B.Math Admission Test 2008

1. $f(x) = \frac{1}{t}\int_0^t f(x+y) - f(y)\,dy$

$\Rightarrow f(x+z) = \frac{1}{t}\int_0^t f(x+y+z) - f(y)\,dy$

$= \frac{1}{t}\int_0^t f(x+y+z) - f(x+y) + f(x+y) - f(y)\,dy$

$= f(x) + f(z)$ let $x+y = u$

now, $f(x+z) = f(x) + f(z)$

Its a famous problem which has three solutions (I know) which led to $f(x) = cx$

2. Process 1 : $f'(x+y) = f'(x)$ keeping y constant

$f'(x+y) = f'(y)$ keeping x constant

$f'(x+y) = f'(x) = f'(y) = c$ (let)

$f(x) = cx + k$

Putting $x=y=0$ in $f(x+y) = f(x) + f(y)$ we get $f(0) = 0 = k$

Process 2 : $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{f(h) + f(x) - f(x)}{h} = \lim_{h\to 0} \frac{f(h)}{h} = f'(0) = c$ (le

$f'(x) = c$

on integrating $f(x) = cx + k$

Putting $x=y=0$ in $f(x+y) = f(x) + f(y)$ we get $f(0) = k = 0$

Process 3: left to you. hint : p/q

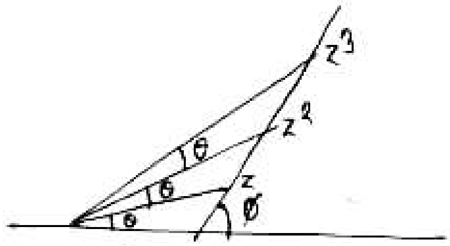
2'. $|P(n)| \le d|n|^3$ degree of $P(x) \le 3$

$|P(n)| \ge d|n|^3$ degree of $P(x) \ge 3$

$\therefore$ degree of $P(x) = 3$

every polynominal with degree 3 must have a real root.

3.



$\arg z = \theta$ (let)

$\arg z^2 = 2\arg z = 2\theta$

$\arg z^3 = 3\arg z = 3\theta$

now from coni's theorem,

$$\arg\left(\frac{z^3-z^2}{z-z^2}\right) = \pi$$

$\Rightarrow \arg(-z) = \pi$

indicates

$-z$ is real

hence, z is real

$$\arg\left(\frac{z^3-z}{z^2-z}\right) = 0$$

$\Rightarrow \arg(z) = 0$

indicates

z is real

4. consider $s_1 = a_1$; $s_2 = a_1+a_2$. $s_3 = a_1+a_2+a_3$; $s_n = a_1+a_2+\dots+a_n$

If any of these is divisible by n then we are done as we can say $r=n$ and $K=$ the specific term which is divisible

hence, ~~many~~ none of them considered divisible by n. let $s_i \equiv l_i \pmod n$

we get $(n-1)$ types of remainder but there are n numbers. Hence, by box principle two of the sums' remainders let s_K and s_r with $K<r$ are equal modulo n.

hence $s_r - s_K = (a_K + a_{K+1} + \dots + a_{K+r})$ is divisible by n.

5. $P(n) = (n-\alpha)(n-\beta)(n-\gamma)$

as α, β, γ, n integer hence

$n-\alpha = n-\beta = n-\gamma$

which leds to $\alpha=\beta=\gamma$

cantradiction as α, β, γ increased.

6. By strong induction,

$$F_{m+2} = \binom{m+1}{0} + \binom{m}{1} + \binom{m-1}{0} + \cdots$$

$$= \binom{m}{0} + \left(\binom{m-1}{0} + \binom{m-1}{1}\right) + \left(\binom{m-2}{1} + \binom{m-2}{2}\right) + \cdots$$

as $\binom{m+1}{0} = \binom{m}{0} = 1$

$$= \left\{\binom{m-1}{0} + \binom{m-2}{1} + \binom{m-3}{2} + \cdots\right\} + \left\{\binom{m}{0} + \binom{m-1}{1} + \binom{m-2}{2} + \cdots\right\}$$

$$= F_m + F_{m+1}$$

7. $\sum_{r=1}^{24} r^2 = \frac{24 \times 25 \times 26}{6} = 2600$

8. let, $a = \sin\theta$ $\quad d = \sin\alpha$
$b = \cos\theta$ $\quad c = \cos\alpha$

hence, given that $\sin\theta\cos\alpha + \cos\theta\sin\alpha = 0$

$\sin(\theta+\alpha) = 0$

$\theta = -\alpha$

hence, $a^2 + c^2 = \sin^2\theta + \cos^2(-\theta)$

$= \sin^2\theta + \cos^2\theta = 1$

$b^2 + d^2 = \cos^2\theta + \sin^2(\alpha) = \cos^2\theta + \sin^2\theta = 1$

$ab + cd = \sin\theta\cos\theta + \sin\alpha\cos\alpha - \sin(\theta+\alpha) = 0$

9. $x_1 + x_2 + \cdots + x_{n-1} = 1/x_n$ — (I)

$x_2 + x_3 + \cdots + x_n = 1/x_1$ — (II)

(I) − (II)

$x_1 - x_n = \left(1/x_n - 1/x_1\right) \Rightarrow x_1 x_2 = 1$

hence we can get by similar attempts that

$x_i x_j = 1$

hence $x_i = 1$

10 by Fermat's little theorem we get,

$a^{p-1} \equiv 1 \pmod{p}$

$a^{p-1} - 1$ is divisible by p

$a^p \equiv a \pmod{p}$

$\Rightarrow a^p - 1 \equiv a - 1 \pmod{p}$