 0** **(d) p < q**

Ans:- (b) |f(x) - f(0)| = $|\frac{x^P}{(sinx)^q} - 0| \leq |x^P| < \epsilon$

Whenever |x-0| < $\epsilon^{\frac{1}{p}} = \delta$ if p > 0.

So, f(x)is continuous for p > 0 at x= 0.

11. The limit $\lim_{x\to\infty} \log(1 - \frac{1}{n^2})^n$ equals

(a) e^{-1} **(b) $e^{-\frac{1}{2}}$** **(c) e^{-2}** **(d) 1**

Ans:- (d) L = $(1-\frac{1}{n^2})^n$

⇨ $logL = nlog(1-\frac{1}{n^2})$

⇨ $\lim_{x\to\infty} logL = \lim_{x\to\infty}[-n\{\frac{1}{n^2}+\frac{1}{2n^4}+\cdots\infty\}] = 0$

∴L = $e^0 = 1$.

12. The minimum value of the function f(x, y)= $4x^2+9y^2-12x-12y+14$ is

(a) 1 (b) 3 (c) 14 (d) none

Ans:- (a) f(x, y) = $4x^2+9y^2-12x-12y+14$

= $(4x^2-12x+9)$+($9y^2-12y+4$)+1

= $(2x-3)^2+(3y-2)^2+1 \geq 1$

So, minimum value of f(x, y) is 1.

13. From a group of 20 persons, belonging to an association, A president, a secretary and there members are to be elected for the executive committee. The number of ways this can be done is

(a) 30000 (b) 310080 (c) 300080 (d) none

Ans:- (b) $20_{c_1} \times 19_{c_1} \times 18_{c_3}$ or $\frac{20!}{1!1!3!15!}$ = 310080

14. The $\lim_{x\to 0}\frac{\cos x-\sec x}{x^2(1+x)}$ is

(a) -1 (b) 1 (c) 0 (d) does not exist

Ans:- (a) $\lim_{x\to 0}\frac{\cos x-\sec x}{x^2(1+x)} = \lim_{x\to 0}\frac{-\sin^2 x}{\cos x\,(x^2)(x+1)}$

= - $\lim_{x\to 0}\frac{1}{\cos x}(\frac{\sin x}{x})^2.\frac{1}{(x+1)}$ = -1. 1. 1 = - 1.

15. Let R = $\frac{48^{52}-46^{52}}{96^{26}+92^{26}}$. Then R satisfies

(a) R < 1 (b) 23^{26}< R < 24^{26} (c) 1 < R < 23^{26} (d) R > 24^{26}

Ans:- (b) R= $\frac{(2.24)^{52}-(2.23)^{52}}{(4.24)^{26}+(4.23)^{26}} = \frac{2^{52}(24^{52}-23^{52})}{4^{26}(24^{26}+23^{26})} = \frac{2^{52}}{2^{52}}.\frac{(24^{26}+23^{26})(24^{26}-23^{26})}{24^{26}+23^{26}}$

= $24^{26}-23^{26} < 24^{26}$

Also, R= $24^{26}-23^{26} = (1+23)^{26}-23^{26}$

= $23^{26}+26_{c_1}.23^{25}+26_{c_2}.23^{24}+\ldots+1-23^{26}$

$= 26.\ 23^{25} + 26_{c_2}\ .23^{24} + \ ... + 1 > 26.\ 23^{25} > 23.\ 23^{25} = 23^{26}$

$\therefore 23^{26} < R < 24^{26}$

16. A function f is said to be odd if f (-x)= -f (x) $\forall\, x$. Which of the following is not odd?

(a) f (x+ y)= f(x)+ f(y) $\forall\, x, y$

(b) f (x)= $\frac{xe^{x/2}}{1+e^x}$

(c) f (x) = x - [x]

(d) f (x) = $x^2 \sin x + x^3 \cos x$

Ans:- (c) f (x+ y)= f(x)+ f(y) $\forall\, x, y$

Let x = y = 0

⇒ f (0) = f (0) + f (0)

∴ f (0)= 0

Replacing y with –x , we have

f (x- x) = f(x) + f (-x)

⇒ f (0) = f(x) + f (-x)

⇒ f(x) + f (-x) = 0

⇒ f (-x) = -f(x)

Thus f is odd.

Again for f (x) = $\frac{xe^{x/2}}{1+e^x}$

f(-x)= $\frac{(-x)(e^{-x/2})}{1+e^{-x}} = \frac{(-x)\left(e^{-x/2}\right).e^x}{1+e^x} = -\frac{xe^{x/2}}{1+e^x}$ = -f (x)

∴ f is odd.

f (x) = x- [x] is not odd.

Counter example:-

f (-2.3) = -2.3 – [-2.3] =-2.3 – (-3) = 3- 2.3 = 0.7

f (2.3) = 2.3 – [2.3] = 2.3 -2 =0.3

∴ f(2.3) ≠ f(-2.3)

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Thus f is not odd

f (x) = $x^2 \sin x + x^3 \cos x$

f(-x) = -$x^2 \sin x - x^3 \cos x$ = -f(x)

∴ f is odd here.

17. Consider the polynomial $x^5 + ax^4 + bx^3 + cx^2 + dx + 4$. If (1+2i) and (3-2i) are two roots of this polynomial then the value of a is

(a) -524/65 (b) 524/65 (c) -1/65 (d) 1/65

Ans:- (a) The polynomial has 5 roots. Since complex root occur in pairs, so there is one real root taking it as m.

So, m, 1+2i, 1-2i, 3+2i, 3-2i are the five roots.

Sum of the roots= $-\frac{a}{1} = 8 + m.$

Product of the roots= (1+4)(9+4)m= 65 m=$\frac{4}{65}$

∴ m = $\frac{4}{65}$.

∴ a= $-8 - \frac{4}{65} = -\frac{524}{65}$.

18. In a special version of chess, a rook moves either horizontally or vertically on the chess board. The number of ways to place 8 rooks of different colors on a 8×8 chess board such that no rook lies on the path of the other rook at the start of the game is

(a) 8× ⌊8 (b) ⌊8 × ⌊8 (c) $2^8 \times$ ⌊8 (d) $2^8 \times \binom{64}{8}$

Ans:- The first rook can be placed in any row in 8 ways & in any column in 8 ways. So, it has 8^2 ways to be disposed off. Since no other rook can be placed in the path of the first rook, a second rook can be placed in 7^2 ways for there now remains only 7 rows and 7 columns. Counting in this manner, the number of ways = $8^2 . 7^2 . 6^2 \dots 1^2 = (8!)^2$

19. The differential equation of all the ellipses centered at the origin is

(a) $y^2 + x(y')^2 - yy' = 0$ (b) x y y″+ $x(y')^2 - yy' = 0$

(c) y y″+ $x(y')^2 - xy' = 0$ (d) none

Ans:- (d) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, after differentiating w.r.t x, we get

$\Rightarrow \frac{2x}{a^2} + \frac{2yy'}{b^2} = 0 \Rightarrow \frac{yy'}{b^2} = -\frac{x}{a^2}$

$\Rightarrow \frac{(y')^2}{b^2} + \frac{y(y'')}{b^2} = -\frac{1}{a^2}$

$\Rightarrow (y')^2 + y(y'')^2 = -\frac{b^2}{a^2}.$

20. If f(x)= x+ sinx, then find $\frac{2}{\pi^2}.\int_{\pi}^{2\pi}(f^{-1}(x) + sinx)dx$

(a) 2 **(b) 3** **(c) 6** **(d) 9**

Ans:- (b) Let x= f(t) ⇒ dx= f′(t)dt

$\Rightarrow \int_{\pi}^{2\pi} f^{-1}(x)dx = \int_{\pi}^{2\pi} t\ f'(t)dt = (t\,[f(t)])^{2\pi}{}_{\pi} - \int_{\pi}^{2\pi} f(t)dt = (4\pi^2 - \pi^2) - \int_{\pi}^{2\pi} f(t)dt$

I= $\int_{\pi}^{2\pi}(f^{-1}(x) + sinx)dx = \int_{\pi}^{2\pi} f^{-1}(x)dx + \int_{\pi}^{2\pi} sinxdx$

$= 3\pi^2 - \int_{\pi}^{2\pi} f(t)dt + \int_{\pi}^{2\pi} sinxdx$

$= 3\pi^2 - \int_{\pi}^{2\pi}(f(x) - sinx)dx$

$= 3\pi^2 - \int_{\pi}^{2\pi} xdx = 3\pi^2 - \frac{1}{2}(4\pi^2 - \pi^2)$

$= \frac{3}{2}\pi^2$

$\Rightarrow \frac{2}{\pi^2} I = 3.$

21. Let P= (a, b), Q= (c, d) and 0 < a < b < c < d, L≡(a, 0), M≡(c, 0), R lies on x-axis such that PR + RQ is minimum, then R divides LM

(a) Internally in the ratio a: b **(b) internally in the ratio b: c**

(c) internally in the ratio b: d **(d) internally in the ratio d: b**

Ans:- (c) Let R = (α, 0). PR+RQ is least

⟹ PQR should be the path of light

⟹ *Δ* PRL and QRM are similar

$\Longrightarrow \frac{LR}{RM} = \frac{PL}{QM} \Longrightarrow \frac{\alpha - a}{c - \alpha} = \frac{b}{d}$

⟹ αd- αd= bc –αb

⟹ $\alpha = \frac{ad + bc}{b + d}$

$\Longrightarrow$ R divides LM internally in the ratio b : d (as $\frac{b}{d} > 0$)

22. A point (1, 1) undergoes reflection in the x-axis and then the co-ordinate axes are roated through an angle of $\frac{\pi}{4}$ in anticlockwise direction. The final position of the point in the new co-ordinate system is-

(a) (0, $\sqrt{2}$) **(b) (0, $-\sqrt{2}$)** **(c) $-\sqrt{2}, 0$** **(d) none of these**

Ans:- . (b) Image of (1, 1) in the x-axis is (1, -1). If (x, y) be the co-ordinates of any point and (x' , y') be its new co-ordinates, then x' = x cos θ+ y sin θ,

y'= y cosθ – x sin θ, where θ is the angle through which the axes have been roated.

Here θ= $\frac{\pi}{4}$, x= 1, y= -1

$\therefore$ x'= 0, y'= -$\sqrt{2}$

23. If a, $x_1, x_2, \ldots, x_k$ and b, $y_1, y_2, \ldots, y_k$ from two A.P. with common difference m and n respectively, then the locus of point (x, y) where x= $\frac{\sum_{i=1}^{k} x_1}{k}$ $is\ and\ y = \frac{\sum_{i=1}^{k} y_1}{k}$ is

(a) (x-a)m= (y-b)n **(b) (x-m) a= (y-n) b**

(c)(x-n)a = (y-m)b **(d) (x-a) n-(y-b) m**

Ans:- (d)

X= $\frac{\frac{k}{2}(x_1+x_k)}{k} = \frac{x_1+x_k}{2} = \frac{a+m+a\ mk}{2}$

$or,\ x = a + \frac{(k+1)m}{2}$

or, 2(x-a)= (k+1)m ……………..(1)

Similarly,

2(y-b)= (k+ 1)n ……………….(2)

We have to eliminate k

From (1) and (2)

$\frac{x-a}{y-b} = \frac{m}{n}$

or, (x- a)n = (y -b)m

24. The remainder on dividing $1234^{567} + 89^{1011}$ by 12 is

(a) 1 **(b) 7** **(c) 9** **(d) none**

Ans:- (c) 1234≡1 (mod 3)⇒ $1234^{567} \equiv 1\ (mod\ 3) and\ 89 \equiv\ -1(mod 3)$

$\Rightarrow 89^{1011} \equiv -1\ (mod\ 3)$

$\therefore 1234^{567} + 89^{1011} \equiv 0\ (mod\ 3)$

Here 1234 is even, so $1234^{567} \equiv 0(mod\ 4) and\ 89 \equiv 1\ (mod\ 4)$

$\Rightarrow 89^{1011} \equiv 1\ (mod\ 4)$

Thus $1234^{567} + 89^{1011} \equiv 1\ (mod\ 4)$

Hence it is 9 (mod 12)

25. The sum of the series 1+$\frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \cdots$ is

(a) e^2 **(b) 3** **(c) $\sqrt{5}$** **(d) $\sqrt{8}$**

Ans. (d) $\sqrt{8} = 2^{\frac{3}{2}} = (\frac{1}{2})^{-\frac{3}{2}} = (1 - \frac{1}{2})^{-\frac{3}{2}}$

$= 1 + (\frac{1}{2})\left(\frac{3}{2}\right) + \frac{\left(-\frac{3}{2}\right)(-\frac{3}{2}-1)}{2!}(-\frac{1}{2})^2 + \cdots$

$= 1 + \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \cdots$

26. If f(x) = cos x+ cos ax is a periodic function, then a is necessarily

(a) an integer **(b) a rational number** **(c) an irrational number** **(d) an event number**

Ans. (b) Period of cos x= 2π and period of cos ax= $\frac{2\pi}{|a|}$

Period of f(x) = L.C.M. of $\frac{2\pi}{1}\ and\ \frac{2\pi}{|a|} = \frac{L.C.M. of\ 2\pi\ and\ 2\pi}{H.C.F. of\ 1\ and\ |a|}$

Since k= H.C.F. of 1 and |a|

$\therefore \frac{1}{k}$= an integer= m (say) and $\frac{|a|}{k}$= an integer = n (say)

$\therefore$ |a|=$\frac{n}{m} \Longrightarrow a = \pm\frac{n}{m}$ = a rational number.

27. Let f : R⟶ R defined by f(x)= $x^3 + x^2 + 100x + 5\sin x$, then f is

(a) many-one onto (b) many-one into (c) one-one onto (d) one-one into

Ans. (c)

$f(x) = x^3 + x^2 + 100x + 5 \sin x$

∴ f'(x)= $3x^2 + 2x + 100 + 5\cos x$

= $3x^2 + 2x + 94 + (6 + 5\, cosx) > 0$

∴ f is an increasing function and consequently a one –one function.

Clearly f($-\infty$)=$-\infty$, f(∞)=∞ and f(x) is continuous, therefore range f= R= co domain f. Hence f is onto.

28. Let f(x) = $\frac{sin^{101}x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}}$, where [x] denotes the integral part of x is

(a) an odd function **(b) an even function**

(c) neither odd nor even function **(d) both odd and even function**

Ans. (a) when x= nπ, n ϵ I, sin x = 0 and $\left[\frac{x}{\pi}\right] + \frac{1}{2} \neq 0$

∴ f(x) = 0

∴ when x = nπ, f(x)= 0 and f(-x)= 0

∴ f(-x)= f(x)

When x $\neq n\pi, n\, \epsilon\, I, \frac{x}{\pi} \neq an\ integer$

$$\therefore \left[\frac{x}{\pi}\right] + \left[-\frac{x}{\pi}\right] = -1 \ \therefore \left[-\frac{x}{\pi}\right] = -1 - \left[\frac{x}{\pi}\right]$$

$$\Rightarrow \left[-\frac{x}{\pi}\right] + \frac{1}{2} = -\left[\frac{x}{\pi}\right] - \frac{1}{2} = -\left(\left[\frac{x}{\pi}\right] + \frac{1}{2}\right)$$

Now $f(-x) = \frac{sin^{101}(-x)}{\left[-\frac{x}{\pi}\right]+\frac{1}{2}} = \frac{-\sin x}{-(\left[\frac{x}{\pi}\right]+\frac{1}{2})} = \frac{\sin x}{\left[\frac{x}{\pi}\right]+\frac{1}{2}} = f(x)$

Hence in all cases f(-x)= f(x)

29 . If k be the value of x at which the function

f(x) = $\int_{-1}^{x} t(e^t - 1)(t-1)(t-2)^3(t-3)^5\,dt$ **has maximum value and sinx + cosecx = k, then for n ∈N,** $sin^n x + cosec^n x = \ldots$

(a) 2 **(b) -2** **(c)** $\frac{\pi}{2}$ **(d)** π

Ans. (a) f'(x) = $x(e^x - 1)(x-1)(x-2)^3(x-3)^5$

By Sign Rule we get

f(x) has max. at x = 2

∴ k = 2

Now sin x + cosec x = k ⟹ sin x + cosec x = 2

$\Longrightarrow(\sin x - 1)^2 = 0 \Longrightarrow \sin x = 1$

∴ cosec x = 1

Hence $sin^n x + cosec^n x = 2$

30. If f(x+ y) = f(x) + f(y) – xy – 1for all x, y ∈R and f(1)=1, then the number of solutions of f(n)= n, n ∈N is

(a) 0 **(b) 1** **(c) 2** **(d) more than 2**

Ans. (b)

Given f(x+ y)= f(x)+f(y) - xy - 1 $\forall\, x, y, \epsilon\, R$ ……………(1)

f(1)= 1 ……………..(2)

f(2) = f(1+1)= f(1)+f(-1)-1-1= 0

f(3)= f(2+1)= f(2)+f(1)-2.1-1= -2

f(n+1) = f(n) +f(1) – n – 1 = f(n)- n< f(n)

Thus f(1) > f(2) > f(3)> …and f(1)= 1

∴f(1)= 1 and f(n)< 1, for n> 1

Hence f(n)= n, n ϵ N has only one solution n= 1