BSTAT/BMATH 2014 SOLVED PAPER

1. (B) Now, $a - b^2 \geq 1/4$

⇨ $a \geq b^2 + 1/4$

Similarly, $b \geq c^2 + 1/4$

⇨ $a \geq (c^2 + 1/4)^2 + 1/4$

Again $c \geq d^2 + 1/4$

⇨ $a \geq \{(d^2 + 1/4)^2 + 1/4\}^2 + 1/4$

Again $d \geq a^2 + 1/4$

⇨ $a \geq [\{(a^2 + 1/4)^2 + 1/4\}^2 + 1/4]^2 + 1/4$

Now, $(a^2 + 1/4)/2 \geq \sqrt{\{a^2*(1/4)\}} = a/2$

⇨ $a^2 + 1/4 \geq a$

⇨ $(a^2 + 1/4)^2 \geq a^2$ .... (A)

⇨ $(a^2 + 1/4)^2 + 1/4 \geq a^2 + 1/4$

⇨ $\{(a^2 + 1/4)^2 + 1/4\}^2 \geq (a^2 + 1/4)^2 \geq a^2$ (using (A))

So going forward in this way we will get,

$a \geq [\{(a^2 + 1/4)^2 + 1/4\}^2 + 1/4]^2 + 1/4 \geq a^2 + 1/4$ (Using (A) again and again)

⇨ $a^2 - a + 1/4 \leq 0$

⇨ $(a - 1/2)^2 \leq 0$

⇨ $a - 1/2 = 0$

⇨ $a = 1/2$

Similarly, $b = 1/2$, $c = 1/2$ and $d = 1/2$

Putting the values backward, we get, $a - b^2 = 1/4$, $b - c^2 = 1/4$, $c - d^2 = 1/4$ and $d - a^2 = 1/4$

⇨ Only one solution.

2. (A) Now, $\log_{12}18 = a$

⇨ $\log18/\log12 = a$

⇨ $\log2\times3^2/\log2^2\times3 = a$

⇨ $(\log2 + 2\log3)/(2\log2 + \log3) = a$

⇨ $\log2 + 2\log3 = 2a\log2 + a\log3$

⇨ $(2a - 1)\log2 = (2 - a)\log3$

⇨ $\log3/\log2 = (2a - 1)/(2 - a)$

Now, $\log_{24}16 = \log16/\log24$

$= \log2^4/\log2^3\times3$

$= 4\log2/(3\log2 + \log3)$

$= 4/(3 + \log3/\log2)$

$= 4/\{3 + (2a - 1)/(2 - a)\}$

$= 4(2 - a)/(6 - 3a + 2a - 1)$

$= (8 - 4a)/(5 - a)$

3. (C) Now, $\sec^2x - \tan^2x = 1$

- ⇨ **$(\sec x - \tan x)(\sec x + \tan x) = 1$**
- ⇨ **$(\sec x - \tan x) = 1/(\sec x + \tan x)$**
- ⇨ **$(\sec x - \tan x) = 1/2\cos x$**
- ⇨ **$(\sec x - \tan x) = \sec x/2$**
- ⇨ **$\sec x/2 = \tan x$**
- ⇨ **$\sin x = ½$**
- ⇨ **$x = \pi/6, 5\pi/6$**

4. (C) Now, last digit must be 2.

3 and 9 both are divisible by 3. We need to consider 3 2's or 6 2's to get the number divided by 3 and hence 6.

We put 3 2's and the rest 3 numbers can be two 3 one 9 or two 9 one 3 or three 3 or three 9.
In first case number of numbers = 5!/(2!×2!) = 30 (note that last digit 2 is fixed, it cannot permutate)

In second case number of numbers = 5!/(2!×2!) = 30

In third case number of numbers = 5!/(3!×2!) = 10

In fourth case number of numbers = 5!/(3!×2!) = 10

And 1 number for 6 2's.

So, total number of six digit numbers with the given criteria = 30 + 30 + 10 + 10 + 1 = 81

5. (B) Let, $I = \int\{(\tan^{-1}x)/x\}dx$ (integration running from 1/2014 to 2014)

⇨ $I = \int[\{\cot^{-1}(1/x)\}/x]dx$ (integration running from 1/2014 to 2014)

⇨ $I = \int[\{\pi/2 - \tan^{-1}(1/x)\}/x]dx$ (integration running from 1/2014 to 2014)

⇨ $I = (\pi/2)\int dx - \int[\{\tan^{-1}(1/x)\}/x]dx$ (integration running from 1/2014 to 2014)

⇨ $I = (\pi/2)\log x|_{1/2014}^{2014} - \int[\{\tan^{-1}(1/x)\}/x]dx$ (integration running from 1/2014 to 2014)

Now, let $x = 1/z$

⇨ $dx = -dz/z^2$, at $x = 1/2014$, $z = 2014$ and $x = 2014$, $z = 1/2014$

⇨ $I = (\pi/2)\{\log(2014) - \log(1/2014)\} - \int\{(z\tan^{-1}z)/(-z^2)\}dz$ (integration running from 2014 to 1/2014)

⇨ $I = (\pi/2)\{\log(2014) - \log(2014)^{-1}\} + \int\{(\tan^{-1}z)/z\}dz$ (integration running from 2014 to 1/2014)

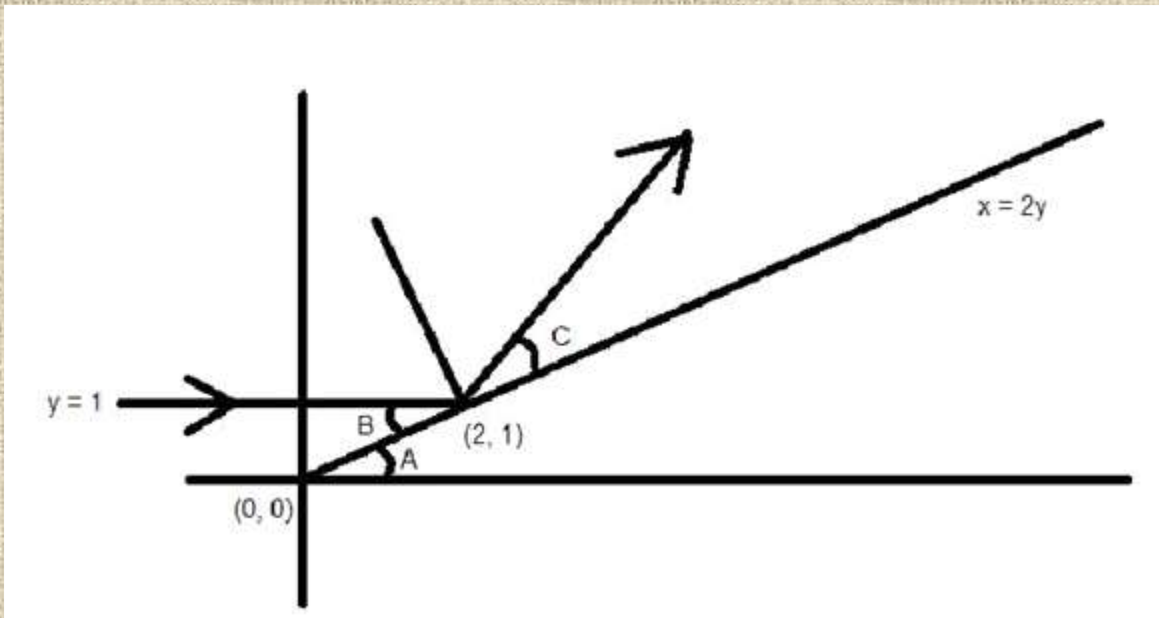
⇨ $I = (\pi/2)\{\log(2014) + \log(2014)\} - \int\{(\tan^{-1}z)/z\}dz$ (integration running from 1/2014 to 2014) (note that – sign came up due to upper limit and lower limit change)

⇨ $I = (\pi/2)*2\log(2014) - I$

⇨ $2I = \pi\log(2014)$

⇨ $I = (\pi/2)\log(2014)$

6. **(A)**



Angle C = Angle B = Angle A = $\tan^{-1}(1/2)$

⇨ **tanC = ½**

⇨ **(m – ½)/(1 + m/2) = ½ (m is slope of the reflected ray line and ½ is slope of x = 2y)**

⇨ **m – ½ = ½ + m/4**

⇨ **3m/4 = 1**

⇨ **m = 4/3**

The equation of reflected ray line is, y – 1 = (4/3)(x – 2)

⇨ **4x – 3y = 5**

7. (D) If x > 0, then 2x – [x] = 4

⇨ **2x = 4 + [x]**

RHS is an integer as [x] is an integer.

Either x is (an odd number)/2 or an integer. First case (x is an odd number/2)

Let x = p/2 where p is odd. Then [x] = (p

– 1)/2 Putting the values we get, p = 4 +

(p – 1)/2

p = 7

x = 3.5

Second case (x is an integer) 2x = 4 + x

⇨ **x = 4**

If x < 0 then [x] – 2x = 4

Similarly 2 cases, x is an odd integer/2 or x is an integer. Third case (x is an odd integer/2)

Let x = p/2 where p is odd integer

⇨ **[x] = (p – 1)/2 Putting the values we get,**

(p – 1)/2 – p = 4

⇨ **p = -9**

⇨ **x = -4.5**

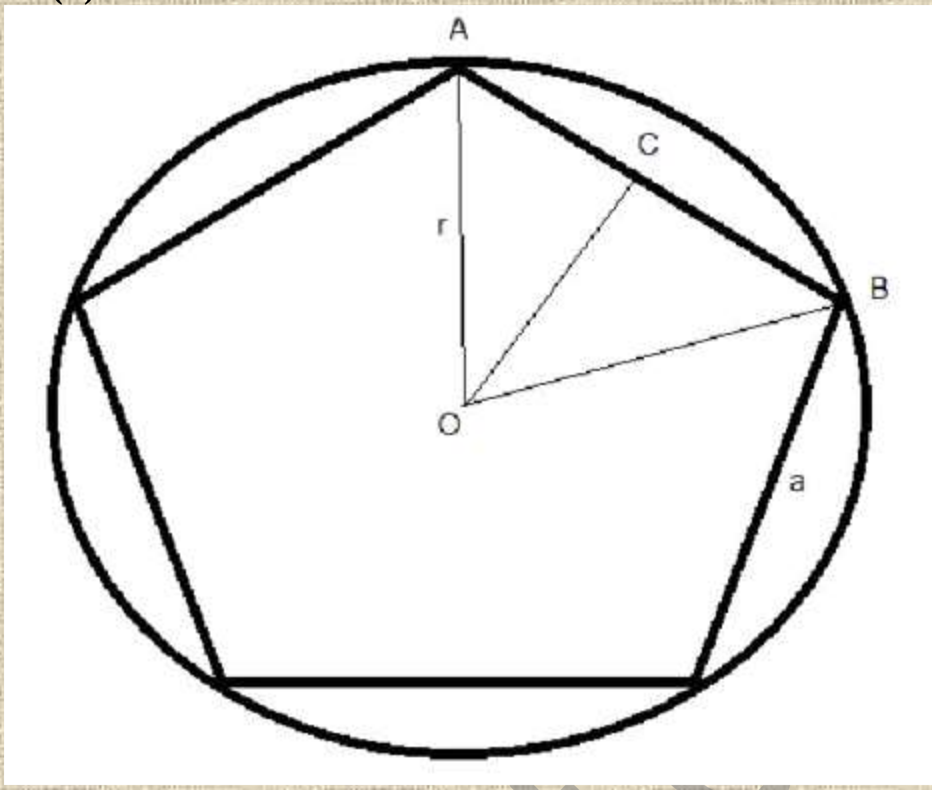
Fourth case (x is integer)

So, x – 2x = 4

⇨ **x = -4**

⇨ **Four solutions viz. 3.5, 4, -4.5, -4**

8. **(B)**



Angle A = (5 – 2)*180°/5 = 108°

⇨ **Angle OAB = 108°/2 = 54°**

From triangle OAC,

⇨ **Cos54° = AC/OA**

⇨ **AC = rcos54°**

⇨ **AB = 2rcos54°**

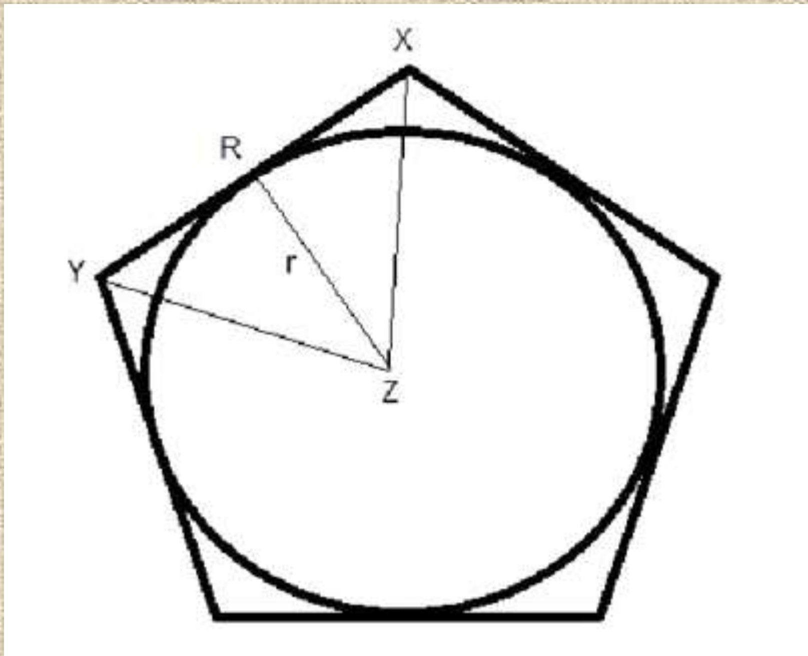
Now, sin54° = OC/OA

⇨ **OC = rsin54°**

Area of triangle OAB = (1/2)×OC×AB

= (1/2)(rsin54°)(2rcos54°) = (r^2/2)sin108°

⇨ **Area of the inscribed pentagon = 5×(r^2/2)sin108°**



Angle X = (5 – 2)×180°/5 = 108°

⇨ **Angle RXZ = 108°/2 = 54°**

Now, RX/RZ = cot54°

⇨ **RX = rcot54°**

⇨ **XY = 2rcot54°**

Now, area of triangle XYZ = (1/2)×XY×RZ = (1/2)×(2rcot54°)×r

= r^2cot54°

⇨ **Area of circumscribed pentagon = 5r^2cot54°**

Now, ratio of area of inscribed and circumscribed pentagons is

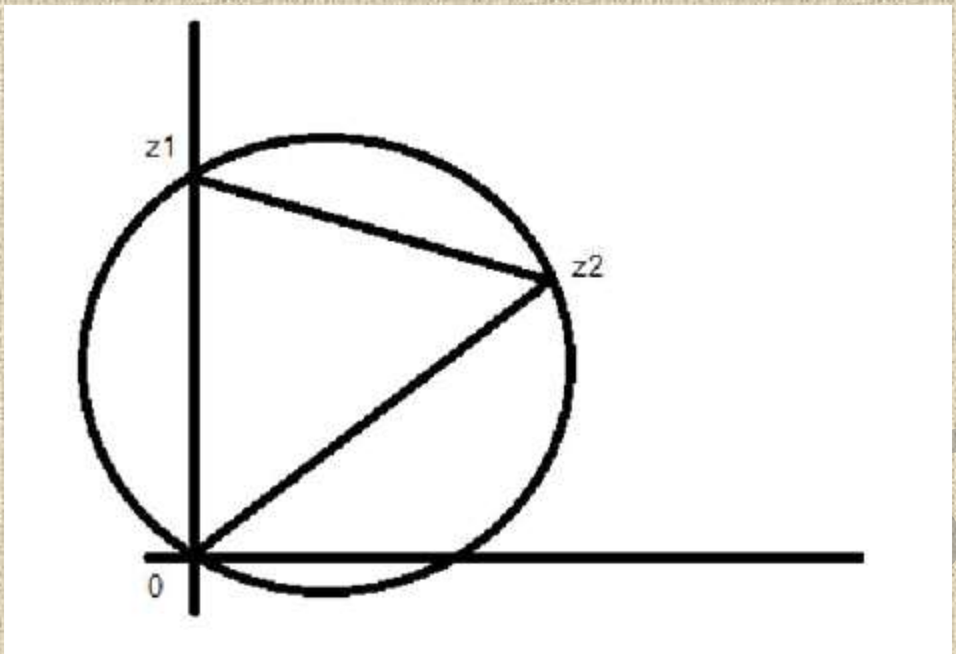
{5×(r^2/2)sin108°}/(5r^2cot54°)

= **(2sin54°cos54°)(sin54°)/2cos54°**

= **$\sin^2$54°**

= **$\cos^2$36°**

9. **(C)**



Let, $z_1 = iy_1$ and $z_2 = x_2 + iy_2$

Now, mid-point of z_1, z_2

$= x_2/2 + i(y_1 + y_2)/2$

Slope of z_1, $z_2 = (y_2 - y_1)/x_2$

Slope of perpendicular bisector of z_1, $z_2 = -x_2/(y_2 - y_1)$

Equation of perpendicular bisector of z_1, z_2 is,
$y - (y_1 + y_2)/2 = -\{x_2/(y_2 - y_1)\}(x - x_2/2)$

Equation of perpendicular bisector of z_1 is, $y = y_1/2$

Solving the two equations we get the circumcentre of triangle z_1, z_2, 0 as the intersection of these two lines is circumcentre.

Now, $y_1/2 - (y_1 + y_2)/2$
$= -(x_2/(y_2 - y_1)\}(x - x_2/2)$

⇨ $y_2(y_2 - y_1)/2x_2 = x - x_2/2$

⇨ $x = y_2(y_2 - y_1)/2x_2 + x_2/2$

Now, we have,

⇨ $|z_1 + z_2| = |z_1 - z_2|$

⇨ $|iy_1 + x_2 + iy_2| = |iy_1 - x_2 - iy_2|$

⇨ $x_2^2 + (y_1 + y_2)^2 = (-x_2)^2 + (y_1 - y_2)^2$

⇨ $4y_1y_2 = 0$

⇨ $y_2 = 0$

⇨ $x = x_2/2$

⇨ The circumcentre $z = x_2/2 + iy_1/2 = (z_1 + z_2)/2$

10. (A)

Everyone gets at least one chocolate and exactly 2 students get at least 2 chocolate. Now, the rest 8 – 2 = 6 students get exactly one chocolate because if they get more than one chocolate then there will be more than 2 students who will get at least 2 chocolates and hence contradicting the statement that exactly 2 students get at least 2 chocolates.

Now, we can choose any 2 students out of 8 students in $^8C_2$

ways. Now, we give one chocolate to 8 students each.

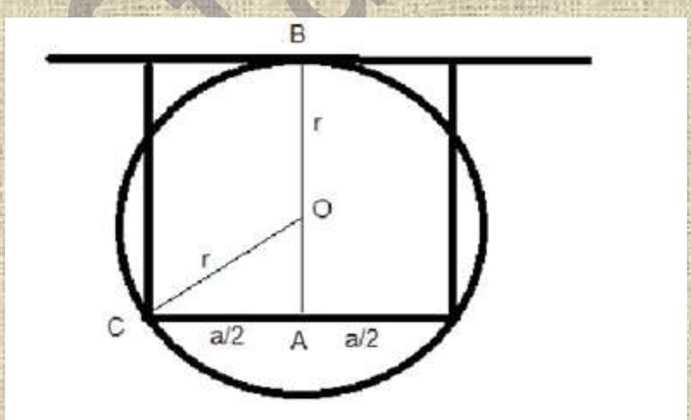
Remaining chocolates = 20 – 8 = 12

Now, the chosen two students will get at least one chocolate among 12 chocolates and we need to distribute rest 12 chocolates among those 2 students only.

Number of ways = $^{12-1}C_{2-1} = {}^{11}C_1$

⇨ **Total number of ways = $^8C_2 * {}^{11}C_1 = (8*7/2)*11 = 308$**

11. (D)



Now, from triangle OAC we get, OA = $\sqrt{(r^2 - a^2/4)}$
Now, AO + OB = a

⇨ $\sqrt{(r^2 - a^2/4)} + r = a$
⇨ $\sqrt{(r^2 - a^2/4)} = a - r$
⇨ $r^2 - a^2/4 = a^2 - 2ar + r^2$
⇨ $5a^2/4 = 2ar$
⇨ $a = 8r/5$

12. (C) The element of P which gets divided by 2^n where n is largest is $2^6\times2^5\times(3\times2^5)\times2^4\times(3\times2^4) = 9\times2^{24}$
⇨ n = 24

13. (B) So, $g(x) = 3ax^2 + 2bx + c - 6ax - 2b + 6a$

Let us put x = -r where r is real and positive.

$g(-r) = 3ar^2 - 2b(r + 1) + c + 6ar + 6a$

Now, $f'(x) = 3ax^2 + 2bx + c$
$f'(-(r + 1)) > 0$ (As f(x) is increasing)
$3a(r + 1)^2 - 2b(r + 1) + c > 0$ ....(A)
Now, $g(-r) = 3a(r^2 + 2r + 1) - 2b(r + 1) + c + 3a$
$= 3a(r + 1)^2 - 2b(r + 1) + c + 3a > 0$ (As a > 0 and from (A))

Now, let r > 0

$g(r) = 3ar^2 - 6ar + 3a + 2b(r - 1) + c +$

$3a = 3a(r - 1)^2 + 2b(r - 1) + c + 6a$

Now, $f'(r - 1) = 3a(r - 1)^2 + 2b(r - 1) + c$ ....(B)
⇨ $g(r) > 0$ (As a > 0 and from (B))
Now, $g(0) = 6a - 2b + c$

Now, $f'(-1) = 3a - 2b + c > 0$ .....(C)
⇨ $g(0) = 3a - 2b + c + 3a > 0$ (As a > 0 and from (C))

g(x) is always positive.

14. (D) We have, (1/2){h(6 – 2) + 5(2 – k) + 3(k – 6)} = 12

⇨ 4h – 2k = 32

⇨ k = 2h – 16

Now, distance of a point in A from (0, 0) is B = $\sqrt{(h^2 + k^2)}$ Let, C = $B^2 = h^2 + k^2$

Now, B will be minimum when C is minimum.

Now, C = $h^2 + (2h - 16)^2$ (Putting value of k from above)

⇨ C = $5h^2 - 64h + 256$

⇨ dC/dh = 10h – 64 = 0

⇨ h = 32/5

Now, $d^2C/dh^2 = 10 > 0$

⇨ C is minimum when h = 32/5

⇨ C is minimum when k = 2×(32/5) – 16 = -16/5

⇨ Cmin = $(32/5)^2 + (-16/5)^2 = (16/5)^2(4 + 1) = 16^2/5$

⇨ Bmin = √Cmin = 16/√5

15. (D) Now, $a^2 + b^2 = c^2$

Let 3 doesn't divide a or b.

Dividing the equation by 4 we get, $(\pm1)^2 + (\pm1)^2 \equiv 0 \pmod 3$

⇨ 1 + 1 ≡ 0 (mod 3)

Which is impossible.

Now, Let c divides one of a, b.

Dividing the equation by 3 again we get, $(\pm1)^2 + 0 \equiv 0 \pmod 3$

⇨ 1 ≡ 0 (mod 3)

Which is impossible.

⇨ 3 divides both a and b.

Now, we have, $(3a_1)^2 + (3b_1)^2 = (3c_1)^2$ where $a = 3a_1$, $b = 3b_1$ and $c = 3c_1$

⇨ $a_1^2 + b_{12}^2 = c_1^2$

Let, 3 doesn't divide any of a_1, b_1, c_1

Dividing the equation by 3 we get,

$1 + 1 \equiv 1 \pmod 3$

Which is impossible

Let, 3 divides both of a_1, b_1

Dividing the equation by 3 we get,

$0 + 0 \equiv 1 \pmod 3$

Which is impossible.

⇨ 3 divides exactly one of a_1, b_1. Let 3 divides b_1 (Without loss of generality)

In that case dividing the equation by 3 we get,

$1 + 0 \equiv 1 \pmod 3$

Which is consistent.

⇨ 3 divides b_1

⇨ 3 divides a, 3^2 divides b and 3 divides c

⇨ 3^4 divides abc

16. (D)

17. (A) Now, $f(x) = 1/(x - 2)$

⇨ $x - 2 = 1/f$

⇨ $x = 1/f + 2$

⇨ $f^{-1} = 1/x + 2 = (2x + 1)/x$

Now, $1/(x - 2) = (2x + 1)/x$

⇨ $x = 2x^2 + x - 4x - 2$

⇨ $x^2 - 2x - 1 = 0$

⇨ $x = \{2 \pm \sqrt{(4 + 4*1*1)}\}/2$

⇨ $x = 1 \pm \sqrt{2}$

When $x = 1 + \sqrt{2}$, $f = 1/(1 + \sqrt{2} - 2) = 1/(\sqrt{2} - 1) = (\sqrt{2} + 1)/\{\sqrt{2} - 1)(\sqrt{2} + 1) = 1 + \sqrt{2}$

When $x = 1 - \sqrt{2}$, $f = 1/(1 - \sqrt{2} - 2) = 1/(-1 - \sqrt{2}) = (-1 + \sqrt{2})/\{(-1 - \sqrt{2})(-1 + \sqrt{2})] = 1 - \sqrt{2}$

So the intersection points are $(1 + \sqrt{2}, 1 + \sqrt{2})$ and $(1 - \sqrt{2}, 1 - \sqrt{2})$

18. (C) $374 = 2\times11\times17$

Now, if we take, 119, 120 and 121 then no one is coprime to 374.

If we take 118, 119, 120, 121, 122 then also no one is coprime to 374. Because there are 2 odd prime factors of 374 and consecutive odd numbers are divisible by 11 and 17.

So, if we take 3 odd numbers then one must be coprime to 373.

So, if we take, 117, 118, 119, 120, 121, 122 then 117 is coprime to 374. Also if we take 118, 119, 120, 121, 122, 123 then 123 is coprime to 371.

⇨ N = 6

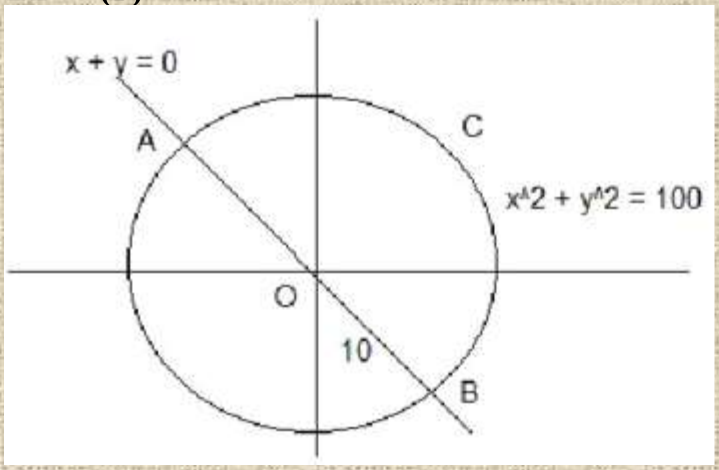
19. (C)

20. (A) Clearly the limit holds for $\alpha \geq 1$

In that case $\lim (\sin^{\alpha}x)/x$ as $x \to 0 = \lim \{(\sin x)/x\}\times\sin^{\alpha-1}x$ as $x \to 0 = 1\times0 = 0$

If $\alpha - 1$ is negative then it makes a 0 in numerator.

21. **(B)**



Now, $\sin(x + y) > 0$

⇨ **$\sin(x + y) > \sin 0$**

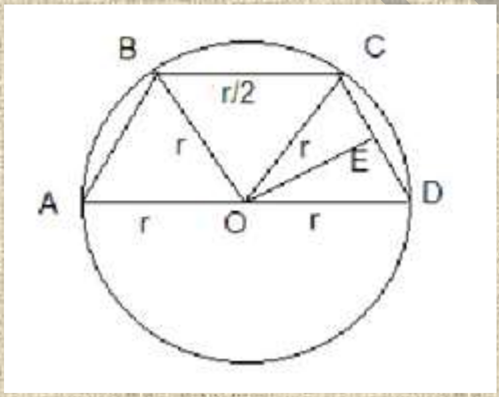
⇨ **$x + y > 0$**

Clearly the area represented by R is the semicircular area AOBCA

Now, radius of the circle = 10

⇨ **Area of the semicircular region AOBCA = $(\pi\times10^2)/2 = 50\pi$**

22. **(A)**



The circumcentre lie on AD.

⇨ **O, the circumcentre must divide AD in equal halves.**

⇨ **BC = CD**

Now, from triangle OBC, let Angle BOC = θ

⇨ **$\cos\theta = (r^2 + r^2 - r^2/4)/2\times r\times r$**

⇨ $\cos\Theta = 7/8$
⇨ Angle DOC = $(180° - \Theta)/2$
⇨ $\cos(DOC) = \cos(90° - \Theta/2) = \sin(\Theta/2)$
⇨ $\sin(\Theta/2) = (r^2 + r^2 - a^2)/2*r*r$
⇨ $\sin(\Theta/2) = 1 - a^2/2r^2$
⇨ $\sin^2(\Theta/2) = (1 - a^2/2r^2)^2$
⇨ $(1 - \cos\Theta)/2 = (1 - a^2/2r^2)^2$
⇨ $(1 - 7/8)/2 = (1 - a^2/2r^2)^2$
⇨ $1/16 = (1 - a^2/2r^2)^2$
⇨ $1 - a^2/2r^2 = 1/4$
⇨ $a^2/2r^2 = 3/4$
⇨ $a^2/r^2 = 3/2$
⇨ $a/r = \sqrt{3} : \sqrt{2}$
⇨ $2a/2r = \sqrt{3} : \sqrt{2}$

23. (B)

24. (B)

The combinations are when none of a, b, c is 1.

$(2, 2, 2\times5^3) = 3!/2! = 3$ $(2, 2^2, 5^3) = 3!$

$= 6$ $(5, 5, 5\times2^3) = 3!/2! = 3$ $(5, 5^2, 2^3) = 3! = 6$

$(2, 2\times5, 2\times5^2) = 3!$

$= 6$ $(2, 2^2\times5, 5^2) = 3!$

$= 6$ $(2, 2^2\times5^2, 5) = 3! = 6$

$(2^2, 2\times5, 5^2) = 3!$

$= 6$ $(2^2\times5, 2\times5, 5) = 3! = 6$

$(5, 2\times5^2, 2^2) = 3!$

$= 6$ $(2\times5, 2\times5, 2\times5) = 1$

Number of cases = 3 + 6 + 3 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 1 = 55

Now, when one of a, b, c is 1.

$(1, 2, 2^2\times5^2) = 3!$

$= 6$ $(1, 2^2, 2\times5^3) = 3! = 6$

$(1, 2^3, 5^3) = 3!$

$= 6\ (1, 2\times5, 2^2\times5^2) = 3! = 6$

$(1, 2\times5^2, 2^2\times5) = 3!$

$= 6\ (1, 2^3\times5, 5^2) = 3!$

$= 6\ (1, 2^3\times5^2, 5) = 3! = 6$

Number of cases = 6 + 6 + 6 + 6 + 6 + 6 + 6 = 42

Now, when 2 of a, b, c are 1.

(1, 1, 1000) = 3!/2! = 3

Therefore, total number of cases = 55 + 42 + 3 = 100

25. (B) Let z = 1/x, then z -> 0 as x -> ∞

The limit is lim $\{f(3 + z)/f(3)\}^{1/z}$

as z -> 0

= lim $[1 + \{f(3 + z) - f(3)\}/f(3)]^{1/z}$

Now, {f(z + 3) – f(3)} /f(3) -> 0 as z -> 0

So, the limit is = e

26. (D)

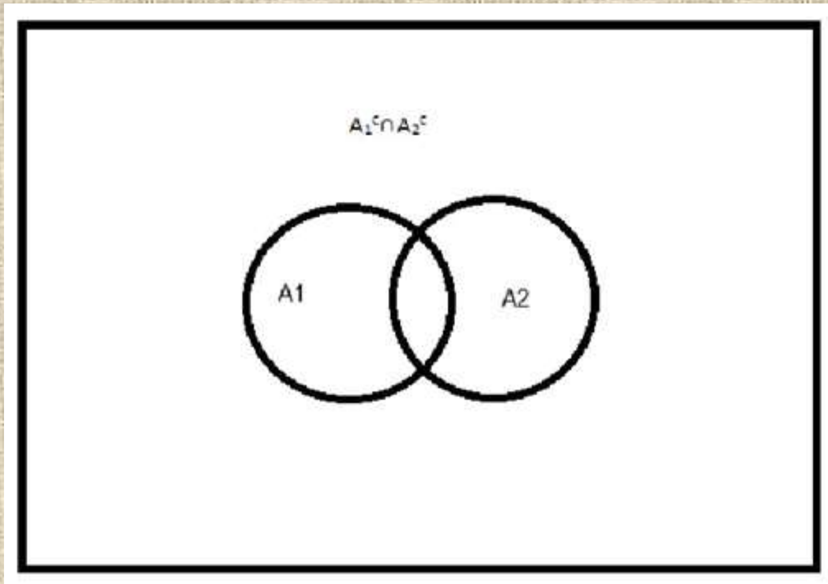
Let $z = re^{i\theta}$

Now, $z - 1/z = re^{i\theta} + 1/re^{i\theta} = (r\cos\theta + \cos\theta/r) + i(r\sin\theta - \sin\theta/r)$

- ⇨ **$|z - 1/z| = \sqrt{(r^2\cos^2\theta + \cos^2\theta/r^2 - 2\cos^2\theta + r^2\sin^2\theta + \sin^2\theta/r^2 - 2\sin^2\theta)}$**
- ⇨ **$2 = \sqrt{(r^2 + 1/r^2 - 2)}$**
- ⇨ **$r^2 + 1/r^2 = 6$**
- ⇨ **$r^2 + 2 + 1/r^2 = 8$**
- ⇨ **$(r + 1/r)^2 = 8$**
- ⇨ **$r + 1/r = 2\sqrt{2}$**
- ⇨ **$r^2 - r2\sqrt{2} + 1 = 0$**
- ⇨ **$r = \{2\sqrt{2} + \sqrt{(8 - 4)}\}/2$**
- ⇨ **$r = (1 + \sqrt{2})$**

27. (B)

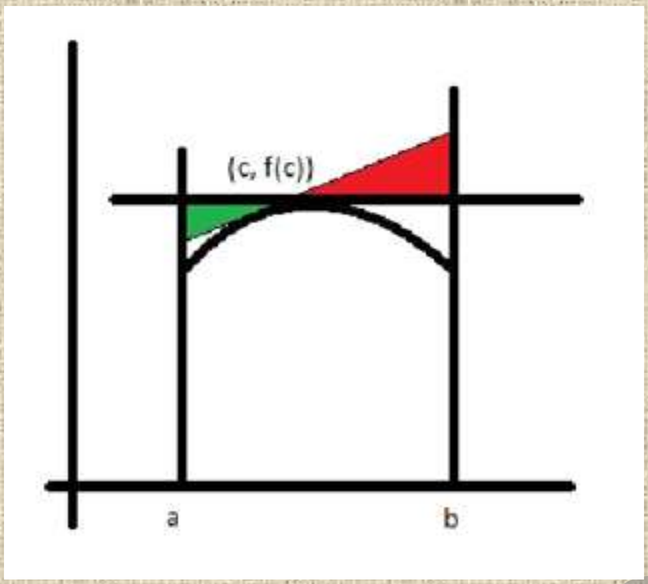
28. (D)



$(f^{-1}(A_1))^c \cup (f^{-1}(A_2))^c$ includes the whole area except $A_1 \cap A_2$
So, this may not be always true.

Second statement holds if the function is one-one and onto. So, option (d) is correct.

29. **(A)**



Clearly when c is shifted from (a + b)/2 then the red area > green area and so A(c) is minimum when c = (a + b)/2.

30. **(C)**