1. (B) Let coordinate of P is (h, k)
Let coordinate of A is (0, a) and coordinate of B is (b, 0)

Now, \( AP : PB = 1 : 2 \)
\[ h = \frac{0 \times 2 + 1 \times b}{3} \]
\[ h = \frac{b}{3} \]
\[ k = \frac{a \times 2 + 0 \times 1}{3} \]
\[ k = \frac{2a}{3} \]

Now, length of the rod remains constant as the rod slides on the wall.
\[ \sqrt{a^2 + b^2} = c \]
\[ \left(\frac{3k}{2}\right)^2 + (3h)^2 = c^2 \]
\[ 4h^2 + k^2 = c_1^2 \]
\[ \text{The locus of P is } 4x^2 + y^2 = c_1^2 \]

2. (A) Now, \( x^2 + y^2 = 2007 \) = odd.
One of \( x \) and \( y \) is even and another is odd.

Let \( x \) is even and \( y \) is odd (without loss of generality) Now, \( x^2 \equiv 0 \) (mod 4) (as \( x \) is even)

And, \( y \equiv \pm 1 \) (mod 4)
\[ y^2 \equiv (\pm 1)^2 = 1 \) (mod 4)

Now, dividing the equation by 4 we get,
\[ 0 + 1 \equiv 3 \) (mod 4)
\[ 1 \equiv 3 \) (mod 4)

Which is impossible.
\[ \text{No solution.} \]
3. (C) Clearly they will intersect at two points. One in (0, 1) and another in \((e, e^2)\) as \(f_1(e) < f_2(e)\) and \(f_1(e^2) > f_2(e^2)\).

4. (B) Now, \(u_n = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \ldots + \frac{n}{2^n}\)
\[\Rightarrow (1/2)u_n = 1/2^2 + 2/2^3 + 4/2^4 + \ldots + (n - 1)/2^n + n/2^{(n + 1)}\]

Now, \(u_n - (1/2)u_n = \frac{1}{2} + \left(\frac{2}{2^2} - \frac{1}{2^2}\right) + \left(\frac{3}{2^3} - \frac{2}{2^3}\right) + \left(\frac{4}{2^4} - \frac{3}{2^4}\right) + \ldots + \left\{\frac{n}{2^n} - \left(\frac{n - 1}{2^n}\right)\right\} - \frac{n}{2^{(n + 1)}}\)
\[\Rightarrow (1/2)u_n = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \ldots + \frac{1}{2^n}\right) - \frac{n}{2^{(n + 1)}}\]
\[\Rightarrow u_n = \left(1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots + \frac{1}{2^{n-1}}\right) - \frac{n}{2^n}\]
\[\Rightarrow u_n = 1*\left\{1 - \left(\frac{1}{2}\right)^n\right\}/\left(1 - \frac{1}{2}\right) - \frac{n}{2^n}\]
\[\Rightarrow u_n = 2(1 - 1/2^n) - \frac{n}{2^n}\]
\[\Rightarrow \lim_{n \to \infty} u_n = 2\]
5. (D) Now, \( \frac{1}{(z - 3)} + \frac{1}{(z - 3w)} + \frac{1}{(z - 3w^2)} \)
\[ = \frac{(z - 3w)(z - 3w^2) + (z - 3)(z - 3w^2) + (z - 3)(z - 3w)}{(z - 3)(z - 3w)(z - 3w^2)} \]
\[ = \frac{z^2 - 3wz - 3w^2z + 9w^3 + (z - 3)(z - 3w^2 + z - 3w)}{(z - 3)(z^2 - 3wz - 3w^2z + 9w^3)} \]
\[ = \frac{z^2 + 3z + 9 + (z - 3)(2z + 3)}{(z - 3)(z^2 + 3z + 9)} \]
\[ = \frac{3z^2}{(z^3 - 27)} \]

6. (C) The sample space looks like below,
\[ \{n, f(n)\} = \{(1, 1); (2, 2); (3, 3); (4, 4)\}; \{(1, 1); (2, 2); (3, 4); (4, 3)\}; \{(1, 2); (2, 1); (3, 4); (4, 3)\}; \{(1, 2); (2, 3); (3, 4); (4, 1)\}; \{(1, 3); (2, 2); (3, 1); (4, 4)\}; \{(1, 3); (2, 2); (3, 4); (4, 1)\}; \{(1, 3); (2, 4); (3, 1); (4, 2)\}; \{(1, 3); (2, 4); (3, 2); (4, 1)\}; \{(1, 4); (2, 1); (3, 2); (4, 3)\}; \{(1, 4); (2, 3); (3, 2); (4, 1)\} \]
\[ \Rightarrow \text{Number of such functions} = 12. \]

7. (D)
Now, \( \lim \left[ \frac{f(x) - f(0)}{(x - 0)} \right] \) as \( x \to 0^+ \)
\[ = \lim e^{-1/x}/x \text{ as } x \to 0^+ \]
Let, \( z = 1/x \). As \( z \to \infty \) as \( x \to 0^+ \)
So, \( \lim (z/e^z) \) as \( z \to \infty \)
\[ = \lim 1/e^z \text{ as } z \to \infty \text{ (Applying L’Hospital rule)} \]
\[ = 0 \]
Now, \( \lim \left[ \frac{f(x) - f(0)}{(x - 0)} \right] \) as \( x \to 0^- \)
\[ = 0 \]
\[ \Rightarrow f \text{ is differentiable.} \]
Now, \( f'(x) = e^{-1/x}/x^2 \) for \( x > 0 \)
\[ = 0 \text{ for } x \leq 0 \]
Now, \( \lim f(x) \) as \( x \to 0^+ \)
\[ = \lim e^{-1/x}/x^2 \]
Let, \( z = 1/x \), As \( z \to \infty \) as \( x \to 0^+ \)
So, \( \lim z^2/e^z \) as \( z \to \infty \)
= \( \lim 2z/e^z \) as \( z \to \infty \) (Applying L’Hospital rule)
= \( \lim 2/e^z \) as \( z \to \infty \) (Again applying L’Hospital rule)
= 0

And \( \lim f'(x) \) as \( x \to 0^- \)
= 0

Also \( f'(0) = 0 \).

8. (B) Now, last digit of 9! is 0.

Now, \( 3^2 \equiv -1 \pmod{10} \)
\[ \Rightarrow (3^2)^{4983} \equiv (-1)^{4983} \pmod{10} \]
\[ \Rightarrow 3^{9966} \equiv -1 \pmod{10} \]
\[ \Rightarrow 3^{9966} \equiv 9 \pmod{10} \]

9. (C) \( f''(x) = \{(4x + 3)(2x - 1) - (2x^2 + 3x + 1)\times 2\}/(2x - 1)^2 = 0 \)
\[ 8x^2 - 4x + 6x - 3 - 4x^2 - 6x - 2 = 0 \]
\[ 4x^2 - 4x - 5 = 0 \]
\[ x = \{4 \pm \sqrt{(16 + 4\times 4\times 5)}\}/2\times 4 \]
\[ x = (4 \pm 4\sqrt{6})/8 \]
\[ x = (1 \pm \sqrt{6})/2 \]

For no value of \( x \) in \([2, 3]\) \( f(x) \) attains maximum or minimum value.

Now, \( f'(x) \) also is not less than 0 for all values of \( x \) in \((2, 3)\).

\[ \Rightarrow f(x) \text{ is not decreasing in } (2, 3). \]

Now, \( f(2) = 5 \) and \( f(3) = 28/5 \)

10. (D)
Now, the angle between the two tangents is 90°.

Now, the angle between the two tangents and their respective normal is 90°.

The two normal meets at focus (a, 0)

Now, the quadrilateral of four points (h, k), (at\(^2\), 2at), (as\(^2\), 2at), (a, 0) have total angle 360°

- The normal meet at 90° at focus (360° - 90° - 90° - 90° = 90°)
- The quadrilateral is a square because the length of the tangents from a point are equal.

So, we have, \((at^2 - a)^2 + (2at - 0)^2 = (as^2 - a)^2 + (2as - 0)^2\)

(distances of \((at^2, 2at)\) and \((as^2, 2as)\) from focus are same as it is a square)

\[
\begin{align*}
(t^2 - 1)^2 + 4t^2 &= (s^2 - 1)^2 + 4s^2 \\
(t^2 + 1)^2 &= (s^2 + 1)^2 \\
t^2 + 1 &= s^2 + 1 \\
t^2 - s^2 &= 0 \\
(t + s)(t - s) &= 0 \\
t + s &= 0 \text{ (As } \ t \neq s) \\
s &= -t
\end{align*}
\]

Now, the points are \((at^2, 2at)\) and \((at^2, -2at)\)

Now, the distances of these two points from P are equal.

- \((at^2 - h)^2 + (2at - k)^2 = (at^2 - h)^2 + (-2at - k)^2\)
- \((2at + k)^2 = (2at - k)^2\)
- \(k = 0\)
- The locus of P is \(y = 0\)
- A straight line.

11. (C)

12. (D) Now, \(2^a - 5^b7^c = 1\) where a, b, c are positive.

Minimum value of b, c = 1

- a > 5.

Dividing the equation by 8 we get,

\(0 - (5 \text{ or } 1)(-1)^c \equiv 1 \pmod{5}\) (If b is odd \(5^b \equiv 5 \pmod{8}\) and if even then \(5^b \equiv 1 \pmod{8}\)

Clearly to hold the equation, b must be even and c must be odd.
Now, dividing the equation by 5 we get, $2^a - 0 \equiv 1 \pmod{5}$

- $a$ is divisible by 4 because $(2^2)^{2m} \equiv (-1)^{2m} \equiv 1 \pmod{5}$

Now, dividing the equation by 3 we get, $(-1)^a - (-1)^b \times 1^c \equiv 1 \pmod{3}$

$\Rightarrow 1 - 1 \equiv 1 \pmod{3}$ (As $a, b$ both even)

Which is impossible.

The equation has no solution.

13. (A) Let $z = h + ik$

We have, $|h + ik - ia| = k + 1$

- $\sqrt{h^2 + (k - a)^2} = k + 1$
- $h^2 + k^2 - 2ak + a^2 = k^2 + 2k + 1$
- $h^2 - 2k(a + 1) + (a^2 - 1) = 0$
- Locus of $z$ is $x^2 - 2y(a + 1) + a^2 - 1 = 0$
- It's a parabola.

14. (B) Now, $\tan(\sin x)$ never attains the value 1.

15. (D) Now, $f'(x) = \frac{2(x - 1) - 2x}{(x - 1)^2} = \frac{-2}{(x - 1)^2} < 0$

$f(x)$ is strictly decreasing.

$f(x)$ is one-one and onto.

16. (C) Now, $g(t) = \int f(x) \, dx$ (integration running from 0 to t)

$= \int f(c) \, dx$ (integration running from 0 to t)

(As $f(x + 1) = f(x) = tf(c)$)

$\Rightarrow g''(t) = f(c)$

Now, $h(t) = \lim \frac{g(t + n)}{n}$ as $n \to \infty$

$= \lim \frac{g'(t + n)}{1}$ as $n \to \infty$ (Applying L’Hospital rule)

$= \lim f(c)$ as $n \to \infty$

$= f(c)$.

So, $h(t)$ is defined for all $t \in \mathbb{R}$ and independent of $t$. 
17. (A)  
Now, \( a_1 = 24^{1/3} \)  
Let \( a_n < 3 \)  
Now, \( a_{n+1} = (a_n + 24)^{1/3} \)  
We have, \( a_n < 3 \)  
\[ \Rightarrow (a_n + 24)^{1/3} < 3 \]  
\[ \Rightarrow a_{n+1} < 3 \]  
The integer part of \( a_{100} \) is 2.  

18. (B) Now, \( \frac{4}{(4 - x^2)} + \frac{9}{(9 - y^2)} \)  
\[ = \frac{4}{(4 - x^2)} + \frac{9}{(9 - 1/x^2)} \]  
\[ = \frac{4}{(4 - x^2)} + \frac{9x^2}{(9x^2 - 1)} \]  
\[ = \frac{4}{(4 - x^2)} + 1/(9x^2 - 1) + 1 \]  
\[ = (36x^2 - 4 + 4 - x^2)/(4 - x^2)(9x^2 - 1) + 1 \]  
\[ = 35x^2/(4 - x^2)(9x^2 - 1) + 1 \]  
Let, \( f(x) = 35x^2/(4 - x^2)(9x^2 - 1) + 1 \)  
f'(x) = \[35\{2x(4 - x^2)(9x^2 - 1) + 2xx^2(9x^2 x^2)^2(9x^2 - 1)^2 \} = 0 \]  
\[ \Rightarrow 36x^2 - 4 - 9x^4 + x^2 + 9x^4 - x^2 - 36x^2 + 9x^4 = 0 \]  
\[ \Rightarrow 9x^4 - 4 = 0 \]  
\[ \Rightarrow x^2 = 2/3 \]  
Now, \( f(\pm\sqrt{2/3}) = 35(2/3)/(4 - 2/3){9x(2/3) - 1} + 1 = 12/5 \)  

19. (A) Clearly the limit is e.  

20. (D) \( f(-1) = 0 \)  
\[ f(x) = x(x^3 + 1) + (x^2 - 1) \]  
\[ = x(x + 1)(x^2 - x + 1) + (x + 1)(x - 1) \]  
\[ = (x + 1)(x^3 - x^2 + x + x - 1) \]  
\[ = (x + 1)(x^3 - x^2 + 2x - 1) \]  
Now, \( x^3 - x^2 + 2x - 1 < 0 \) for all \( x < 0 \)
There cannot be any root for \( x < 0 \)
Option (b) and (c) cannot be true.

Now, \( f(x) = x^4 + x^2 + x - 1 \)
\[ f'(x) = 4x^3 + 2x + 1 \]
Now, \( f'(-1) = -5 \) and \( f'(0) = 1 \)
The sign changes.
\[ f'(x) \text{ has at least one root in } (-1, 0) \]
Now, \( f''(x) = 12x^2 + 2 > 0 \)
\[ f \text{ has a local minimum in } (-1, 0) \]
Now, \( f'(x) > 0 \) for all \( x \).
- \( f'(x) \) is strictly increasing.
- \( f'(x) \) has only one root and other two roots are complex.
- \( f \) has exactly one local minima in \((-1, 0)\)

21. (A) We take the arrangement
\[ A()()B,()()A()()B()()A()B()A()()B()() \]
We can put rest 6 A’s in 4 places in \( 6 + 4 - 1 C_4 - 1 \) ways = \( 9C_3 \) ways.
We can put rest 4 B’s in 4 places in \( 4 + 4 - 1 C_4 - 1 \) ways = \( 7C_3 \) ways.
So, this can be done in \( 9C_3 \times 7C_3 \) ways.
Now, we have started with A. We can also start with B. So same another case appear.
- Total number of ways = \( 2 \times 9C_3 \times 7C_3 \)

22. (C)
\[ f(x) = x^{n+1} \text{ if } x > 0 \]
\[ f(x) = -x^{n+1} \text{ if } x < 0 \]
\[ f(x) = 0 \text{ if } x = 0 \]
\[ \lim \{ f(x) - f(0) \}/(x - 0) \text{ as } x \to 0^+ \]
\[ = \lim \{ (x^{n+1} - 0)/x \} \text{ as } x \to 0^+ \]
\[ = \lim x^n \text{ as } x \to 0^+ \]
\[ = 0 \]
\[
\lim \left[ \frac{f(x) - f(0)}{x - 0} \right] \text{ as } x \to 0^-
\]
\[
= \lim \{(-x^{n+1} - 0)/x\} \text{ as } x \to 0^-
\]
\[
= \lim -x^n \text{ as } x \to 0^-
\]
\[
= 0
\]
\[
\Rightarrow f \text{ is differentiable everywhere.}
\]

23. (C) The equation of the given line is, \(2x + 3y - k = 0\)
\[
\Rightarrow x/(k/2) + y/(k/3) = 1
\]
\[
A = (k/2, 0) \text{ and } B = (0, k/3)
\]
Mid-point of AB i.e., centre of the circle = \((k/4, k/6)\)

Radius = \(\sqrt{(k/2 - k/4)^2 + (0 - k/6)^2}\)
\[
= \sqrt{(k^2/16 + k^2/36)} = (k/12)\sqrt{9 + 4}
\]
\[
= (k\sqrt{13})/12
\]
Equation of the circle is, \((x - k/4)^2 + (y - k/6)^2 = (k\sqrt{13})^2/12\)
i.e. \(x^2 + y^2 - kx/2 - ky/3 = 13k^2/144 - k^2/16 - k^2/36\)
i.e. \(x^2 + y^2 - kx/2 - ky/3 = 0\)

24. (D) If \(0 < \alpha < \frac{1}{2}\)

Then, \(2\alpha < 1\)
\[
\Rightarrow (2\alpha)^n \to 0 \text{ as } n \to \infty
\]
\[
x_n \to \infty
\]

If \(\alpha > \frac{1}{2}\)

Then \(2\alpha > 1\)
\[
\Rightarrow (2\alpha)^n \to \infty \text{ as } n \to \infty
\]
\[
x_n \to 0
\]

25. (C)

Now, option (a) cannot be true as \(\Theta \in (0, n/2)\) and \(\sin\Theta \in (0, 1)\)
Now, option (b) cannot be true. Let \(\Theta = n/4\), then \(\cos(\sin\Theta) > \cos\Theta\)
Option (d) is same as option (a). Let \(\cos\Theta = x\) then it becomes, \(x < \sin x\)
26.  (A) Clearly shortest distance between A and B’ along the surface is \(\sqrt{5^2 + 2^2} = \sqrt{29}\).

27.  (B)
Let, \(\sum \frac{1}{\sqrt{k}}\) summation running from \(k = 2\) to \(k = 9999 = S\)

Now, \(\frac{1}{2\sqrt{k + 1}} < \sqrt{k + 1} - \sqrt{k}\)
\(\frac{1}{2\sqrt{2}} < \sqrt{2} - \sqrt{1}\)
\(\frac{1}{2\sqrt{3}} < \sqrt{3} - \sqrt{2}\)
\(\frac{1}{2\sqrt{4}} < \sqrt{4} - \sqrt{3}\)
...
...
\(\frac{1}{2\sqrt{9999}} < \sqrt{9999} - \sqrt{9998}\)

Adding we get, \((1/2) \sum_{k=2}^{9999} \frac{1}{\sqrt{k}} \sum_{k=2}^{9999} \frac{1}{\sqrt{k}} < \sqrt{9999} - \sqrt{1}\)
\(S < 2(\sqrt{9999} - 1)\)
\(S < 197.99\)
Now,
\(\sqrt{k + 1} - \sqrt{k} < 1/(2\sqrt{k})\)
\(\sqrt{3} - \sqrt{2} < 1/2\sqrt{2}\)
\(\sqrt{4} - \sqrt{3} < 1/2\sqrt{3}\)
\(\sqrt{5} - \sqrt{4} < 1/2\sqrt{4}\)
...
...
\(\sqrt{10000} - \sqrt{9999} < 1/2\sqrt{9999}\)

Adding the inequalities we get,
\(\sqrt{10000} - \sqrt{2} < \sum (1/2\sqrt{k})\) summation running from \(k = 2\) to \(k = 9999\)
\(\Rightarrow S > 2(100 - \sqrt{2})\)
\(\Rightarrow S > 197.17\)
The integer part of \(S\) is 197.
28. (B)
Now, \( x^6 + y^4 - (x^4 + y^2) \)
\[
= x^4(x^2 - 1) + y^2(y^2 - 1)
\]
Now, \( x \) and \( y \) both less than or equal to 1.
\( x^2 \) and \( y^2 \) both less than or equal to 1.
\( x^6 + y^4 \leq x^4 + y^2 \leq 1 \)
All \((x, y)\) that satisfies \( x^4 + y^2 \leq 1 \) also satisfies \( x^6 + y^4 \leq 1 \)
\( A \) is a subset of \( B \).

29. (A) Now, \( AM \geq GM \)
\[
\left( \binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \ldots + \binom{10}{10} \right)/11
> \left\{ \left( \binom{10}{0} \right) \left( \binom{10}{1} \right) \ldots \left( \binom{10}{10} \right) \right\}^{1/11}
\]
(Equality cannot hold as all the numbers are not equal)
\[
\Rightarrow (2^{10/11})^{11} > \left\{ \left( \binom{10}{0} \right) \left( \binom{10}{1} \right) \ldots \left( \binom{10}{10} \right) \right\}^{1/11}
= \left( \binom{10}{1} \right)^2 \left( \binom{10}{2} \right)^2 \left( \binom{10}{3} \right)^2 \left( \binom{10}{4} \right)^2 \left( \binom{10}{5} \right)^2
\]

30. (D) Let the roots of the equation are \( s, sr, sr^2, sr^3 \)
Now, \( -a = s + sr + sr^2 + sr^3 = s(r^4 - 1)/(r - 1) \)
\( b = s*sr + s*sr^2 + s*sr^3 + sr*sr^2 + sr*sr^3 + sr^2*sr^3 \)
\( \Rightarrow b = s^2r + s^2r^2 + 2s^2r^3 + s^2r^4 + s^2r^5 = s^2r(r^5 - 1)/(r - 1) + s^2r^3 \)
\( c = s^3r^3 + s^3r^6 + s^3r^5 + s^3r^4 \)
\( \Rightarrow c = s^3r^3(r^4 - 1)/(r - 1) \)
\( d = s*sr*sr^2*sr^3 \)
\( \Rightarrow d = s^4r^6 \)
Now, \( c^2 = s^6r^6 \{(r^4 - 1)/(r - 1)\}^4 \)
\( = (s^4r^6)[s(r^4 - 1)/(r - 1)]^2 \)
\( = dxa^2 \)
\( \Rightarrow c^2 = a^2d \)