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Probabilistic AutoRegressive Neural Networks for Accurate Long-range Forecasting

Presented by

Tanujit Chakraborty

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Joint work with

Madhurima Panja, Uttam Kumar, and Abdenour Hadid

Time Series Forecasting

Time series is a set of observations, each one being recorded at a specific time (e.g., weekly dengue cases in India).

Components of time series

- Trend
- Seasonal
- Cyclical
- Irregular Fluctuations

Stationary time series is roughly horizontal, constant variance and no patterns predictable in the long-term.

Forecasting is the method of estimating how the sequence of observations will continue into the future.

AutoRegressive Integrated Moving Average (ARIMA)

- The ARIMA model, introduced by Box and Jenkins (1976), is a linear regression model indulged in tracking linear tendencies in stationary time series data.
- AR: autoregressive (lagged observations as inputs) I: integrated (differencing to make series stationary) MA: moving average (lagged errors as inputs).
- The model is expressed as ARIMA(p, d, q) where p, d, and q are integer parameter values that decide the structure of the model.
- The mathematical expression of the ARIMA model is as follows:

$$y_t = \theta_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \cdots - \theta_q \varepsilon_{t-q},$$

where y_t is the actual value, ε_t is the random error at time t, ϕ_t and θ_t are the coefficients of the model.

- It is assumed that ε_t has zero mean with constant variance, and satisfies the i.i.d condition.
- Three basic Steps: Model identification, Parameter Estimation, and Diagnostic Checking.

AutoRegressive Neural Network (ARNN)

- ARNN framework is a modification of the artificial neural network (ANN) for time series data sets.
- ARNN model is a feed-forward neural network time series model which uses lagged values of the time series as inputs to the neural network.
- The, architecture models p lagged inputs to predict the future trajectories of the series using a single-hidden layer with k hidden nodes.
- The mathematical expression of the ARNN model is as follows:

$$\hat{y}_t = \phi_0 \left\{ w_{c_0} + \sum_h w_{h_0} \phi_h \left(w_{c_h} + \sum_i w_{i_h} y_{t-j_i} \right) \right\}$$

wherew_{c_h} denotes the connecting weights and ϕ_i is the activation function.

- An ARNN(p, k) model uses p as the optimal number of lags (calculated based on the (AIC) for an AR(p) model and k is set to $k = \left\lceil \frac{p+1}{2} \right\rceil$ for non-seasonal data sets.

Hybrid Forecasting Techniques

Hybridization is generally performed due to the lack of the comprehensive individual forecasters in capturing various patterns in the data, concurrently.

Need for hybrid forecasters:

- Improving forecasting accuracy due to comprehensive pattern detection and modeling.
- Reducing the risk of using inappropriate model due to the combination of forecasts.
- Simplifying the procedure of model selection due to the use of different components.

Hybrid Methods
- Parallel
  - Linear combination function
  - Nonlinear combination function
- Series
  - Nonlinear - Linear
  - Linear - Nonlinear
- Parallel-Series
  - Architecture – 1 [$f_{comb,t} = f_2(\hat{e}_t, y_t)$]
  - Architecture – 2 [$f_{comb,t} = f_2(\hat{L}_t, y_t)$]
  - Architecture – 3 [$f_{comb,t} = f_2(\hat{e}_t, \hat{L}_t, y_t)$]

Motivation

- Individual forecasting models from different paradigms suffer in modeling the complexities of real-world time series.
- Hybrid forecasting models are based on several data-level assumptions, such as,
  - linear and nonlinear patterns of a series can be modeled separately or that the residuals comprise only the nonlinear trends,
  - there exists an additive or multiplicative relationship between the linear and nonlinear segments of the datasets,

  the violation of which might substantially degrade the forecast accuracy of hybrid models.

- The proposed Probabilistic AutoRegressive Neural Networks (PARNN) approach aims to overcome these limitations of hybrid time series forecasting models while improving their predictive accuracies.

PARNN Framework

- PARNN is a modification of the artificial neural network (ANN) and the recurrent neural network (RNN) models for modeling complex non-linear time series.
- This assumption-free hybrid framework blends the classical ARIMA model with the scalable neural network architecture.
- PARNN (m, k, l) model considers the future values of the time series to be a nonlinear function of m-lagged values of the target time series (y_t) and l-lagged values of ARIMA residuals (e_t) (feedback errors).

$$y_t = f(y_{t-1}, y_{t-2}, \dots, y_{t-m}, e_{t-1}, e_{t-2}, \dots, e_{t-l}),$$

where nonlinear function f is a single hidden-layered autoregressive neural network having $(m + l)$ input neurons and $k = \left\lceil \frac{m+l+1}{2} \right\rceil$ hidden neurons.

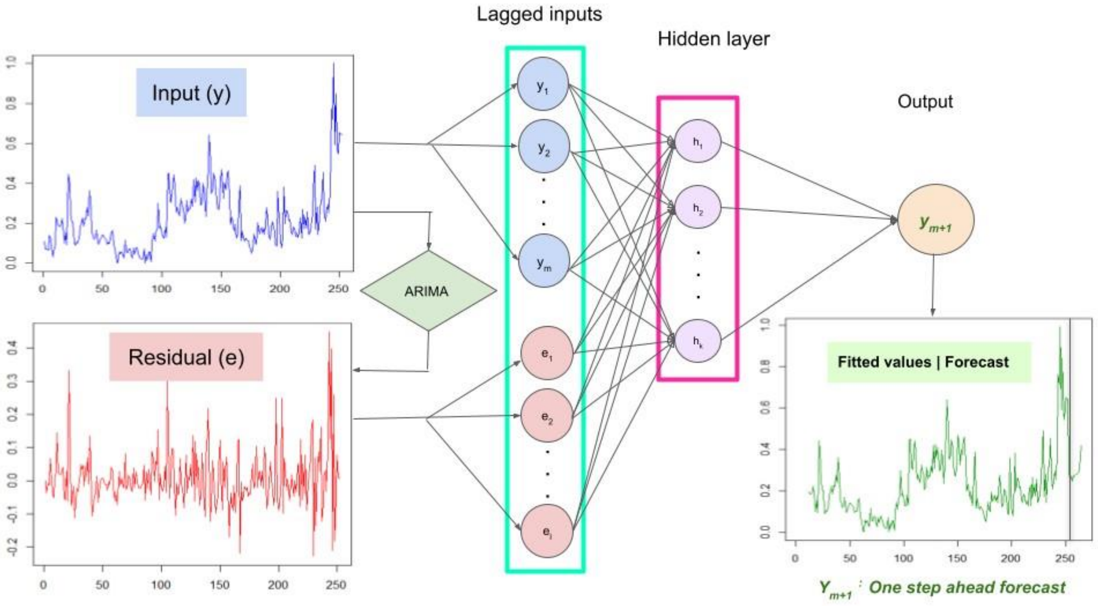

Fig: Architecture of the PARNN model

PARNN Model Workflow

- The learning of the proposed PARNN model comprise of two stages:
    - Stage 1: The original time series y_t is standardized and the ARIMA residuals e_t are generated by using the best-fitted ARIMA model with minimum Akaike Information Criterion (AIC).
    - Stage 2: Lagged values of standardized series and the ARIMA residuals are fitted with a single-hidden layered feed-forward neural network to model the nonlinear and linear relationships in the dataset.
- The fitted PARNN model generates one-step ahead forecast $\hat{y}_t$ using the lagged inputs as

$$\hat{y}_t = \alpha_0 + \sum_{j=1}^{k} \alpha_j \, G\left(\alpha_{0,j} + \sum_{i=1}^{m} \alpha_{i,j} y_{t-i} + \sum_{i=m+1}^{m+l} \alpha_{i,j} e_{t+m-i} \right) + \epsilon_t,$$

where α denotes the connection weights and G is a bounded nonlinear sigmoidal activation function of the neural network. Multi-step ahead forecasts are iteratively generated in the PARNN model.

PARNN Prediction Intervals

- The PARNN framework quantifies the uncertainty of the generated forecast using two approaches:
  - Conformal prediction
  - Simulating future paths with bootstrapped residuals.
- Conformal prediction transforms point estimates into prediction regions, ensuring convergence in a distribution-free and model-agnostic manner by analyzing residual distributions.
- In the bootstrapped residuals approach, we simulate future model paths by drawing 1000 random samples from the Gaussian error distribution of ϵ_t.
- These simulations help us calculate 80% prediction intervals based on percentiles for the model's future values.

Application of PARNN

- For the experimental evaluation of the proposal we consider twelve real-world applied time series of varied domains and frequency.
- The global characteristics of these time series are summarized below:

Datasets	Frequency	Time Span	Length	LTD	Stationary	Linear	Seasonal	Trend	Gaussian	Chaotic
GOOG Stock	Daily	2020-2022	504	🟢	🔴	🔴	🔴	🟢	🔴	🟢
MSFT Stock		2020-2022	504	🟢	🔴	🟢	🔴	🟢	🔴	🔴
AMZN Stock		2020-2022	504	🟢	🔴	🟢	🔴	🟢	🔴	🟢
Births		1968-1988	7305	🟢	🔴	🔴	🟢	🟢	🔴	🔴
Colombia Dengue	Weekly	2005-2016	626	🟢	🔴	🔴	🟢	🟢	🔴	🟢
Colombia Malaria		2005-2016	626	🟢	🔴	🟢	🟢	🟢	🔴	🟢
Venezuela Dengue		2002-2014	660	🟢	🔴	🟢	🟢	🟢	🔴	🟢
Venezuela Malaria		2002-2014	669	🟢	🔴	🔴	🟢	🟢	🔴	🟢
US EPU Index	Monthly	2000-2021	264	🟢	🔴	🟢	🔴	🟢	🔴	🟢
UK unemployment		1971-2016	552	🟢	🔴	🟢	🔴	🔴	🔴	🔴
Russia Exchange		2000-2021	264	🟢	🔴	🔴	🔴	🟢	🔴	🟢
Tourism	Quarterly	1998-2017	80	🟢	🔴	🟢	🔴	🟢	🔴	🟢

Table: Global characteristics of time series.

(LTD – long term dependency, 🟢 indicates presence of the feature, and 🔴 indicates absence of the feature)

Forecast Evaluations

- Performance metrics such as mean absolute scaled error (MASE), root mean square error (RMSE), and symmetric Mean Absolute Percent Error (sMAPE) are used to evaluate the performances of different forecasting models for the time series data sets:

$$MASE = \frac{\sum_{i=M+1}^{M+n} |y_i - \hat{y}_i|}{\frac{n}{M-S} \sum_{i=S+1}^{M} |y_i - y_{i-s}|}; \quad RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2};$$

$$sMAPE = \frac{1}{n} \sum_{i=1}^{n} \frac{|y_i - \hat{y}_i|}{(|y_i| + |\hat{y}_i|)/2} * 100\%,$$

where y_i is the actual output, $\hat{y}_i$ is the predicted output, and n denotes the forecast horizon, M denotes the training data size, and S is the seasonality of the dataset.

- By definition, the lower the value of these performance metrics, the better is the performance of the concerned forecasting model.

Long-term Experimental Evaluation

Dataset	Metric	RWD	ETS	ARIMA	TBATS	MLP	ARNN	Hybrid-1	Hybrid-2	NBeats	DeepAR	TCN	Transformers	Hybrid-3	PARNN
MSFT Stock	*MASE*	10.02	9.865	9.947	5.549	10.98	4.741	9.950	9.950	7.792	29.46	15.31	18.78	9.849	**3.240**
		(0)	(0)	(0)	(0)	(0.54)	(0.05)	(0.002)	(0.001)	(0.68)	(19.7)	(20.4)	(10.1)	(0.08)	(0.02)
	RMSE	54.51	53.76	54.20	31.09	59.36	26.73	54.21	54.21	43.20	134.1	81.25	86.97	53.74	**17.59**
		(0)	(0)	(0)	(0)	(2.86)	(0.28)	(0.01)	(0.003)	(4.41)	(87.2)	(100)	(44.7)	(0.38)	(0.33)
	SMAPE	13.52	13.32	13.43	7.841	14.67	6.761	13.43	13.43	10.75	61.63	26.93	32.81	13.31	**4.660**
		(0)	(0)	(0)	(0)	(0.65)	(0.07)	(0.002)	(0.001)	(0.83)	(50.9)	(47.1)	(23.9)	(0.09)	(0.03)
Births	*MASE*	2.383	1.492	1.462	1.422	2.301	2.289	1.426	1.397	3.862	10.94	$62E^3$	1.539	1.424	**0.554**
		(0)	(0)	(0)	(0)	(0.001)	(0.01)	(1.87)	(0.04)	(2.51)	(1.67)	$(19E^3)$	(0.01)	(0.03)	(0.003)
	RMSE	2432	1480	1442	1404	2187	2199	1407	1427	4165	9877	$15E^6$	1540	1426	**640.5**
		(0)	(0)	(0)	(0)	(0.03)	(0.15)	(19.2)	(25.2)	(2859)	(2176)	$(49E^5)$	(5.35)	(19.4)	(2.42)
	SMAPE	22.03	13.13	12.86	12.49	22.01	21.41	13.98	12.25	28.56	176.8	90.49	13.57	12.51	**4.832**
		(0)	(0)	(0)	(0)	(0.12)	(0.35)	(0.67)	(0.35)	(13.4)	(42.8)	(69.2)	(0.04)	(0.22)	(0.03)
Colombia Dengue	*MASE*	5.337	4.862	4.865	4.860	4.324	4.096	4.864	4.865	18.535	10.483	7.645	5.928	4.935	**3.228**
		(0)	(0)	(0)	(0)	(0.09)	(0.17)	(0.01)	(0.003)	(4.84)	(0.85)	(3.47)	(2.16)	(0.03)	(0.04)
	RMSE	1111	997.2	998.3	997.5	845.3	840.1	997.8	998.5	3610	1914	1727	1125	1015	**611.0**
		(0)	(0)	(0)	(0)	(22.8)	(39.1)	(1.68)	(0.74)	(1094)	(127)	(788)	(405)	(7.17)	(7.98)
	SMAPE	43.40	40.98	40.99	40.96	37.99	36.42	40.99	40.99	83.74	146.6	59.81	61.39	41.37	**28.94**
		(0)	(0)	(0)	(0)	(0.51)	(1.03)	(0.03)	(0.01)	(8.62)	(24.9)	(29.1)	(37.5)	(0.16)	(0.36)
US-EPU Index	*MASE*	1.595	1.685	1.671	1.948	1.681	1.617	1.658	1.655	1.837	2.511	2.758	2.421	1.591	**1.483**
		(0)	(0)	(0)	(0)	(0.02)	(0.03)	(0.002)	(0.001)	(0.15)	(1.06)	(1.28)	(0.67)	(0.06)	(0.003)
	RMSE	81.41	85.73	85.03	93.35	85.53	82.19	84.50	84.34	89.77	110.2	124.9	108.9	80.93	**68.41**
		(0)	(0)	(0)	(0)	(0.70)	(1.99)	(0.05)	(0.02)	(4.64)	(34.5)	(57.5)	(20.9)	(2.75)	(0.15)
	SMAPE	29.24	31.36	31.01	38.27	31.27	29.77	30.72	30.64	35.53	59.35	59.99	53.67	29.19	**26.95**
		(0)	(0)	(0)	(0)	(0.46)	(0.76)	(0.03)	(0.01)	(4.01)	(38.7)	(27.2)	(25.6)	(1.27)	(0.07)
Russia Exchange	*MASE*	3.749	5.376	4.676	4.833	4.754	5.120	4.625	4.648	4.199	8.792	10.97	7.442	4.622	**3.162**
		(0)	(0)	(0)	(0)	(0.20)	(0.05)	(0.003)	(0.001)	(0.83)	(5.96)	(11.9)	(5.60)	(0.11)	(0.69)
	RMSE	7.616	10.78	9.402	9.717	9.561	10.28	9.297	9.340	8.550	17.34	24.45	14.67	9.320	**6.555**
		(0)	(0)	(0)	(0)	(0.39)	(0.10)	(0.01)	(0.001)	(1.58)	(10.8)	(27.3)	(10.4)	(0.17)	(1.32)
	SMAPE	10.04	14.75	12.69	13.15	12.92	13.99	12.55	12.61	11.36	27.37	31.26	22.78	12.55	**8.405**
		(0)	(0)	(0)	(0)	(0.58)	(0.16)	(0.01)	(0.001)	(2.37)	(23.9)	(34.7)	(21.5)	(0.31)	(1.92)

Table: Long-term forecast performance of different models for selected dataset. Mean values and (standard deviations) of 10 repetitions are reported with best performing models marked **bold.**

Medium-term Experimental Evaluation

Dataset	Metric	RWD	ETS	ARIMA	TBATS	MLP	ARNN	Hybrid-1	Hybrid-2	NBeats	DeepAR	TCN	Transformers	Hybrid-3	PARNN
MSFT Stock	*MASE*	2.828	**1.684**	2.968	1.713	3.774	3.084	2.968	2.968	7.013	20.49	11.63	12.06	2.952	1.845
		(0)	(0)	(0)	(0)	(0.14)	(0.01)	(0.001)	(0.001)	(0.58)	(17.6)	(18.9)	(8.11)	(0.11)	(0.03)
	RMSE	17.38	**10.94**	18.08	11.11	22.37	18.84	18.08	18.08	37.99	104.4	70.89	61.68	17.84	11.77
		(0)	(0)	(0)	(0)	(0.73)	(0.04)	(0.01)	(0.004)	(2.67)	(87.9)	(112)	(40.7)	(0.59)	(0.16)
	SMAPE	4.740	**2.867**	4.966	2.916	6.257	5.155	4.966	4.966	11.27	50.36	22.13	24.29	4.942	3.136
		(0)	(0)	(0)	(0)	(0.23)	(0.01)	(0.002)	(0.001)	(0.87)	(51.3)	(41.5)	(21.4)	(0.18)	(0.05)
Births	*MASE*	2.447	1.362	1.352	1.344	2.169	2.180	1.254	1.276	6.439	11.25	9.102	1.571	1.313	**0.636**
		(0)	(0)	(0)	(0)	(0.01)	(0.001)	(0.01)	(0.01)	(0.56)	(13.8)	(11.7)	(0.002)	(0.03)	(0.05)
	RMSE	2553	1354	1376	1369	2229	2227	1365	1349	7417	$10E^3$	$10E^3$	1597	1352	**760.8**
		(0)	(0)	(0)	(0)	(0.001)	(0.01)	(4.87)	(5.07)	(83.9)	(97.4)	($14E^3$)	(1.98)	(20.6)	(94.1)
	SMAPE	23.00	12.09	12.01	11.93	20.53	20.59	11.25	11.30	39.54	191.2	64.20	14.05	11.64	**5.610**
		(0)	(0)	(0)	(0)	(0.1)	(0.01)	(0.57)	(0.06)	(1.95)	(11.7)	(68.2)	(0.02)	(0.24)	(0.42)
Colombia Dengue	*MASE*	8.720	8.718	9.069	8.613	10.33	10.69	9.075	9.098	10.17	10.09	56.87	6.197	9.118	**1.762**
		(0)	(0)	(0)	(0)	(0.13)	(0.59)	(0.003)	(0.08)	(0.99)	(1.48)	(68.5)	(2.76)	(0.12)	(0.23)
	RMSE	922.4	917.2	949.8	906.0	1083	1122	953.0	955.2	1089	1043	7421	680.4	957.5	**222.6**
		(0)	(0)	(0)	(0)	(14.7)	(63.6)	(0.39)	(6.16)	(136)	(136)	(9200)	(259)	(13.3)	(20.1)
	SMAPE	53.29	53.33	54.79	52.90	59.58	60.87	54.79	54.86	58.36	126.1	109.6	49.47	54.96	**14.76**
		(0)	(0)	(0)	(0)	(0.47)	(2.11)	(0.01)	(0.32)	(3.25)	(32.7)	(36.9)	(32.4)	(0.49)	(1.76)
Colombia Malaria	*MASE*	4.436	4.997	4.661	4.916	3.330	2.733	4.652	4.631	4.599	8.834	10.87	3.911	4.481	**2.543**
		(0)	(0)	(0)	(0)	(0.23)	(0.24)	(0.01)	(0.04)	(1.29)	(0.91)	(4.24)	(1.94)	(0.04)	(0.002)
	RMSE	591.1	655.3	617.0	646.8	444.8	404.8	615.3	610.9	637.1	1059	1507	519.7	595.6	**361.7**
		(0)	(0)	(0)	(0)	(26.4)	(27.7)	(0.56)	(5.64)	(156)	(99.1)	(526)	(211)	(4.24)	(0.15)
	SMAPE	38.59	42.03	39.99	41.54	31.28	26.55	39.94	39.81	38.53	143.8	71.52	42.79	38.87	**25.26**
		(0)	(0)	(0)	(0)	(1.63)	(1.89)	(0.03)	(0.26)	(8.75)	(27.5)	(27.4)	(31.9)	(0.25)	(0.02)
US EPU Index	*MASE*	4.939	4.716	4.590	2.264	3.927	5.076	4.729	4.712	6.132	2.147	6.125	**1.567**	4.772	1.727
		(0)	(0)	(0)	(0)	(0.36)	(1.00)	(0.004)	(0.003)	(1.04)	(1.74)	(3.81)	(1.33)	(0.22)	(0.27)
	RMSE	104.9	100.3	97.62	51.93	84.43	110.5	100.5	100.1	136.4	48.42	145.6	**39.42**	101.3	40.84
		(0)	(0)	(0)	(0)	(7.18)	(20.8)	(0.08)	(0.05)	(26.0)	(35.2)	(82.7)	(25.8)	(4.48)	(6.51)
	SMAPE	52.22	50.49	49.49	27.82	43.90	52.56	50.59	50.45	59.06	39.58	65.66	26.63	50.91	**22.36**
		(0)	(0)	(0)	(0)	(3.13)	(8.05)	(0.03)	(0.02)	(6.41)	(41.6)	(42.5)	(30.7)	(1.71)	(2.98)
Russia Exchange	*MASE*	4.816	4.172	3.632	2.926	2.527	5.567	3.631	3.642	12.07	13.06	23.79	13.89	3.399	**1.250**
		(0)	(0)	(0)	(0)	(0.19)	(0.47)	(0.01)‘	(0.004)	(8.85)	(13.7)	(36.1)	(10.7)	(0.09)	(0.08)
	RMSE	4.912	4.197	3.874	3.028	2.646	5.750	3.873	3.883	12.31	12.81	25.89	13.47	3.766	**1.492**
		(0)	(0)	(0)	(0)	(0.17)	(0.48)	(0.01)	(0.004)	(8.78)	(13.1)	(36.4)	(10.2)	(0.10)	(0.11)
	SMAPE	6.088	5.301	4.627	3.750	3.248	7.573	4.626	4.640	14.01	20.97	32.26	21.32	4.335	**1.626**
		(0)	(0)	(0)	(0)	(0.24)	(0.66)	(0.01)	(0.01)	(9.47)	(26.7)	(49.9)	(20.8)	(0.12)	(0.12)

Table: Medium-term forecast performance of different models for selected dataset. Mean values and (standard deviations) of 10 repetitions are reported with best performing models marked **bold.**

Short-term Experimental Evaluation

Dataset	Metric	RWD	ETS	ARIMA	TBATS	MLP	ARNN	Hybrid-1	Hybrid-2	NBeats	DeepAR	TCN	Transformers	Hybrid-3	PARNN
MSFT Stock	*MASE*	2.759	3.213	2.683	3.198	2.159	2.594	2.684	2.682	2.266	20.26	8.649	11.77	2.629	**1.385**
		(0)	(0)	(0)	(0)	(0.08)	(1.03)	(0.001)	(0.001)	(0.64)	(17.9)	(18.5)	(6.73)	(0.06)	(0.07)
	RMSE	15.02	17.84	14.61	17.76	11.64	14.12	14.61	14.61	12.66	98.65	44.55	57.54	14.54	**8.302**
		(0)	(0)	(0)	(0)	(0.45)	(5.49)	(0.004)	(0.003)	(3.57)	(85.2)	(87.9)	(32.0)	(0.32)	(0.22)
	SMAPE	4.536	5.305	4.408	5.279	3.536	4.284	4.409	4.407	3.719	47.19	23.52	21.98	4.319	**2.282**
		(0)	(0)	(0)	(0)	(0.13)	(1.78)	(0.002)	(0.001)	(1.07)	(49.7)	(59.9)	(15.9)	(0.10)	(0.11)
Births	*MASE*	2.416	1.370	1.210	1.222	1.487	1.552	1.893	1.173	2.117	11.35	3.520	1.625	1.205	**0.940**
		(0)	(0)	(0)	(0)	(0.01)	(0.02)	(0.97)	(0.01)	(0.05)	(2.83)	(3.16)	(0.003)	(0.004)	(0.29)
	RMSE	2538	1376	1293	1286	1686	1700	1374	1283	2247	3648	3873	1671	1302	**1087**
		(0)	(0)	(0)	(0)	(18.7)	(24.6)	(4.98)	(5.55)	(17.8)	(78.5)	(2963)	(3.40)	(2.93)	(309)
	SMAPE	22.41	12.05	10.59	10.72	14.98	14.05	10.67	10.24	20.16	19.1	37.09	14.42	10.55	**8.255**
		(0)	(0)	(0)	(0)	(0.19)	(0.18)	(0.07)	(0.04)	(7.83)	(23.9)	(55.6)	(0.03)	(0.03)	(2.48)
Colombia Dengue	*MASE*	2.129	2.096	1.754	1.388	2.214	1.868	1.751	1.683	**1.349**	7.467	4.113	6.851	1.737	2.030
		(0)	(0)	(0)	(0)	(0.06)	(0.34)	(0.003)	(0.08)	(0.49)	(1.68)	(2.12)	(5.23)	(0.08)	$(3E^{-15})$
	RMSE	254.3	250.4	227.3	196.3	260.0	199.5	227.1	227.5	**188.2**	699.0	454.6	658.4	227.4	245.9
		(0)	(0)	(0)	(0)	(5.13)	(41.9)	(0.16)	(6.72)	(36.3)	(150)	(180)	(471)	(5.46)	$(2E^{-13})$
	SMAPE	19.76	19.50	16.84	**13.90**	20.39	20.01	16.81	16.24	14.13	110.2	44.11	51.95	16.69	19.01
		(0)	(0)	(0)	(0)	(0.44)	(4.03)	(0.02)	(0.61)	(5.54)	(39.4)	(43.1)	(33.7)	(0.60)	$(2E^{-14})$
Colombia Malaria	*MASE*	1.475	1.481	1.472	1.617	1.424	1.546	1.488	1.463	1.670	6.470	3.945	3.099	1.478	**1.093**
		(0)	(0)	(0)	(0)	(0.04)	(0.09)	(0.001)	(0.03)	(0.19)	(0.86)	(1.69)	(1.04)	(0.03)	(0.02)
	RMSE	258.8	276.4	266.2	306.3	259.0	288.0	265.9	263.0	237.8	822.9	588.4	446.3	259.4	**181.8**
		(0)	(0)	(0)	(0)	(14.9)	(22.3)	(0.10)	(3.82)	(14.7)	(102)	(246)	(121)	(4.01)	(2.01)
	SMAPE	21.22	21.12	21.11	22.37	20.59	21.74	21.31	21.05	24.45	132.4	45.94	41.70	21.24	**16.59**
		(0)	(0)	(0)	(0)	(0.39)	(0.86)	(0.004)	(0.29)	(2.81)	(31.6)	(10.7)	(23.5)	(0.34)	(0.36)
US-EPU Index	*MASE*	0.799	0.753	0.849	0.823	0.615	0.637	0.871	0.868	2.128	2.139	2.745	1.332	0.970	**0.534**
		(0)	(0)	(0)	(0)	(0.03)	(0.04)	(0.01)	(0.003)	(0.52)	(1.97)	(1.88)	(1.37)	(0.11)	(0.06)
	RMSE	17.24	15.88	19.75	17.90	13.73	13.84	20.40	20.41	50.29	41.42	55.65	27.17	22.49	**12.91**
		(0)	(0)	(0)	(0)	(0.55)	(0.45)	(0.09)	(0.04)	(10.7)	(34.6)	(34.6)	(24.1)	(2.19)	(0.48)
	SMAPE	10.14	9.523	10.49	10.45	7.773	8.040	10.74	10.70	24.32	35.46	43.46	19.96	11.84	**6.671**
		(0)	(0)	(0)	(0)	(0.35)	(0.47)	(0.05)	(0.03)	(5.71)	(40.8)	(47.8)	(26.3)	(1.27)	(0.67)
Russia Exchange	*MASE*	1.865	0.928	0.878	**0.645**	0.748	1.078	0.856	0.818	2.141	8.808	10.37	10.46	1.392	0.875
		(0)	(0)	(0)	(0)	(0.07)	(0.17)	(0.004)	(0.01)	(0.93)	(14.1)	(11.6)	(9.68)	(0.24)	(0.02)
	RMSE	2.078	0.981	1.129	**0.792**	0.860	1.266	1.113	1.084	2.431	8.811	11.74	10.35	1.741	0.973
		(0)	(0)	(0)	(0)	(0.08)	(0.18)	(0.003)	(0.002)	(1.015)	(13.8)	(12.5)	(9.54)	(0.27)	(0.03)
	SMAPE	2.497	1.258	1.191	**0.874**	1.012	1.465	1.161	1.109	2.849	15.14	17.15	16.41	1.886	1.184
		(0)	(0)	(0)	(0)	(0.09)	(0.23)	(0.01)	(0.01)	(1.22)	(27.2)	(26.1)	(18.5)	(0.32)	(0.02)

Table: Short-term forecast performance of different models for selected dataset. Mean values and (standard deviations) of 10 repetitions are reported with best performing models marked **bold.**

Forecast Curves

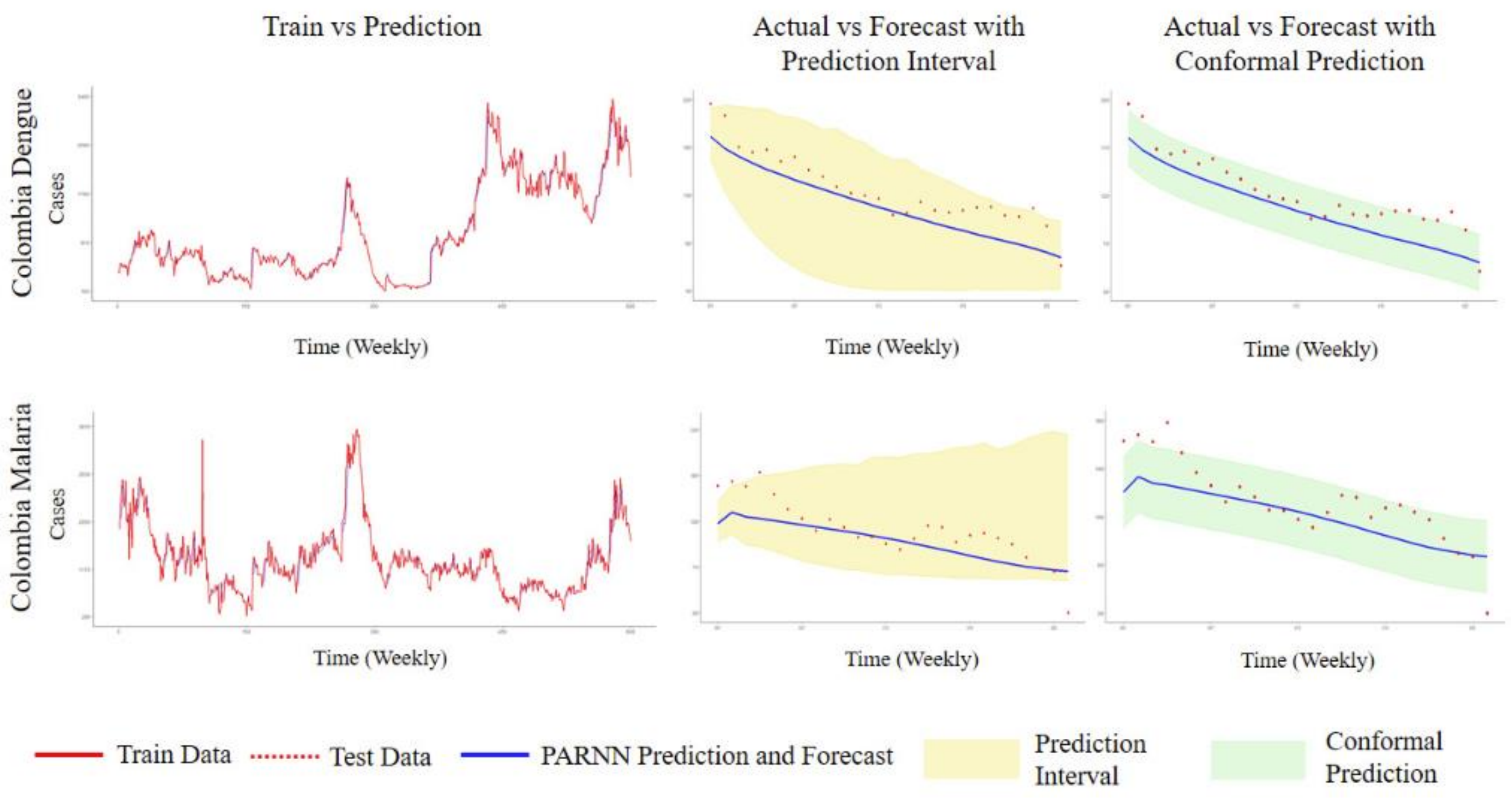

Fig: The plot shows input series (red line), ground truth (red points), 80% prediction interval (yellow shaded region), 80% conformal prediction (green shaded region), predictions (blue), and point forecasts (blue) generated by the PARNN model for the selected datasets.

Statistical Significance of Results

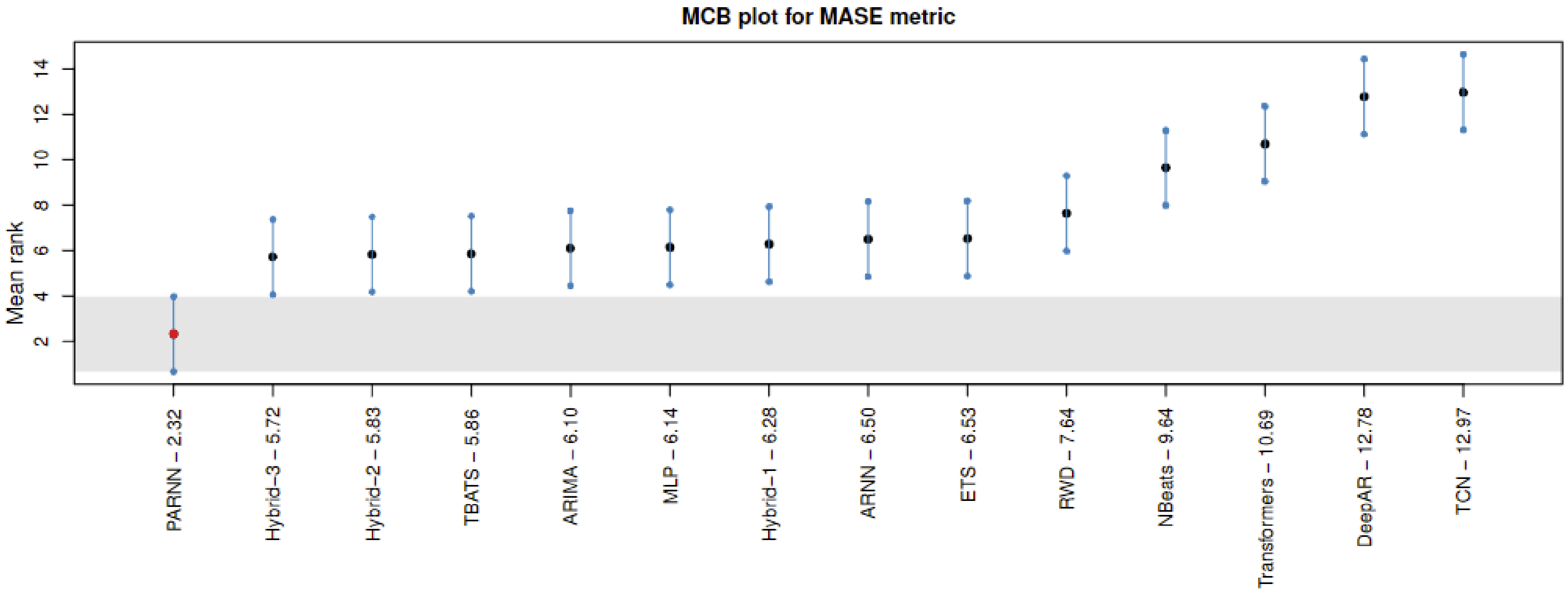

Fig: Schematic visualization of the multiple comparisons with the best (MCB) test. The plot provides the result for the MASE metric. For example, in the figure, PARNN – 2.32 specifies the rank of the PARNN model. The blue lines indicate the critical distance of the model, the middle point of this interval, which is denoted by black (significant) or red (not significant), represents the mean rank, and the shaded region marks the reference value.

Overall Performance Analysis

The overall experimental evaluation of the proposed model and the benchmark forecasters reveal some interesting observations.

- The traditional forecasting models like random walk and ARIMA fails to handle the complexities of real-world time series. Although the TBATS and ETS models demonstrate high accuracy for short-term forecasting task in linear time series, but their performance diminishes for medium-term and long-term projection.
- In case of the data-driven machine learning and deep learning approaches the nonlinear relationship is modeled more precisely, however, their overall performance is unsatisfactory for long-term forecasts. Data set size creates a barrier to the performance of the deep learners.
- The proposed PARNN approach can optimally model the complex non-stationary, non-linear, and non-Gaussian structure of the data generating process, thus resulting in improved forecasts. Unlike the deep learning and hybrid approaches, the stable architecture of our proposal limits the number of training parameters, hence restricting the model over-fitting and computational complexity.
- Moreover, the augmentation of the original time series with the ARIMA residuals in the PARNN framework allows it overcome the limitations of hybrid approaches and allows it to generate significantly accurate long-term forecasts.

Concluding Remarks

- We have proposed a new forecasting model that is suitable for non-stationary, non-linear, and non-Gaussian forecasting problems with limited historical data.
- The PARNN framework adds a new horizon to hybrid forecasting paradigm due to its assumption-free approach.
- Simple and easily interpretable model, fast in implementation due to pre-defined architecture (multivariate set-up is yet to be explored).
- Experimental results suggest a significant improvement in long-range forecast accuracy owing to the ARIMA residuals.
- The prediction intervals generated in this framework provides a suitable way for uncertainty quantification in real-world forecasting problems.
- Data and code: https://github.com/mad-stat/PARNN.

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THANK
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