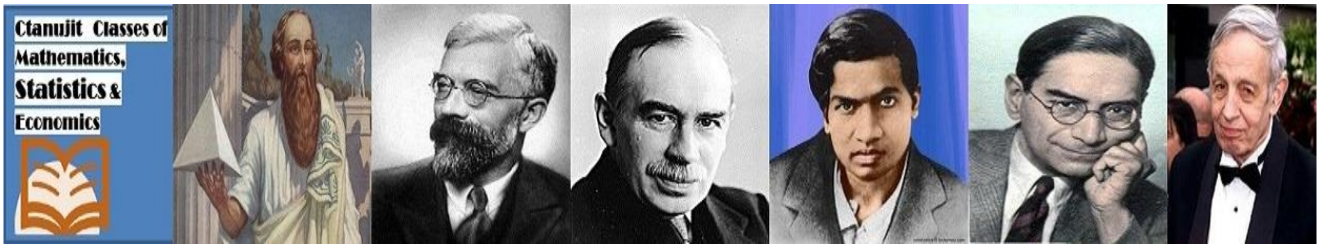

❖ Ctanujit Classes Of Mathematics, Statistics & Economics

- **ISI MSQE Previous Years Sample Papers**

Questions are complied By Ctanujit Classes

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Test code: ME I/ME II, 2004
Syllabus for ME I

Matrix Algebra: Matrices and Vectors, Matrix Operations, Determinants,

Nonsingularity, Inversion, Cramer's rule.

Calculus: Limits, Continuity, Differentiation of functions of one or more variables, Product rule, Partial and total derivatives,Derivatives of implicit functions, Unconstrained optimization (first and second order conditions for extrema of a single variable and several variables). Taylor Series, Definite and Indefinite Integrals: standard formulae, integration by parts and integration by substitution. Differential equations. Constrained optimization of functions of a single variable.

Theory of Sequence and Series:

Linear Programming: Formulations, statements of Primal and Dual problems. Graphical Solutions.

Theory of Polynomial Equations (up to third degree)

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory.

Sample Questions for ME I (Mathematics), 2004

For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question and write it in your answer book.

1. $X \sim B(n, p)$. The maximum value of Var (X) is

   (A) $\frac{n}{4}$; (B) n ; (C) $\frac{n}{2}$; (D) $\frac{1}{n}$.

2. P (x) is a quadratic polynomial such that P (1) = - P (2). If one root of the equation is -1, the other root is

   (A) $-\frac{4}{5}$; (B) $\frac{8}{5}$; (C) $\frac{4}{5}$; (D) $-\frac{8}{5}$.

3. $f(x) = (x-a)^3 + (x-b)^3 + (x-c)^3$, $a < b < c$.

   The number of real roots of $f(x) = 0$ is

   (A) 3; (B) 2; (C) 1; (D) 0.

4. A problem of statistics is given to the three students A, B and C. Their probabilities of solving it independently are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$, respectively.

   The probability that the problem will be solved is

   (A) $\frac{3}{5}$; (B) $\frac{2}{5}$; (C) $\frac{1}{5}$; (D) $\frac{4}{5}$.

5. Suppose correlation coefficients between x and y are computed from

   (i) $y = 2 + 3x$ and (ii) $2y = 5 + 8x$. Call them ρ_1 and ρ_2, respectively.

   Then

(A) $\rho_1 > \rho_2$; (B) $\rho_2 > \rho_1$; (C) $\rho_1 = \rho_2$; (D) either $\rho_1 > \rho_2$ or $\rho_1 < \rho_2$.

6. In the linear regression of y on x, the estimate of the slope parameter is given by $\dfrac{Cov(x,y)}{V(x)}$. Then the slope parameter for the linear regression of x on y is given by

(A) $\dfrac{V(x)}{Cov(x,y)}$; (B) $\dfrac{Cov(x,y)}{V(x)}$; (C) $\dfrac{Cov(x,y)}{\sqrt{V(x)V(y)}}$; (D) none of these.

7. Suppose $f(x) = e^x$ then

(A) $f\left(\dfrac{x_1 + x_2}{2}\right) > \dfrac{f(x_1) + f(x_2)}{2}$ for all x_1 and x_2 and $x_1 \neq x_2$;

(B) $f\left(\dfrac{x_1 + x_2}{2}\right) < \dfrac{f(x_1) + f(x_2)}{2}$ for all x_1 and x_2 and $x_1 \neq x_2$;

(C) $f\left(\dfrac{x_1 + x_2}{2}\right) > \dfrac{f(x_1) + f(x_2)}{2}$ for some values of x_1 and x_2 and $x_1 \neq x_2$;

$f\left(\dfrac{x_1 + x_2}{2}\right) < \dfrac{f(x_1) + f(x_2)}{2}$ for some values of x_1 and x_2 and $x_1 \neq x_2$;

(D) there exists at least one pair (x_1, x_2), $x_1 \neq x_2$ such that

$$f\left(\frac{x_1 + x_2}{2}\right) = \frac{f(x_1) + f(x_2)}{2}$$

8. Consider the series (i) and (ii) defined below:

(i) $\dfrac{1}{2} + \dfrac{2}{2^2} + \dfrac{3}{2^3} + \dfrac{4}{2^4} + \dots$

and

(ii) $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+\ldots$

Then,

(A) the first series converges, but the second series does not converge;

(B) the second series converges, but the first series does not converge;

(C) both converge;

(D) both diverge.

9. The function $x\left|x\right|$ is

(A) discontinuous at $x = 0$;

(B) continuous but not differentiable at $x = 0$;

(C) differentiable at $x = 0$;

(D) continuous everywhere but not differentiable anywhere.

10. The sequence $(-1)^{n+1}$ has

(A) no limit;

(B) 1 as the limit;

(C) –1 as the limit;

(D) 1 and –1 as the limits.

11. The series

$1.\frac{1}{2}+\frac{1}{2}.\frac{1}{3}+\frac{1}{3}.\frac{1}{4}+\ldots+\frac{1}{n}.\frac{1}{n+1}+\ldots$

(A) diverges;

(B) converges to a number between 0 and 1;

(C) converges to a number greater than 2;

(D) none of these.

12. Consider the function

$$f(x)=\frac{x^t-1}{x^t+1}, \qquad (x>0)$$

The limit of the function as t tends to infinity;

(A) does not exist; (B) exists and is everywhere continuous;

(C) exists and is discontinuous at exactly one point;

(D) exists and is discontinuous at exactly two points.

13. If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial x}$ is equal to

(A) *sin u cos u* ; (B) *cot u* ; (C) *tan u* ; (D) none of these.

14. The deviative of $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ with respect to x is

(A) $-\frac{x}{2}$; (B) $\frac{x}{2}$; (C) $\frac{1}{2}$; (D) $-\frac{1}{2}$.

15. The sum of the infinite series $1+\frac{3}{4}+\frac{7}{16}+\frac{15}{64}+\frac{31}{256}+\ldots$ is

(A) $\frac{2}{3}$; (B) $\frac{4}{3}$; (C) $\frac{8}{3}$; (D) none of these.

16. Five boys and four girls are to be seated in a row for a photograph. It is desired that no two girls sit together. The number of ways in which they can be so arranged is

(A) $6_{P_4}\times \lfloor 5$; (B) $4_{P_2}\times \lfloor 5$; (C) $\lfloor 4\times \lfloor 5$; (D) none of these.

17. A point moves so that the ratio of its distance from the points *(-a, 0)* and *(a, 0)* is 2:3. The equation of its locus is

(A) $x^2+y^2+10ax+a^2=0$;

(B) $5x^2+5y^2+26ax+5a^2=0$;

(C) $5x^2+5y^2-26ax+5a^2=0$;

(D) $x^2 + y^2 - 10ax + a^2 = 0$;

18. If the sum, $\sum_{x=1}^{100} \lfloor x$, is divided by 36, the remainder is

(A) 3; (B) 6; (C) 9; (D) none of these.

19. If a, b, c, d are in G.P., then $(a^3 + b^3)^{-1}, (b^3 + c^3)^{-1}, (c^3 + d^3)^{-1}$ are in

(A) A. P.; (B) G. P.; (C) H.P.; (D) none of these.

20. The solution set of the inequality $\big| |x| - 1 \big| < 1 - x$ is

(A) $(-\infty, 0)$; (B) $(-\infty, \infty)$; (C) $(0, \infty)$; (D) $(-1, 1)$.

21. The distance of the curve, $y = x^2$, from the straight line $2x - y = 4$ is minimum at the point

(A) $(-1, 1)$; (B) $(1, 1)$; (C) $(2, 4)$; (D) $\left(\frac{1}{2}, \frac{1}{4}\right)$

22. The dual to the following linear program:

$$\begin{array}{ll} \text{maximise} & x_1 + x_2 \\ \text{subject to} & -3x_1 + 2x_2 \leq -1 \\ & x_1 - x_2 \leq 2 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

has

(A) a unique optimal solution;

(B) a feasible solution, but no optimal solution;

(C) multiple optimal solutions;

(D) no feasible solution.

23. The number of real roots of the equation

$x^2 - 3\,|x| + 2 = 0$

is

(A) 1; (B) 2; (C) 3; (D) 4.

24. There are four letters and four directed envelopes. The number of ways in which the letters can be put into the envelopes so that every letter is in a wrong envelope is

(A) 9; (B) 12; (C) 16; (D) 64.

25. If $a^2x^2 + 2bx + c = 0$ has one root greater than unity and the other less than unity, then

(A) $a^2 + 2b + c = 0$; (B) $a^2 + 2b + c > 0$;

(C) $2b + c < 0$; (D) $2b + c > 0$;

26. Given the two sequences $a_n = \frac{1}{n}$ and

$b_n = \frac{1}{n+1}$, the sum, $\sum_{n=1}^{99} \frac{(a_n - b_n)^2}{a_n b_n}$, is

(A) 1; (B) $1 - \frac{1}{99}$; (C) $\frac{99}{100}$; (D) none of these.

27. If $A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$, then $A^{100} + A^5$ is

(A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$; (B) $\begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$; (C) $\begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix}$; (D) none of these.

28. $\lim_{x \to 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$ is

(A) 0; (B) 1; (C) $e^{-\frac{1}{2}}$; (D) $e^{-\frac{1}{6}}$.

29. The system of equations

$$x + y + z = 6$$
$$x + ay - z = 1$$
$$2x + 2y + bz = 12$$

has a unique solution if and only if

(A) $a \neq 1$; (B) $b \neq 2$; (C) $ab \neq 2a + b + 2$; (D) none of these.

30. The number of times $y = x^3 - 3x + 3$ intersects the x – axis is

(A) 0; (B) 1; (C) 2; (D) 3.

Syllabus for ME II (Economics)

Microeconomics: Theory of consumer behaviour, Theory of producer behaviour, Market forms and Welfare economics.

Macroeconomics: National income accounting, Simple model of income determination and Multiplier, IS – LM model, Aggregate demand and aggregate supply model, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2004

$\boxed{\text{NO. 1.}}$ ***Instruction for question numbers 1 (i) – 1 (vi)***

For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question and write it in you answer book.

(i) A consumer consumes only two goods x and y. Her utility function is $U(x, y) = x + y$. Her budget constraint is $px + y = 10$ where p is the price of good x. If $p = \frac{1}{2}$, then the (absolute) own – price elasticity of good x is

(A) 0; (B) $\frac{1}{2}$; (C) 1; (D) ∞

(ii) A consumer consumes only two goods x and y. The price of good x in the local market is p and that in a distant market is q, where $p > q$. However, to go to the distant market, the consumer has to incur a fixed cost C. Suppose that the price of good y is unity in both markets. The consumer's income is I and $I > C$. Let x_0 be the equilibrium consumption of good x. If the consumer has smooth downward sloping and convex indiference curves, then

(A) $(p-q)x_0 = C$ always holds;

(B) $(p-q)x_0 = C$ never holds;

(C) $(p-q)x_0 = C$ may or may not hold depending on the consumer's preferences;

(D) none of the above.

(iii) Consider the following production function $Q = \min\left(\frac{L}{2a}, \frac{K}{4b}\right)$.

Let w and r be the wage and rental rate respectively. The cost function associated with this production function is

(A) 2 awQ;

(B) 4 brQ;

(C) $(wa + 2br)Q$;

(D) none of the above.

(iv) During a period net loan from abroad of an economy is positive. This necessarily implies that during this period

(A) trade balance is positive;

(B) net factor income from abroad is negative;

(C) current account surplus is negative;

(D) change in foreign exchange reserve is positive.

(v) Consider a simple Keynesian economy in which the government expenditure (G) exactly equals its total tax revenue: $G = tY$ where t is the tax rate and Y is the national income. Suppose that the government raises t. Then

(A) Y increases;

(B) Y decreases;

(C) Y remains unchanged;

(D) Y may increase or decrease.

(vi) Which one of the following statements is FALSE?

Interest on pubic debt is not a part of

(A) both personal income and national income;

(B) government consumption expenditure;

(C) national income;

(D) personal income.

No. 2 Indicate whether the following statements are TRUE or FALSE, adding a few lines to justify your answer in each case:

(i) A barrel of crude oil yields a fixed number of gallons of gasoline. Therefore, the price per gallon of gasoline divided by the price per barrel of crude oil is independent of crude oil production.

(ii) If there is no money illusion, once you know all the price elasticities of demand for a commodity, you can calculate its income elasticity.

(iii) If two agents for an Edgeworth box diagram have homothetic preferences then the contract curve is a straight line joining the two origins.

No. 3 Consider a duopoly situation where the inverse market demand function is P(Q) = 10- Q $(\text{where } Q = q_1 + q_2)$. The cost function of firm 1 is $(1 + 2q_1)$ and that of firm 2 is $(1 + 4q_2)$. The firms do not incur any fixed cost if they produce nothing. Calculate the Cournot equilibrium output and profit of the two firms. If, ceteris paribus, the fixed cost of firm 2 is Rs. 2 (instead of Re. 1), what happens to the Cournot equilibrium?

No. 4 A simple Keynesian model has two groups of income earners. The income of group 1 (Y_1) is fixed at Rs. 800. Both groups have proportional consumption function; the average propensity to consume is 0.8 for group 1 and 0.5 for group 2. Group 2 consumes only domestically produced goods. However, group 1 consumes both domestically produced as well as imported goods, their marginal propensity to import being 0.4. investment goods are produced domestically and Investment (I) is autonomously given at Rs. 600.

(i) Compute gross domestic product (Y)

(ii) Suppose group 2 makes an income transfer of Rs. 100 to group 1. However, imports are restricted and cannot exceed Rs. 250 (that is, import function ceases to be operative at this value). How does Y change?

(iii) How does your answer to part (ii) change, if the upper limit of imports is raised to Rs. 400?

No. 5 A consumer consumes electricity (X_E) and other goods (X_O). The price of other goods is unity. To consume electricity the consumer has to pay a rental charge R and a per unit price p. However, p increases with the quantity of electricity consumed according to the function $p = \frac{1}{2} X_E$. The utility function of the consumer is $U = X_E + X_O$ and his income is $I > R$.

(i) Draw the budget line of the consumer.

(ii) If $R = 0$ and $I = 1$, find the optimum consumption bundle.

(iii) Find the maximum *R* that the electricity company can extract from the consumer.

$\boxed{\text{No. 6}}$ A consumer has Rs. 25 to spend on two goods x and y. The price of good *x* is Rs. 3 and that of good *y* is Rs. 4. The continuously differentiable utility function of the consumer is $U(x, y) = 12x + 16y - x^2 - y^2$ where $x \geq 0$ and $y \geq 0$. What happens to the optimum commodity bundle if, instead of Rs. 25, the consumer has Rs. 50 or more to spend on the two goods?

$\boxed{\text{No. 7}}$ Consider an IS – LM model with the following elements:

$$s = s\left(y - t.\theta.y\right), \qquad 0 < s'(.) < 1 \tag{1}$$

$$i = i\,(r), \quad i'(.) < 0 \tag{2}$$

$$l = l\,(y, r), \quad l_1(.) > 0, \quad l_2(.) < 0 \tag{3}$$

where s is the desired private saving, *y* is the real GNP, θ is the (exogenously given) labour's share in GNP, *t* is the proportionate tax rate on labour earnings, *i* is the desired real physical investment, *r* is the interest rate and *l* is the desired real money holdings. The real money balance $\frac{M}{P}$, together with the real government spending g and the tax – rate *t*, are exogenous.

(i) The Laffer curve plots equilibrium tax collections on the vertical axis against *t* on the horizontal axis. Laffer's famous formula was that the curve slopes downwards. Show analytically whether or not that can happen here.

(ii) Suppose *t* is imposed on all factor payments, that is, on GNP. Reformulate equation (1) for this case. Does your answer to part (i) change?

Test code: ME I/ME II, 2005

Syllabus for ME I, 2005

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of a single variable, Theory of Sequence and Series**.**

Linear Programming: Formulations, statements of Primal and Dual problems. Graphical Solutions.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2005

For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question and write it in your answer book.

1. $X \sim B(n, p)$. The maximum value of Var (X) is

   (A) $\frac{n}{4}$; (B) n ; (C) $\frac{n}{2}$; (D) $\frac{1}{n}$.

2. The function $x|x|$ is

   (A) discontinuous at $x = 0$;

   (B) continuous but not differentiable at $x = 0$;

   (C) differentiable at $x = 0$;

   (D) continuous everywhere but not differentiable anywhere.

3. If $A = \begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$, then $A^{100} + A^{5}$ is

   (A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$; (B) $\begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$; (C) $\begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix}$; (D) none of these.

4. The maximum and minimum values of the function

   $f(x) = |x^2 + 2x - 3| + 1.5\, log_e x$, over the interval $\left[\frac{1}{2}, 4\right]$, are

   (A) $(21 + 3\ log_e 2, -1.5\ log_e 2)$; (B) $(21 + log_e 1.5, 0)$;

   (C) $(21 + 3\ log_e 2, 0)$; (D) $(21 + log_e 1.5, -1.5\ log_e 2)$.

5. Let α and β be the roots of the equation $x^2 - px + q = 0$.
Define the sequence $x_n = \alpha^n + \beta^n$. Then x_{n+1} is given by

(A) $px_n - qx_{n-1}$; (B) $px_n + qx_{n-1}$;
(C) $qx_n - px_{n-1}$; (D) $qx_n + p\ x_{n-1}$.

6. Let $f : [-1,1] \to R$ be twice differentiable at $x = 0$, $f(0) = f'(0) = 0$, and $f''(0) = 4$. Then the value of $\underset{x \to 0}{Lt} \dfrac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is

(A) 11 ; (B) 2 ; (C) 12 ; (D) none of these.

7. For $e < x_1 < x_2 < \infty$, $\dfrac{log_e x_2}{log_e x_1}$ is

(A) less than $\dfrac{x_2}{x_1}$; (B) greater than $\dfrac{x_2}{x_1}$, but less than $\left(\dfrac{x_2}{x_1}\right)^2$;

(C) greater than $\left(\dfrac{x_2}{x_1}\right)^3$; (D) greater than $\left(\dfrac{x_2}{x_1}\right)^2$, but less than $\left(\dfrac{x_2}{x_1}\right)^3$.

8. The value of the expression $\dfrac{1}{\sqrt{1}+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \ldots + \dfrac{1}{\sqrt{99}+\sqrt{100}}$ is

(A) a rational number lying in the interval (0,9) ;
(B) an irrational number i lying in the interval (0,9) ;
(C) a rational number lying in the interval (0,10) ;
(D) an irrational number lying in the interval (0,10).

9. Consider a combination lock consisting of 3 buttons that can be pressed in any combination (including multiple buttons at a time), but in such a way that each number is pressed exactly once. Then the total number of possible combination locks with 3 buttons is

(A) 6 ; (B) 9 ; (C) 10 ; (D) 13 .

10. Suppose the correlation coefficient between x and y is denoted by R, and that between x and (y + x), by R_1.

Then, (A) $R_1 > R$; (B) $R_1 = R$;

(C) $R_1 < R$ (D) none of these.

11. The value of $\int_{-1}^{1} (x + |x|)\, dx$ is

(A) 0; (B) -1; (C) 1; (D) none of these.

12. The values of $x_1 \geq 0$ and $x_2 \geq 0$ that maximize $\Pi = 45x_1 + 55x_2$

subject to $6x_1 + 4x_2 \leq 120$ and

$3x_1 + 10x_2 \leq 180$

are

(A) (10,12) ; (B) (8,5) ; (C) (12,11) ; (D) none of the above.

Syllabus for ME II (Economics), 2005

Microeconomics: Theory of consumer behaviour, Theory of producer behaviour, Market forms (Perfect competition, Monopoly, Price Discrimination, Duopoly – Cournot and Bertrand) and Welfare economics.

Macroeconomics: National income accounting, Simple model of income determination and Multiplier, IS – LM model, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2005

1. (a) A divisible cake of size 1 is to be divided among n (>1) persons. It is claimed that the only allocation which is Pareto optimal allocation is $(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n})$. Do you agree with this claim ? Briefly justify your answer.

   (b) Which of the following transactions should be included in GDP? Explain whether the corresponding expenditure is a consumption expenditure or an investment expenditure.

   (i) Mr. Ramgopal , a private investment banker, hires Mr. Gopi to do cooking and cleaning at home.
   (ii) Mr. Ramgopal buys a new Maruti Esteem.
   (iii) Mr. Ramgopal flies to Kolkata from Delhi to see Durga Puja celebration.

(iv) Mr. Ramgopal directly buys (through the internet) 100 stocks of Satyam Ltd..

(v) Mr. Ramgopal builds a house.

2. Roses, once in full bloom, have to be picked up and sold on the same day. On any day the market demand function for roses is given by

$$P = \alpha - Q \qquad \text{(Q is number of roses ; P is price of a rose).}$$

It is also given that the cost of growing roses, having been incurred by any owner of a rose garden long ago, is *not* a choice variable for him now.

(a) Suppose, there is only one seller in the market and he finds 1000 roses in full bloom on a day. How many roses should he sell on that day and at what price?

(b) Suppose there are 10 sellers in the market, and each finds in his garden 100 roses in full bloom ready for sale on a day. What will be the equilibrium price and the number of roses sold on that day? (To answer this part assume $\alpha \geq 1100$).

(c) Now suppose, the market is served by a large number of price taking sellers. However, the total availability on a day remains unchanged at 1000 roses. Find the competitive price and the total number of roses sold on that day.

3. Laxmi is a poor agricultural worker. Her consumption basket comprises three commodities: rice and two vegetables - *cabbage* and *potato*. But

there are occasionally very hard days when her income is so low that she can afford to buy only rice and no vegetables. However, there never arises a situation when she buys only vegetables and no rice. But when she can afford to buy vegetables, she buys only one vegetable, namely the one that has the lower price per kilogram on that day. Price of each vegetable fluctuates day to day while the price of rice is constant.

Write down a suitable utility function that would represent Laxmi's preference pattern. Explain your answer.

4. Consider a simple Keynesian model for a closed economy without Government. Suppose, saving is proportional to income (y), marginal propensity to invest with respect to y is 0.3 and the system is initially in equilibrium. Now, following a parallel downward shift of the saving function the equilibrium level of saving is found to increase by 12 units. Compute the change in the equilibrium income.

5. Consider an IS–LM model. In the commodity market let the consumption function be given by $C = a + b Y$, $a>0$, $0< b <1$. Investment and government spending are exogenous and given by I_0 and G_0 respectively. In the money market, the real demand for money is given by $L = kY - gr$, $k> 0$, $g >0$. The nominal money supply and price level are exogenously given at M_0 and P_0 respectively. In these relations C, Y and r denote consumption, real GDP and interest rate respectively.

(i) Set up the IS – LM equations.

(ii) Determine how an increase in the price level P_1, where $P_1 > P_0$, would affect real GDP and the interest rate.

Test code: ME I/ME II, 2006

Syllabus for ME I, 2006

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables**.**

Linear Programming: Formulations, statements of Primal and Dual problems, Graphical solutions.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2006

For each of the following questions four alternative answers are provided. Choose the answer that you consider to be the most appropriate for a question.

1. If $f(x) = \log\left(\dfrac{1+x}{1-x}\right)$, $\quad 0 < x < 1$, then $f\left(\dfrac{2x}{1+x^2}\right)$ equals

   (A) $2f(x)$; (B) $\dfrac{f(x)}{2}$; (C) $(f(x))^2$; (D) none of these.

2. If $u = \phi(x-y, y-z, z-x)$, then $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z}$ equals

(A) 0; (B) 1; (C) u; (D) none of these.

3. Let A and B be disjoint sets containing m and n elements, respectively, and let $C = A \bigcup B$. The number of subsets S of C that contain k elements and that also have the property that $S \bigcap A$ contains i elements is

(A) $\binom{m}{i}$; (B) $\binom{n}{i}$; (C) $\binom{m}{k-i}\binom{n}{i}$; (D) $\binom{m}{i}\binom{n}{k-i}$.

4. The number of disjoint intervals over which the function $f(x) = \left|0.5x^2 - |x|\right|$ is decreasing is

(A) one; (B) two; (C) three; (D) none of these.

5. For a set of real numbers $x_1, x_2, \ldots\ldots, x_n$, the root mean square (RMS) defined as RMS = $\left\{\frac{1}{N}\sum_{i=1}^{n} x_i^2\right\}^{1/2}$ is a measure of central tendency. If AM denotes the arithmetic mean of the set of numbers, then which of the following statements is correct?

(A) RMS < AM always; (B) RMS > AM always;
(C) RMS < AM when the numbers are not all equal;
(D) RMS > AM when numbers are not all equal.

6. Let $f(x)$ be a function of real variable and let Δf be the function $\Delta f(x) = f(x+1) - f(x)$. For $k > 1$, put $\Delta^k f = \Delta(\Delta^{k-1} f)$. Then $\Delta^k f(x)$ equals

(A) $\sum_{j=0}^{k} (-1)^j \binom{k}{j} f(x+j)$; (B) $\sum_{j=0}^{k} (-1)^{j+1} \binom{k}{j} f(x+j)$;
(C) $\sum_{j=0}^{k} (-1)^j \binom{k}{j} f(x+k-j)$; (D) $\sum_{j=0}^{k} (-1)^{j+1} \binom{k}{j} f(x+k-j)$.

7. Let $I_n = \int_0^\infty x^n e^{-x} dx$, where n is some positive integer. Then I_n equals

(A) $n! - nI_{n-1};$ (B) $n! + nI_{n-1};$ (C) $nI_{n-1};$ (D) none of these.

8. If $x^3 = 1$, then

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \text{ equals}$$

(A) $(cx^2 + bx + a) \begin{vmatrix} 1 & b & c \\ x & c & a \\ x^2 & a & b \end{vmatrix}$; (B) $(cx^2 + bx + a) \begin{vmatrix} x & b & c \\ 1 & c & a \\ x^2 & a & b \end{vmatrix}$;

(C) $(cx^2 + bx + a) \begin{vmatrix} x^2 & b & c \\ x & c & a \\ 1 & a & b \end{vmatrix}$; (D) $(cx^2 + bx + a) \begin{vmatrix} 1 & b & c \\ x^2 & c & a \\ x & a & b \end{vmatrix}$.

9. Consider any integer $I = m^2 + n^2$, where m and n are any two odd integers. Then

(A) I is never divisible by 2;
(B) I is never divisible by 4;
(C) I is never divisible by 6;
(D) none of these.

10. A box has 10 red balls and 5 black balls. A ball is selected from the box. If the ball is red, it is returned to the box. If the ball is black, it and 2 additional black balls are added to the box. The probability that a second ball selected from the box will be red is

(A) $\frac{47}{72}$; (B) $\frac{25}{72}$; (C) $\frac{55}{153}$; (D) $\frac{98}{153}$.

11. Let $f(x) = \dfrac{\log\left(1+\dfrac{x}{p}\right)-\log\left(1-\dfrac{x}{q}\right)}{x}$, $x \neq 0$. If f is continuous at $x = 0$, then the value of $f(0)$ is

(A) $\dfrac{1}{p}-\dfrac{1}{q}$; (B) $p+q$; (C) $\dfrac{1}{p}+\dfrac{1}{q}$; (D) none of these.

12. Consider four positive numbers x_1, x_2, y_1, y_2 such that $y_1\, y_2 > x_1\, x_2$. Consider the number $S = (x_1 y_2 + x_2 y_1) - 2x_1 x_2$. The number S is

(A) always a negative integer;
(B) can be a negative fraction;
(C) always a positive number;
(D) none of these.

13. Given $x \geq y \geq z$, and $x + y + z = 12$, the maximum value of $x+3y+5z$ is

(A) 36; (B) 42; (C) 38; (D) 32.

14. The number of positive pairs of integral values of (x, y) that solves $2xy-4x^2+12x-5y=11$ is

(A) 4; (B) 1; (C) 2; (D) none of these.

15. Consider any continuous function f : [0, 1] $\rightarrow$ [0, 1]. Which one of the following statements is ***incorrect***?

(A) f always has at least one maximum in the interval [0, 1];
(B) f always has at least one minimum in the interval [0, 1];
(C) $\exists$ x $\in$ [0, 1] such that $f(x) = x$;
(D) the function f must always have the property that $f(0) \in \{0, 1\}$, $f(1) \in \{0, 1\}$ *and* $f(0) + f(1) = 1$.

Syllabus for ME II (Economics), 2006

Microeconomics: Theory of consumer behaviour, Theory of production, Market forms (Perfect competition, Monopoly, Price Discrimination, Duopoly – Cournot and Bertrand (elementary problems)) and Welfare economics.

Macroeconomics: National income accounting, Simple model of income determination and Multiplier, IS – LM model (with comparative statics), Harrod – Domar and Solow models, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2006

1.(a) There are two sectors producing the same commodity. Labour is perfectly mobile between these two sectors. Labour market is competitive and the representative firm in each of the two sectors maximizes profit. If there are 100 units of labour and the production function for sector i is: $F(L_i) = 15\sqrt{L_i}$, $i = 1,2$, find the allocation of labour between the two sectors.

(b) Suppose that prices of all variable factors and output double. What will be its effect on the short-run equilibrium output of a competitive firm? Examine whether the short-run profit of the firm will double.

(c) Suppose in year 1 economic activities in a country constitute only production of wheat worth Rs. 750. Of this, wheat worth Rs. 150 is exported and the rest remains unsold. Suppose further that in year 2 no production takes place, but the unsold wheat of year 1 is sold domestically and residents of the country import shirts worth Rs. 250. Fill in, with adequate explanation, the following chart :

Year	GDP	= Consumption	+ Investment	+ Export	- Import
1	____	____	____	____	____
2	____	____	____	____	____

2. A price-taking farmer produces a crop with labour L as the only input. His production function is: $F(L) = 10\sqrt{L} - 2L$. He has 4 units of labour

in his family and he cannot hire labour from the wage labour market. He does not face any cost of employing family labour.

(a) Find out his equilibrium level of output.

(b) Suppose that the government imposes an income tax at the rate of 10 per cent. How does this affect his equilibrium output?

(c) Suppose an alternative production technology given by: $F(L) = 11\sqrt{L} - L - 15$ is available. Will the farmer adopt this alternative technology? Briefly justify your answer.

3. Suppose a monopolist faces two types of consumers. In type I there is only one person whose demand for the product is given by : $Q_I = 100 - P$, where P represents price of the good. In type II there are n persons, each of whom has a demand for one unit of the good and each of them wants to pay a maximum of Rs. 5 for one unit. Monopolist cannot price discriminate between the two types. Assume that the cost of production for the good is zero. Does the equilibrium price depend on n ? Give reasons for your answer.

4. The utility function of a consumer is: $U(x, y) = xy$. Suppose income of the consumer (M) is 100 and the initial prices are $P_x = 5$, $P_y = 10$. Now suppose that P_x goes up to 10, P_y and M remaining unchanged. Assuming Slutsky compensation scheme, estimate price effect, income effect and substitution effect.

5. Consider an *IS-LM* model for a closed economy. Private consumption depends on disposable income. Income taxes (T) are lump-sum. Both private investment and speculative demand for money vary inversely with interest rate (r). However, transaction demand for money depends not on income (y) but on disposable income (y_d). Argue how the equilibrium values of private investment, private saving, government saving, disposable income and income will change, if the government raises T.

6. An individual enjoys bus ride. However, buses emit smoke which he dislikes. The individual's utility function is: $U = U(x, s)$, where x is the distance (in km) traveled by bus and s is the amount of smoke consumed from bus travel.

(a) What could be the plausible alternative shapes of indifference curve between x and s?

(b) Suppose, smoke consumed from bus travel is proportional to the distance traveled: $s = \alpha x$ (α is a positive parameter). Suppose further that the bus fare per km is p and that the individual has money income M to spend on bus travel. Show the budget set of the consumer in an (s, x) diagram.

(c) What can you say about an optimal choice of the individual? Will he necessarily exhaust his entire income on bus travel?

7. (a) Suppose the labour supply (l) of a household is governed by maximization of its utility (u): $u = c^{2/3} h^{1/3}$, where c is the household's consumption and h is leisure enjoyed by the household (with $h + l = 24$). Real wage rate (w) is given and the household consumes the entire labour income (wl). What is the household's labour supply? Does it depend on w?

(b) Consider now a typical Keynesian (closed) economy producing a single good and having a single household. There are two types of final expenditure – viz., investment autonomously given at 36 units and household consumption (c) equalling the household's labour income (wl). It is given that $w = 4$. Firms produce aggregate output (y) according to the production function: $y = 24\sqrt{l}$. Find the equilibrium level of output and employment. Is there any involuntary unemployment? If so, how much?

8. Suppose an economic agent's life is divided into two periods, the first period constitutes her youth and the second her old age. There is a single consumption good, C, available in both periods and the agent's utility function is given by

$$u(C_1, C_2) = \frac{C_1^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\,\frac{C_2^{1-\theta} - 1}{1-\theta}, \qquad 0 < \theta < 1, \rho > 0,$$

where the first term represents utility from consumption during youth. The second term represents discounted utility from consumption in old age, $1/(1+\rho)$ being the discount factor. During the period, the agent has a unit of labour which she supplies inelastically for a wage rate w. Any savings (i.e., income minus consumption during the first period) earns a rate of interest r, the proceeds from which are available in old age in units of the only consumption good available in the economy. Denote savings by s. The agent maximizes utility subjects to her budget constraint.

i) Show that θ represents the elasticity of marginal utility with respect to consumption in each period.

ii) Write down the agent's optimization problem, i.e., her problem of maximizing utility subject to the budget constraint.

iii) Find an expression for s as a function of w and r.

iv) How does s change in response to a change in r? In particular, show that this change depends on whether θ exceeds or falls short of unity.

v) Give an intuitive explanation of your finding in (iv)

9. A consumer consumes only two commodities x_1 and x_2. Suppose that her utility function is given by $\mathrm{U}(x_1, x_2) = \min\ (2x_1, x_2)$.

(i) Draw a representative indifference curve of the consumer.

(ii) Suppose the prices of the commodities are Rs.5 and Rs.10 respectively while the consumer's income is Rs. 100. What commodity bundle will the consumer purchase?

(iii) Suppose the price of commodity 1 now increases to Rs. 8. Decompose the change in the amount of commodity 1 purchased into income and substitution effects.

10. A price taking firm makes machine tools Y using labour and capital according to the production function Y = $K^{0.25}L^{0.25}$. Labour can be hired at the beginning of every week while capital can be hired only at the beginning of every month. Let one month be considered as long run period and one week as short run period. Further assume that one month equals four weeks. The wage rate per week and the rental rate of capital per month are both 10.

(i) Given the above information, find the short run and the long run cost functions of the firm.

(ii) At the beginning of the month of January, the firm is making long run decisions given that the price of machine tools is 400. What is the long run profit maximizing number of machine tools? How many units of labour and capital should the firm hire at the beginning of January?

11. Consider a neo-classical one-sector growth model with the production function $Y = \sqrt{KL}$. If 30% of income is invested and capital stock depreciates at the rate of 7% and labour force grows at the rate of 3%, find out the level of per capita income in the steady-state equilibrium.

Test code: ME I/ME II, 2007

Syllabus for ME I, 2007

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Linear Programming: Formulations, statements of Primal and Dual problems, Graphical solutions.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2007

1. Let α and β be any two positive real numbers. Then

   $Lim_{x\to 0} \dfrac{(1+x)^{\alpha}-1}{(1+x)^{\beta}-1}$ equals

   (A) $\dfrac{\alpha}{\beta}$; (B) $\dfrac{\alpha+1}{\beta+1}$; (C) $\dfrac{\alpha-1}{\beta-1}$; (D) 1.

2. Suppose the number X is odd. Then X^2-1 is

   (A) odd; (B) not prime;

   (C) necessarily positive; (D) none of the above.

3. The value of k for which the function $f(x) = ke^{kx}$ is a probability density function on the interval [0, 1] is

(A) $k = \log 2$; (B) $k = 2\log 2$; (C) $k = 3\log 3$; (D) $k = 3\log 4$.

4. p and q are positive integers such that $p^2 - q^2$ is a prime number. Then, $p - q$ is

(A) a prime number; (B) an even number greater than 2;

(C) an odd number greater than 1 but not prime; (D) none of these.

5. Any non-decreasing function defined on the interval $[a,b]$

(A) is differentiable on (a, b);

(B) is continuous in $[a,b]$ but not differentiable;

(C) has a continuous inverse;

(D) none of these.

6. The equation $\begin{vmatrix} x & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 8 & 1 \end{vmatrix} = 0$ is satisfied by

(A) $x = 1$; (B) $x = 3$; (C) $x = 4$; (D) none of these.

7. If $f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots,}}}}$, then $f'(x)$ is

(A) $\dfrac{x}{2f(x)-1}$; (B) $\dfrac{1}{2f(x)-1}$; (C) $\dfrac{1}{x\sqrt{f(x)}}$; (D) $\dfrac{1}{2f(x)+1}$.

8. If $P = \log_x(xy)$ and $Q = \log_y(xy)$, then $P + Q$ equals

(A) PQ; (B) P/Q; (C) Q/P; (D) $(PQ)/2$.

9. The solution to $\int \dfrac{2x^3+1}{x^4+2x}\,dx$ is

(A) $\dfrac{x^4+2x}{4x^3+2}$ + constant; (B) $\log x^4 + \log 2x$ + constant;

(C) $\frac{1}{2}\log\left|x^4+2x\right|$ + constant; (D) $\left|\frac{x^4+2x}{4x^3+2}\right|$ + constant.

10. The set of all values of x for which $x^2-3x+2>0$ is

(A) $(-\infty,1)$; (B) $(2,\infty)$; (C) $(-\infty,2)\cap(1,\infty)$; (D) $(-\infty,1)\cup(2,\infty)$.

11. Consider the functions $f_1(x)=x^2$ and $f_2(x)=4x^3+7$ defined on the real line. Then

(A) f_1 is one-to-one and onto, but not f_2;

(B) f_2 is one-to-one and onto, but not f_1;

(C) both f_1 and f_2 are one-to-one and onto;

(D) none of the above.

12. If $f(x)=\left(\frac{a+x}{b+x}\right)^{a+b+2x}$, $a>0,\ b>0$, then $f'(0)$ equals

(A) $\left(\frac{b^2-a^2}{b^2}\right)\left(\frac{a}{b}\right)^{a+b-1}$; (B) $\left(2\log\left(\frac{a}{b}\right)+\frac{b^2-a^2}{ab}\right)\left(\frac{a}{b}\right)^{a+b}$;

(C) $2\log\left(\frac{a}{b}\right)+\frac{b^2-a^2}{ab}$; (D) $\left(\frac{b^2-a^2}{ba}\right)$.

13. The linear programming problem

$$\max_{x,y}\ z=0.5x+1.5y$$

$$\begin{aligned}\text{subject to:}\quad & x+y\le 6\\ & 3x+y\le 15\\ & x+3y\le 15\\ & x,y\ge 0\end{aligned}$$

has

(A) no solution; (B) a unique non-degenerate solution;

(C) a corner solution; (D) infinitely many solutions.

14. Let $f(x;\,\theta) = \theta f(x;1) + (1-\theta) f(x;0)$, where θ is a constant satisfying $0 < \theta < 1$. Further, both $f(x;1)$ and $f(x;0)$ are probability density functions (*p.d.f.*). Then

(A) $f(x;\theta)$ is a *p.d.f.* for all values of θ;

(B) $f(x;\theta)$ is a *p.d.f.* only for $0 < \theta < \frac{1}{2}$;

(C) $f(x;\theta)$ is a *p.d.f.* only for $\frac{1}{2} \le \theta < 1$;

(D) $f(x;\theta)$ is not a *p.d.f.* for any value of θ.

15. The correlation coefficient r for the following five pairs of observations

x	5	1	4	3	2
y	0	4	2	0	-1

satisfies

(A) $r > 0$; (B) $r < -0.5$; (C) $-0.5 < r < 0$; (D) $r = 0$.

16. An n-coordinated function f is called homothetic if it can be expressed as an increasing transformation of a homogeneous function of degree one. Let $f_1(x) = \sum_{i=1}^{n} x_i^r$,

and $f_2(x) = \sum_{i=1}^{n} a_i x_i + b$, where $x_i > 0$ for all *i*, $0 < r < 1$, $a_i > 0$ and *b* are constants. Then

(A) f_1 is not homothetic but f_2 is; (B) f_2 is not homothetic but f_1 is;

(C) both f_1 and f_2 are homothetic; (D) none of the above.

17. If $h(x) = \frac{1}{1-x}$, then $h(h(h(x)))$ equals

(A) $\frac{1}{1-x}$; (B) x; (C) $\frac{1}{x}$; (D) 1- x.

18. The function $x|x| + \left(\frac{|x|}{x}\right)^3$ is

(A) continuous but not differentiable at $x = 0$;

(B) differentiable at $x = 0$;

(C) not continuous at $x = 0$;

(D) continuously differentiable at $x = 0$.

19. $\int \frac{2dx}{(x-2)(x-1)x}$ equals

(A) $\log\left|\frac{x(x-2)}{(x-1)^2}\right|$ + constant;

(B) $\log\left|\frac{(x-2)}{x(x-1)^2}\right|$ + constant;

(C) $\log\left|\frac{x^2}{(x-1)(x-2)}\right|$ + constant;

(D) $\log\left|\frac{(x-2)^2}{x(x-1)}\right|$ + constant.

20. Experience shows that 20% of the people reserving tables at a certain restaurant never show up. If the restaurant has 50 tables and takes 52 reservations, then the probability that it will be able to accommodate everyone is

(A) $1-\frac{209}{552}$; (B) $1-14\times\left(\frac{4}{5}\right)^{52}$; (C) $\left(\frac{4}{5}\right)^{50}$; (D) $\left(\frac{1}{5}\right)^{50}$.

21. For any real number x, define $[x]$ as the highest integer value not greater than x. For example, $[0.5] = 0$, $[1] = 1$ and $[1.5] = 1$. Let $I = \int_0^{3/2} \{[x] + [x^2]\} dx$. Then I equals

(A) 1; (B) $\frac{5 - 2\sqrt{2}}{2}$;

(C) $2\sqrt{2}$; (D) none of these.

22. Every integer of the form $(n^3 - n)(n^2 - 4)$ (for $n = 3, 4, \ldots$) is

(A) divisible by 6 but not always divisible by 12;

(B) divisible by 12 but not always divisible by 24;

(C) divisible by 24 but not always divisible by 120;

(D) divisible by 120 but not always divisible by 720.

23. Two varieties of mango, A and B, are available at prices Rs. p_1 and Rs. p_2 per kg, respectively. One buyer buys 5 kg. of A and 10 kg. of B and another buyer spends Rs. 100 on A and Rs. 150 on B. If the average expenditure per mango (irrespective of variety) is the same for the two buyers, then which of the following statements is the most appropriate?

(A) $p_1 = p_2$; (B) $p_2 = \frac{3}{4} p_1$;

(C) $p_1 = p_2$ or $p_2 = \frac{3}{4} p_1$; (D) $\frac{3}{4} \le \frac{p_2}{p_1} < 1$.

24. For a given bivariate data set $(x_i, y_i;\ i = 1, 2, \ldots, n)$, the squared correlation coefficient (r^2) between x^2 and y is found to be 1. Which of the following statements is the most appropriate?

(A) In the (x, y) scatter diagram, all points lie on a straight line.

(B) In the (x, y) scatter diagram, all points lie on the curve $y = x^2$.

(C) In the (x, y) scatter diagram, all points lie on the curve $y = a + bx^2$, $a > 0$, $b > 0$.

(D) In the (x, y) scatter diagram, all points lie on the curve $y = a + bx^2$, a, b any real numbers.

25. The number of possible permutations of the integers 1 to 7 such that the numbers 1 and 2 always precede the number 3 and the numbers 6 and 7 always succeed the number 3 is

(A) 720; (B) 168;

(C) 84; (D) none of these.

26. Suppose the real valued continuous function f defined on the set of non-negative real numbers satisfies the condition $f(x) = xf(x)$, then $f(2)$ equals

(A) 1; (B) 2;

(C) 3; (D) $f(1)$.

27. Suppose a discrete random variable X takes on the values $0, 1, 2, \ldots, n$ with frequencies proportional to binomial coefficients $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$ respectively. Then the mean (μ) and the variance (σ^2) of the distribution are

(A) $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{2}$;

(B) $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{4}$;

(C) $\mu = \frac{n}{2}$ and $\sigma^2 = \frac{n}{4}$;

(D) $\mu = \frac{n}{4}$ and $\sigma^2 = \frac{n}{2}$.

28. Consider a square that has sides of length 2 units. *Five* points are placed anywhere inside this square. Which of the following statements is **incorrect**?

(A) There cannot be any two points whose distance is more than $2\sqrt{2}$.

(B) The square can be partitioned into four squares of side 1 unit each such that at least one unit square has two points that lies on or inside it.

(C) At least two points can be found whose distance is less than $\sqrt{2}$.

(D) Statements (A), (B) and (C) are all incorrect.

29. Given that f is a real-valued differentiable function such that $f(x)f'(x) < 0$ for all real x, it follows that

(A) $f(x)$ is an increasing function;

(B) $f(x)$ is a decreasing function;

(C) $|f(x)|$ is an increasing function;

(D) $|f(x)|$ is a decreasing function.

30. Let p, q, r, s be four arbitrary positive numbers. Then the value of $\dfrac{(p^2+p+1)(q^2+q+1)(r^2+r+1)(s^2+s+1)}{pqrs}$ is at least as large as

(A) 81;

(B) 91;

(C) 101.

(D) None of these.

Syllabus for ME II (Economics), 2007

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2007

1. (a) There is a cake of size 1 to be divided between two persons, 1 and 2. Person 1 is going to cut the cake into two pieces, but person 2 will select one of the two pieces for himself first. The remaining piece will go to person 1. What is the optimal cutting decision for player 1? Justify your answer.

(b) Kamal has been given a free ticket to attend a classical music concert. If Kamal had to pay for the ticket, he would have paid up to Rs. 300/- to attend the concert. On the same evening, Kamal's alternative entertainment option is a film music and dance event for which tickets are priced at Rs. 200/- each. Suppose also that Kamal is willing to pay up to Rs. X to attend the film music and dance event. What does Kamal do, i.e., does he attend the classical music concert, or does he attend the film music and dance show, or does he do neither? Justify your answer.

2. Suppose market demand is described by the equation P = 300 – Q and competitive conditions prevail. The short-run supply curve is P = -180 +5Q. Find the initial short-run equilibrium price and quantity. Let the long-run supply curve be P = 60 + 2Q. Verify whether the market is also in the long-run equilibrium at the initial short-run equilibrium that you have worked out. Now suppose that the market demand at every price is

doubled. What is the new market demand curve? What happens to the equilibrium in the *very* short-run? What is the new short-run equilibrium? What is the new long-run equilibrium? If a price ceiling is imposed at the old equilibrium, estimate the perceived shortage. Show all your results in a diagram.

3. (a) Suppose in year 1 economic activities in a country constitute only production of wheat worth Rs. 750. Of this, wheat worth Rs. 150 is exported and the rest remains unsold. Suppose further that in year 2 no production takes place, but the unsold wheat of year 1 is sold domestically and residents of the country import shirts worth Rs. 250. Fill in, with adequate explanation, the following chart :

Year	GDP	= Consumption	+ Investment	+ Export	- Import
1	____	____	____	____	____
2	____	____	____	____	____

(b) Consider an IS-LM model for a closed economy with government where investment (I) is a function of rate of interest (r) only. An increase in government expenditure is found to crowd out 50 units of private investment. The government wants to prevent this by a minimum change in the supply of real money balance. It is given that $\frac{dI}{dr} = -50$, slope of the LM curve, $\frac{dr}{dy}(LM) = \frac{1}{250}$, slope of the IS curve, $\frac{dr}{dy}(IS) = -\frac{1}{125}$, and all relations are linear. Compute the change in y from the initial to the final equilibrium when all adjustments have been made.

4. (a) Consider a consumer with income W who consumes *three* goods, which we denote as $i = 1, 2, 3$. Let the amount of good i that the consumer consumes be x_i and the price

of good i be p_i. Suppose that the consumer's preference is described by the utility function $U(x_1, x_2, x_3) = x_1\sqrt{x_2 x_3}$.

(i) Set up the utility maximization problem and write down the Lagrangian.

(ii) Write down the first order necessary conditions for an interior maximum and then obtain the Marshallian (or uncompensated) demand functions.

(b) The production function, $Y = F(K, L)$, satisfies the following properties: (i) CRS, (ii) symmetric in terms of inputs and (iii) $F(1,1) = 1$. The price of each input is Rs. 2/- per unit and the price of the product is Rs. 3/- per unit. *Without using calculus* find the firm's optimal level of production.

5.(a) A monopolist has contracted to sell as much of his output as he likes to the government at Rs.100/- per unit. His sale to the government is positive. He also sells to private buyers at Rs 150/- per unit. What is the price elasticity of demand for the monopolist's products in the private market?

(b) Mrs. Pathak is very particular about her consumption of tea. She always takes 50 grams of sugar with 20 grams of ground tea. She has allocated Rs 55 for her spending on tea and sugar per month. (Assume that she doesn't offer tea to her guests or anybody else and she doesn't consume sugar for any other purpose). Sugar and tea are sold at 2 paisa per 10 grams and 50 paisa per 10 grams respectively. Determine how much of tea and sugar she demands per month.

(c) Consider the IS-LM model with government expenditure and taxation. A change in the income tax rate changes the equilibrium from $(y = 3000,\ r = 4\%)$ to $(y = 3500,\ r = 6\%)$, where y, r denote income and rate of interest, respectively. It is given that a unit increase in y increases demand for real money balance by 0.25 of a unit. Compute the change in real money demand that results from a 1% increase in the rate of interest. (Assume that all relationships are linear**.**)

6. (a) An economy produces two goods, corn and machine, using for their production only labor and some of the goods themselves. Production of one unit of corn requires 0.1 units of corn, 0.3 machines and 5 man-hours of labor. Similarly, production of one machine requires 0.4 units of corn, 0.6 machines and 20 man-hours of labor.

(i) If the economy requires 48 units of corn but no machine for final consumption, how much of each of the two commodities is to be produced? How much labor will be required?

(ii) If the wage rate is Rs. 2/- per man-hour, what are the prices of corn and machines, if price of each commodity is equated to its average cost of production?

(b) Consider two consumers A and B, each with income W. They spend their entire budget over the two commodities, X and Y. Compare the demand curves of the two consumers under the assumption that their utility functions are $U_A = x + y$ and $U_B = x^2 + y^2$ respectively.

7. Consider a Simple Keynesian Model without government for an open economy, where both consumption and import are proportional functions of income (Y). Suppose that average propensities to consume and import are 0.8 and 0.3, respectively. The investment (I) function and the level of export (X) are given by $I = 100 + 0.4Y$ and $X = 100$.

(i) Compute the aggregate demand function if the maximum possible level of imports is 450. Can there be an equilibrium for this model? Show your result graphically.

(ii) How does your answer to part (i) change if the limit to import is raised to 615? What can you say about the stability of equilibrium if it exists?

8. Suppose an economic agent's life is divided into two periods, the first period constitutes her youth and the second her old age. There is a single consumption good, C, available in both periods and the agent's utility function is given by

$$u(C_1, C_2) = \frac{C_1^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho} \frac{C_2^{1-\theta} - 1}{1-\theta}, \qquad 0 < \theta < 1, \rho > 0,$$

where the first term represents utility from consumption during youth. The second term represents discounted utility from consumption in old age, $1/(1+\rho)$ being the discount factor. During the period, the agent has a unit of labour which she supplies inelastically for a wage rate w. Any savings (i.e., income minus consumption during the first period) earns a rate of interest r, the proceeds from which are available in old age in units of the only consumption good available in the economy. Denote savings by s. The agent maximizes utility subjects to her budget constraint.

i) Show that θ represents the elasticity of marginal utility with respect to consumption in each period.

ii) Write down the agent's optimization problem, i.e., her problem of maximizing utility subject to the budget constraint.

iii) Find an expression for s as a function of w and r.

(iv)How does s change in response to a change in r? In particular, show that this change depends on whether θ exceeds or falls short of unity.

(v)Give an intuitive explanation of your finding in (iv)

9. Consider a neo-classical one-sector growth model with the production function $Y = \sqrt{KL}$. If 30% of income is invested and capital stock depreciates at the rate of 7% and labour force grows at the rate of 3%, find out the level of per capita income in the steady-state equilibrium.

Test code: ME I/ME II, 2008

Syllabus for ME I, 2008

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables**.**

Linear Programming: Formulations, statements of Primal and Dual problems, Graphical solutions.

Theory of Polynomial Equations (up to third degree).
Elementary Statistics: Measures of central tendency; dispersion, correlation, Elementary probability theory, Probability mass function, Probability density function and Distribution function.

Sample Questions for ME I (Mathematics), 2008

1. $\int \frac{dx}{x + x\log x}$ equals

   (a) $\log|x + x\log x| + \text{constant}$

   (b) $\log|1 + x\log x| + \text{constant}$

   (c) $\log|\log x| + \text{constant}$

   (d) $\log|1 + \log x| + \text{constant}$.

2. The inverse of the function $\sqrt{-1+x}$ is

   (a) $\frac{1}{\sqrt{x-1}}$, (b) $x^2 + 1$, (c) $\sqrt{x-1}$, (d) none of these.

3. The domain of continuity of the function $f(x) = \sqrt{x} + \frac{x+1}{x-1} - \frac{x+1}{x^2+1}$ is

   (a) $[0,1)$, (b) $(1,\infty)$, (c) $[0,1) \cup (1,\infty)$, (d) none of these

4. Consider the following linear programme:

   minimise $x - 2y$

   subject to
   $x + 3y \geq 3$
   $3x + y \geq 3$
   $x + y \leq 3$

   An optimal solution of the above programme is given by

   (a) $x = \frac{3}{4}$, $y = \frac{3}{4}$.

   (b) $x = 0$, $y = 3$

   (c) $x = -1$, $y = 3$.

   (d) none of (a), (b) and (c).

5. Consider two functions $f_1 : \{a_1, a_2, a_3\} \to \{b_1, b_2, b_3, b_4\}$ and $f_2 : \{b_1, b_2, b_3, b_4\} \to \{c_1, c_2, c_3\}$. The function f_1 is defined by $f_1(a_1) = b_1$, $f_1(a_2) = b_2$, $f_1(a_3) = b_3$ and the function f_2 is defined by $f_2(b_1) = c_1$, $f_2(b_2) = c_2$, $f_2(b_3) = f_2(b_4) = c_3$. Then the mapping $f_2 \circ f_1 : \{a_1, a_2, a_3\} \to \{c_1, c_2, c_3\}$ is

   (a) a composite and one – to – one function but not an onto function.
   (b) a composite and onto function but not a one – to – one function.
   (c) a composite, one – to – one and onto function.
   (d) not a function.

6. If $x = t^{\frac{1}{t-1}}$ and $y = t^{\frac{t}{t-1}}$, $t > 0$, $t \neq 1$ then the relation between x and y is

(a) $y^x = x^{\frac{1}{y}}$, (b) $x^{\frac{1}{y}} = y^{\frac{1}{x}}$, (c) $x^y = y^x$, (d) $x^y = y^{\frac{1}{x}}$.

7. The maximum value of $T = 2x_B + 3x_S$ subject to the constraint $20x_B + 15x_S \leq 900$,

where $x_B \geq 0$ and $x_S \geq 0$, is

(a) 150, (b) 180, (c) 200, (d) none of these.

8. The value of $\int_0^2 [x]^n f'(x)\,dx$, where $[x]$ stands for the integral part of x, n is a positive integer and f' is the derivative of the function f, is

(a) $(n+2^n)(f(2)-f(0))$, (b) $(1+2^n)(f(2)-f(1))$,
(c) $2^n f(2)-(2^n-1)f(1)-f(0)$, (d) none of these.

9. A surveyor found that in a society of 10,000 adult literates 21% completed college education, 42% completed university education and remaining 37% completed only school education. Of those who went to college 61% reads newspapers regularly, 35% of those who went to the university and 70% of those who completed only school education are regular readers of newspapers. Then the percentage of those who read newspapers regularly completed only school education is

(a) 40%, (b) 52%, (c) 35%, (d) none of these.

10. The function $f(x) = x|x|e^{-x}$ defined on the real line is
(a) continuous but not differentiable at zero,
(b) differentiable only at zero,
(c) differentiable everywhere,
(d) differentiable only at finitely many points.

11. Let X be the set of positive integers denoting the number of tries it takes the Indian cricket team to win the World Cup. The team has equal odds for winning or losing any match. What is the probability that they will win in odd number of matches?

(a) 1/4, (b) 1/2, (c) 2/3, (d) 3/4.

12. Three persons X, Y, Z were asked to find the mean of 5000 numbers, of which 500 are unities. Each one did his own simplification.

$X's$ method: Divide the set of number into 5 equal parts, calculate the mean for each part and then take the mean of these.
$Y's$ method: Divide the set into 2000 and 3000 numbers and follow the procedure of A.
$Z's$ method: Calculate the mean of 4500 numbers (which are $\neq 1$) and then add 1.
Then

(a) all methods are correct,
(b) $X's$ method is correct, but Y and $Z's$ methods are wrong,
(c) $X's$ and $Y's$ methods are correct but $Z's$ methods is wrong,
(d) none is correct.

13. The number of ways in which six letters can be placed in six directed envelopes such that exactly four letters are placed in correct envelopes and exactly two letters are placed in wrong envelopes is
(a) 1, (b) 15, (c) 135. (d) None of these.

14. The set of all values of x for which the inequality $|x-3|+|x+2|<11$ holds is
(a) (– 3, 2), (b) (– 5, 2), (c) (–5, 6), (d) none of these.

15. The function $f(x)=x^4-4x^3+16x$ has
(a) a unique maximum but no minimum,
(b) a unique minimum but no maximum,
(c) a unique maximum and a unique minimum,
(d) neither a maximum nor a minimum.

16. Consider the number $K(n)=(n+3)(n^2+6n+8)$ defined for integers n. Which of the following statements is correct?
(a) *K (n)* is always divisible by 4,
(b) *K (n)* is always divisible by 5,
(c) *K (n)* is always divisible by 6,
(d) Statements (a), (b) and(c) are incorrect.

17. 25 books are placed at random on a shelf. The probability that a particular pair of books shall be always together is
(a) $\frac{2}{25}$, (b) $\frac{1}{25}$, (c) $\frac{1}{300}$, (d) $\frac{1}{600}$.

18. $P(x)$ is a quadratic polynomial such that $P(1) = -P(2)$. If –1 is a root of the equation, the other root is

(a) $\frac{4}{5}$, (b) $\frac{8}{5}$, (c) $\frac{6}{5}$, (d) $\frac{3}{5}$.

19. The correlation coefficients between two variables X and Y obtained from the two equations $2x+3y-1=0$ and $5x-2y+3=0$ are

(a) equal but have opposite signs,

(b) $-\frac{2}{3}$ and $\frac{2}{5}$,

(c) $\frac{1}{2}$ and - $\frac{3}{5}$,

(d) Cannot say.

20. If *a, b, c, d* are positive real numbers then $\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{a}$ is always

(a) less than $\sqrt{2}$,
(b) less than 2 but greater than or equal to $\sqrt{2}$,
(c) less than 4 but greater than or equal 2,
(d) greater than or equal to 4.

21. The range of value of x for which the inequality $\log_{(2-x)}(x-3) \geq -1$ holds is

(a) $2 < x < 3$, (b) $x > 3$, (c) $x < 2$, (d) no such x exists.

22. The equation $5x^3 - 5x^2 + 2x - 1$ has
(a) all roots between 1 and 2,
(b) all negative roots,
(c) a root between 0 and 1,
(d) all roots greater than 2.

23. The probability density of a random variable is

$$f(x) = ax^2 \exp^{-kx} \qquad (k > 0, 0 \leq x \leq \infty)$$

Then, a equals

(a) $\frac{k^3}{2}$, (b) $\frac{k}{2}$, (c) $\frac{k^2}{2}$, (d) k.

24. Let $x = r$ be the mode of the distribution with probability mass function $p(x) = \binom{n}{x} p^x (1-p)^{n-x}$. Then which of the following inequalities hold.

(a) $(n+1)p - 1 < r < (n+1)p$,

(b) $r < (n+1)p - 1$,

(c) $r > (n+1)p$,

(d) $r < np$.

25. Let $y=(y_1,\ldots,y_n)$ be a set of n observations with $y_1 \le y_2 \le \ldots \le y_n$. Let $y'=(y_1,y_2,\ldots,y_j+\delta,\ldots,y_k-\delta,\ldots,y_n)$ where $y_k-\delta > y_{k-1} > \ldots. > y_{j+1} > y_j+\delta$, $\delta>0$. Let σ : standard deviation of y and σ' : standard deviation of y'. Then
(a) $\sigma<\sigma'$, (b) $\sigma'<\sigma$, (c) $\sigma'=\sigma$, (d) nothing can be said.

26. Let x be *a r.v.* with *pdf* $f(x)$ and let $F(x)$ be the distribution function. Let $r(x)=\dfrac{xf(x)}{1-F(x)}$. Then for $x<e^{\mu}$ and $f(x)=\dfrac{e^{-\frac{(\log x-\mu)^2}{2}}}{x\sqrt{2\pi}}$, the function $r(x)$ is
(a) increasing in x,
(b) decreasing in x,
(c) constant,
(d) none of the above.

27. A square matrix of order n is said to be a bistochastic matrix if all of its entries are non-negative and each of its rows and columns sum to 1. Let $y_{n\times1}=P_{n\times n}.x_{n\times1}$ where elements of y are some rearrangements of the elements of x. Then
(a) P is bistochastic with diagonal elements 1,
(b) P cannot be bistochastic,
(c) P is bistochastic with elements 0 and 1,
(d) P is a unit matrix.

28. Let $f_1(x)=\dfrac{x}{x+1}$. Define $f_n(x)=f_1(f_{n-1}(x))$, where $n\ge2$. Then $f_n(x)$ is
(a) decreasing in n, (b) increasing in n, (c) initially decreasing in n and then increasing in n, (d) initially increasing in n and then decreasing n.

29. $\lim\limits_{n\to\infty}\dfrac{1-x^{-2n}}{1+x^{-2n}}$, $x>0$ equals

(a) 1, (b) - 1 , (c) 0. (d) The limit does not exist.

30. Consider the function $f(x_1,x_2)=\max\{6-x_1,7-x_2\}$. The solution (x_1^*,x_2^*) to the optimization problem minimize $f(x_1,x_2)$ subject to $x_1+x_2=21$ is
(a) $(x_1^*=10.5,\ x_2^*=10.5)$,
(b) $(x_1^*=11,\ x_2^*=10)$,
(c) $(x_1^*=10,\ x_2^*=11)$,
(d) None of these.

Syllabus for ME II (Economics), 2008

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2008

1. There are two individuals A and B and two goods X and Y. The utility functions of A and B are given by $U_A = X_A$ and $U_B = X_B^2 + Y_B^2$ respectively where X_i, Y_i are consumption levels of the two goods by individual i, $i = A, B$.

   (a) Draw the indifference curves of A and B.
   (b) Suppose A is endowed with 10 units of Y and B with 10 units of X. Indicate the endowment point in a box diagram.
   (c) Draw the set of Pareto optimal allocation points in the box diagram.

2. Suppose an economy's aggregate output (Y) is given by the following production function:

$$Y = U N^{\alpha}, \ (0 < \alpha < 1)$$

   where U, a random variable, represents supply shock. Employment of labour (N) is determined by equating its marginal product to $\frac{W}{P}$, where W is nominal wage and P is price level.

   Use the notations: $u = \log \alpha + \frac{1}{\alpha} \log U$; $p = \log P$; $w = \log W$ and $y = \log Y$.

(a) Obtain the aggregate supply function (y) in terms of p, w, and u.
(b) Add the following relations:

   Wages are indexed: $w = \theta p, \ (0 \le \theta \le 1)$
   Aggregate demand: $y = m - p$, (m = logarithm of money, a policy variable)

   Find the solution of y in terms of m and u.

(c) Does monetary policy affect output (i) if indexation is partial $(0 < \theta < 1)$, (ii) indexation is full $(\theta = 1)$?
(d) Does the real shock affect output <u>more</u> when indexation is higher? Explain.

3. Two firms 1 and 2 sell a single, homogeneous, infinitely divisible good in a market. Firm 1 has 40 units to sell and firm 2 has 80 units to sell. Neither firm can produce any more units. There is a demand curve: $p = a - q$, where q is the total amount placed by the firms in the market. So if q_i is the amount placed by firm i, $q = q_1 + q_2$ and p is the price that emerges. a is positive and a measure of market size. It is known that a is either 100 or 200. The value of a is observed by both firms. After they observe the value of a, each firm decides whether or not to destroy a part of its output. This decision is made simultaneously and independently by the firms. Each firm faces a constant per unit cost of destruction

equal to 10. Whatever number of units is left over after destruction is sold by the firm in the market.

Show that a firm's choice about the amount it wishes to destroy is independent of the amount chosen by the other firm. Show also that the amount destroyed by firm 2 is always positive, while firm 1 destroys a part of its output if and only if a = 100.

4. (a) Two commodities, X and Y, are produced with identical technology and are sold in competitive markets. One unit of labour can produce one unit of each of the two commodities. Labour is the only factor of production; and labour is perfectly mobile between the two sectors. The representative consumer has the utility function: $U = \sqrt{XY}$; and his income is Rs. 100/-. If 10 units of labour are available, find out the equilibrium wage in the competitive labour market.

(b) Consider an economy producing a single good by a production function

$$Y = \min\{K, L\}$$

where Y is the output of the final good. K and L are input use of capital and labour respectively. Suppose this economy is endowed with 100 units of capital and labour supply L_s is given by the function

$$L_s = 50\,w,$$

where w is the wage rate.

Assuming that all markets are competitive find the equilibrium wage and rental rate.

5. The following symbols are used: Y = output, N = employment, W = nominal wage, P = price level, P^e = expected price level.

The Lucas supply function is usually written as:

$$\log Y = \log Y^* + \lambda\ (\log P - \log P^e)$$

where Y^* is the natural level of output. Consider an economy in which labour supply depends positively on the expected real wage:

$$\frac{W}{P^e} = N^{\sigma},\ (\sigma > 0) \quad \text{(labour supply)}$$

Firms demand labour up to the point where its marginal product equals the given (actual) real wage $\left(\frac{W}{P}\right)$ and firm's production function is:

$$Y = N^{\alpha},\ (0 < \alpha < 1)$$

(a) Find the labour demand function.
(b) Equate labour demand with labour supply to eliminate W. You will get an expression involving P, P^e and N. Derive the Lucas supply function in the form given above and find the expressions for λ and Y^*.
(c) How is this type of model referred to in the literature? Explain

6. Consider an IS – LM model given by the following equations

$$C = 200 + .5\,Y_D$$
$$I = 150 - 1000\,r$$
$$T = 200$$
$$G = 250$$
$$\left(\frac{M}{P}\right)^d = 2Y - 4000\,i$$
$$\left(\frac{M}{P}\right)^s = 1600$$
$$i = r - \Pi^e$$

where C is consumption, Y_D is disposable income, I is investment, r is real rate of interest, i is nominal rate of interest, T is tax, G is government expenditure, $\left(\frac{M}{P}\right)^d$ and $\left(\frac{M}{P}\right)^s$ are real money demand and real money supply respectively and and Π^e is the expected rate of inflation. The current price level P remains always rigid.

(a) Assuming that Π^e = 0, i.e., the price level is expected to remain unchanged in future, determine the equilibrium levels of income and the rates of interest.
(b) Suppose there is a *temporary* increase in nominal money supply by 2%. Find the new equilibrium income and the rates of interest.
(c) Now assume that the 2% increase in nominal money supply is *permanent* leading to a 2% increase in the expected future price level. Work out the new equilibrium income and the rates of interest.

7. A firm is contemplating to hire a salesman who would be entrusted with the task of selling a washing machine. The hired salesman is efficient with probability 0.25 and inefficient with probability 0.75 and there is no way to tell, by looking at the salesman, if he is efficient or not. An efficient salesman can sell the washing machine with probability 0.8 and an inefficient salesman can sell the machine with probability 0.4. The firm makes

a profit of Rs. 1000 if the machine is sold and gets nothing if it is not sold. In either case, however, the salesman has to be paid a wage of Rs. 100.

(a) Calculate the expected profit of the firm.
(b) Suppose instead of a fixed payment, the firm pays a commission of t % on its profit to the salesman (i.e., if the good is sold the salesman gets Rs. $1000\times\frac{t}{100}$ and nothing if the good remains unsold). A salesman, irrespective of whether he is efficient or inefficient, has an alternative option of working for Rs. 80. A salesman knows whether he is efficient or not and cares only about the expected value of his income: find the value of t that will maximize the expected profit of the firm.

8. (a) On a tropical island there are 100 boat builders, numbered 1 through 100. Each builder can build up to 12 boats a year and each builder maximizes profit given the market price. Let y denote the number of boats built per year by a particular builder, and for each i, from 1 to 100, boat builder has a cost function $C_i(y) = 11 + i\,y$. Assume that in the cost function the fixed cost, 11, is a quasi-fixed cost, that is, it is only paid if the firm produces a positive level of output. If the price of a boat is 25, how many builders will choose to produce a positive amount of output and how many boats will be built per year in total?

(b) Consider the market for a particular good. There are two types of customers: those of type 1 are the low demand customers, each with a demand function of the form $p = 10 - q_1$, and those of type 2, who are the high demand customers, each with a demand function of the form $p = 2(10 - q_2)$. The firm producing the product is a monopolist in this market and has a cost function $C(q) = 4q^2$ where $q = q_1 + q_2$.

(i) Suppose the firm is unable to prevent the customers from selling the good to one another, so that the monopolist cannot charge different customers different prices. What prices per unit will the monopolist charge to maximize its total profit and what will be the equilibrium quantities to be supplied to the two groups in equilibrium?

(ii) Suppose the firm realizes that by asking for IDs it can identify the types of the customers (for instance, type 1's are students who can be identified using their student IDs). It can thus charge different per unit prices to the two groups, if it is optimal to do so. Find the profit maximizing prices to be charged to the two groups.

9. Consider the following box with 16 squares:

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

There are two players 1 and 2, and the game begins with player 1 selecting one of the boxes marked 1 to 16. Following such a selection, the selected box, as well as all boxes in the square of which the selected box constitutes the *leftmost and lowest* corner, will be deleted. For example, if he selects box 7, then all the boxes, 3, 4, 7 and 8 are deleted. Similarly, if he selects box 9, then all boxes 1 to 12 are deleted. Next it is player 2's turn to select a box from the remaining boxes. The same deletion rule applies in this case. It is then player 1's turn again, and so on. Whoever deletes the last box loses the game? What is a winning strategy for player 1 in this game?

10. (i) Mr. *A's* yearly budget for his car is Rs. 100,000, which he spends completely on petrol (P) and on all other expenses for his car (M). All other expenses for car (M) is measured in Rupees, so you can consider that price of M is Re. 1. When price of petrol is Rs. 40 per liter, Mr. A buys 1,000 liters per year.
The petrol price rises to Rs. 50 per liter, and to offset the harm to Mr. A, the government gives him a cash transfer of Rs. 10,000 per year.

(a) Write down Mr. A's yearly budget equation under the 'price rise plus transfer' situation.
(b) What will happen to his petrol consumption – increase, decrease, or remain the same?
(c) Will he be better or worse off after the price rise plus transfer than he was before?

[Do not refer to any utility function or indifference curves to answer (b) and (c)]

(iii) Mr. B earns Rs. 500 today and Rs. 500 tomorrow. He can save for future by investing today in bonds that return tomorrow the principal plus the interest. He can also borrow from his bank paying an interest. When the interest rates on both bank loans and bonds are 15% Mr. B chooses neither to save nor to borrow.

(a) Suppose the interest rate on bank loans goes up to 30% and the interest rate on bonds fall to 5%. Write down the equation of the new budget constraint and draw his budget line.
(b) Will he lend or borrow? By how much?

Test code: ME I/ME II, 2009

Syllabus for ME I, 2009

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Algebra: Binomial Theorem, AP, GP, HP, exponential, logarithmic series.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation and regression, Elementary probability theory, Probability distributions.

Sample Questions for ME I (Mathematics), 2009

1. An infinite geometric series has first term 1 and sum 4. Its common ratio is

   A $\frac{1}{2}$

   B $\frac{3}{4}$

   C 1

   D $\frac{1}{3}$

2. A continuous random variable X has a probability density function $f(x) = 3x^2$ with $0 \leq x \leq 1$. If $P(X \leq a) = P(x > a)$, then a is:

   A $\frac{1}{\sqrt{6}}$

   B $\left(\frac{1}{3}\right)^{\frac{1}{2}}$

   C $\frac{1}{2}$

   D $\left(\frac{1}{2}\right)^{\frac{1}{3}}$

3. If $f(x) = \sqrt{e^x + \sqrt{e^x + \sqrt{e^x + \ldots}}}$, then $f'(x)$ equals to

   A $\frac{f(x)-1}{2f(x)+1}$.

   B $\frac{f^2(x)-f(x)}{2f(x)-1}$.

   C $\frac{2f(x)+1}{f^2(x)+f(x)}$.

   D $\frac{f(x)}{2f(x)+1}$.

4. $\lim_{x \to 4} \frac{\sqrt{x+5}-3}{x-4}$ is

   A $\frac{1}{6}$

   B 0

   C $\frac{1}{4}$

   D not well defined

5. If $X = 2^{65}$ and $Y = 2^{64} + 2^{63} + \ldots + 2^1 + 2^0$, then

   A $Y = X + 2^{64}$.

B $X = Y$.

C $Y = X + 1$.

D $Y = X - 1$.

6. $\int_0^1 \frac{e^x}{e^x+1} dx =$

A $log(1 + e)$.

B $log 2$.

C $log \frac{1+e}{2}$.

D $2 log(1 + e)$.

7. There is a box with ten balls. Each ball has a number between 1 and 10 written on it. No two balls have the same number. Two balls are drawn (simultaneously) at random from the box. What is the probability of choosing two balls with odd numbers?

A $\frac{1}{9}$.

B $\frac{1}{2}$.

C $\frac{2}{9}$.

D $\frac{1}{3}$.

8. A box contains 100 balls. Some of them are white and the remaining are red. Let X and Y denote the number of white and red balls respectively. The correlation between X and Y is

A 0.

B 1.

C -1.

D some real number between $-\frac{1}{2}$ and $\frac{1}{2}$.

9. Let f, g and h be real valued functions defined as follows: $f(x) = x(1 - x)$, $g(x) = \frac{x}{2}$ and $h(x) = \min\{f(x), g(x)\}$ with $0 \leq x \leq 1$. Then h is

A continuous and differentiable

B is differentiable but not continuous

C is continuous but not differentiable

D is neither continuous nor differentiable

10. In how many ways can three persons, each throwing a single die once, make a score of 8?

A 5

B 15

C 21

D 30

11. If $f(x)$ is a real valued function such that

$$2f(x) + 3f(-x) = 55 - 7x,$$

for every $x \in \Re$, then $f(3)$ equals

A 40

B 32

C 26

D 10

12. Two persons, A and B, make an appointment to meet at the train station between 4 P.M. and 5 P.M.. They agree that each is to wait not more than 15 minutes for the other. Assuming that each is independently equally likely to arrive at any point during the hour, find the probability that they meet.

A $\frac{15}{16}$

B $\frac{7}{16}$

C $\frac{5}{24}$

D $\frac{22}{175}$

13. If x_1, x_2, x_3 are positive real numbers, then

$$\frac{x_1}{x_2} + \frac{x_2}{x_3} + \frac{x_3}{x_1}$$

is always

A ≤ 3

B $\leq 3^{\frac{1}{3}}$

C ≥ 3

D 3

14. $\lim_{n\to\infty} \frac{1^2+2^2+\ldots+n^2}{n^3}$ equals

A 0

B $\frac{1}{3}$

C $\frac{1}{6}$

D 1.

15. Suppose b is an odd integer and the following two polynomial equations have a common root.

$$x^2 - 7x + 12 = 0$$
$$x^2 - 8x + b = 0.$$

The root of $x^2 - 8x + b = 0$ that is not a root of $x^2 - 7x + 12 = 0$ is

A 2

B 3

C 4

D 5

16. Suppose $n \geq 9$ is an integer. Let $\mu = n^{\frac{1}{2}} + n^{\frac{1}{3}} + n^{\frac{1}{4}}$. Then, which of the following relationships between n and μ is correct?

A $n = \mu$.

B $n > \mu$.

C $n < \mu$.

D None of the above.

17. Which of the following functions $f : \Re \to \Re$ satisfies the relation $f(x+y) = f(x) + f(y)$?

A $f(z) = z^2$

B $f(z) = az$ for some real number a

C $f(z) = \log z$

D $f(z) = e^z$

18. For what value of a does the following equation have a unique solution?

$$\begin{vmatrix} x & a & 2 \\ 2 & x & 0 \\ 2 & 1 & 1 \end{vmatrix} = 0$$

A 0

B 1

C 2

D 4

19. Let

$$y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

where l, m, n, a, b, c are non-zero numbers. Then $\frac{dy}{dx}$ equals

A

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

B

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ 0 & 0 & 0 \\ a & b & c \end{vmatrix}$$

C

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

D

$$\begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l-a & m-b & n-c \\ 1 & 1 & 1 \end{vmatrix}$$

20. If $f(x) = |x-1| + |x-2| + |x-3|$, then $f(x)$ is differentiable at

A 0

B 1

C 2

D 3

21. If $(x-a)^2+(y-b)^2=c^2$, then $1+\left[\frac{dy}{dx}\right]^2$ is independent of

A a

B b

C c

D Both b and c.

22. A student is browsing in a second-hand bookshop and finds n books of interest. The shop has m copies of each of these n books. Assuming he never wants duplicate copies of any book, and that he selects at least one book, how many ways can he make a selection? For example, if there is one book of interest with two copies, then he can make a selection in 2 ways.

A $(m+1)^n-1$

B nm

C $2^{nm}-1$

D $\frac{nm!}{(m!)(nm-m)!}-1$

23. Determine all values of the constants A and B such that the following function is continuous for all values of x.

$$f(x)=\begin{cases} Ax-B & \text{if } x\le -1 \\ 2x^2+3Ax+B & \text{if } -1<x\le 1 \\ 4 & \text{if } x>1 \end{cases}$$

A $A=B=0$

B $A=\frac{3}{4}$, $B=-\frac{1}{4}$

C $A=\frac{1}{4}$, $B=\frac{3}{4}$

D $A=\frac{1}{2}$, $B=\frac{1}{2}$

24. The value of $\lim_{x\to\infty}(3^x+3^{2x})^{\frac{1}{x}}$ is

A 0

B 1

C e

D 9

25. A computer while calculating correlation coefficient between two random variables X and Y from 25 pairs of observations obtained the following results: $\sum X = 125$, $\sum X^2 = 650$, $\sum Y = 100$, $\sum Y^2 = 460$, $\sum XY = 508$. It was later discovered that at the time of inputing, the pair $(X = 8, Y = 12)$ had been wrongly input as $(X = 6, Y = 14)$ and the pair $(X = 6, Y = 8)$ had been wrongly input as $(X = 8, Y = 6)$. Calculate the value of the correlation coefficient with the correct data.

A $\frac{4}{5}$

B $\frac{2}{3}$

C 1

D $\frac{5}{6}$

26. The point on the curve $y = x^2 - 1$ which is nearest to the point $(2, -0.5)$ is

A $(1, 0)$

B $(2, 3)$

C $(0, -1)$

D None of the above

27. If a probability density function of a random variable X is given by $f(x) = kx(2 - x), 0 \leq x \leq 2$, then mean of X is

A $\frac{1}{2}$

B 1

C $\frac{1}{5}$

D $\frac{3}{4}$

28. Suppose X is the set of all integers greater than or equal to 8. Let $f : X \to \Re$. and $f(x + y) = f(xy)$ for all $x, y \geq 4$. If $f(8) = 9$, then $f(9) =$

A 8.

B 9.

C 64.

D 81.

29. Let $f : \Re \to \Re$ be defined by $f(x) = (x-1)(x-2)(x-3)$. Which of the following is true about f?

A It decreases on the interval $[2 - 3^{-\frac{1}{2}}, 2 + 3^{-\frac{1}{2}}]$

B It increases on the interval $[2 - 3^{-\frac{1}{2}}, 2 + 3^{-\frac{1}{2}}]$

C It decreases on the interval $(-\infty, 2 - 3^{-\frac{1}{2}}]$

D It decreases on the interval $[2, 3]$

30. A box with no top is to be made from a rectangular sheet of cardboard measuring 8 metres by 5 metres by cutting squares of side x metres out of each corner and folding up the sides. The largest possible volume in cubic metres of such a box is

A 15

B 12

C 20

D 18

Syllabus for ME II (Economics), 2009

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2009

1. Consider the following model of the economy:

$$\begin{aligned} C &= c_0 + c_1 Y_D \\ T &= t_0 + t_1 Y \\ Y_D &= Y - T. \end{aligned}$$

C denotes consumption, $c_0 > 0$ denotes autonomous consumption, $0 < c_1 < 1$ is the marginal propensity to consume, Y, denotes income, T denotes taxes, Y_D denotes disposable income and $t_0 > 0$, $t_1 > 0$. Assume a closed economy where government spending G, and investment I, are exogenously given by $\bar{G}$ and $\bar{I}$ respectively.

- (i) Interpret t_1 in words. Is it greater or less than 1? Explain your answer.
- (ii) Solve for equilibrium output, Y^*.
- (iii) What is the multiplier ? Does the economy respond more to changes in autonomous spending (such as changes in $c_0, \bar{G}$, and, $\bar{I}$) when t_1 is zero or when t_1 is positive ? Explain.

[5]+[5]+[10]

2. Consider an agent who values consumption in periods 0 and 1 according to the utility function

$$u(c_0, c_1) = \log c_0 + \delta \log c_1$$

where $0 < \delta < 1$. Suppose that the agent has wealth ω in period 0 of which she can save any portion in order to consume in period 1. If she saves Re. 1, she is paid interest r so that her budget constraint is

$$c_0 + \frac{c_1}{1+r} = \omega$$

- (i) Derive the agent's demand for c_0 and c_1 as a function of r and ω.
- (ii) What happens to c_0 and c_1 as r increases? Interpret.
- (iii) For what relationship between ω and r will she consume the same amount in both periods?

  [8]+[6]+[6]

3. Consider a firm with production function $F(x_1, x_2) = \min(2x_1, x_1 + x_2)$ where x_1 and x_2 are amounts of factors 1 and 2.

- (i) Draw an isoquant for output level 10.

- (ii) Show that the production function exhibits constant returns to scale.
- (iii) Suppose that the firm faces input prices $w_1 = w_2 = 1$. What is the firms' cost function?

  [8]+[6]+[6]

4. Consider an exchange economy consisting of two individuals 1 and 2, and two goods X and Y. The utility function of individual i, $U_i = X_i + Y_i$. Individual 1 has 3 units of X and 7 units of Y to begin with. Similarly, individual 2 has 7 units of X and 3 units of Y to begin with.

- (i) What is the set of Pareto optimal outcomes in this economy? Justify your answer.
- (ii) What is the set of perfectly competitive (Walrasian) outcomes? You may use diagrams for parts (i) and (ii).
- (iii) Are the perfectly competitive outcomes Pareto optimal? Does this result hold generally in all exchange economies?

  [8]+[8]+[4]

5. A monopoly sells its product in two separate markets. The inverse demand function in market 1 is given by $q_1 = 10 - p_1$, and the inverse demand function in market 2 is given by $q_2 = a - p_2$, where $10 < a \leq 20$. The monopolist's cost function is $C(q) = 5q$, where q is aggregate output.

- (i) Suppose the monopolist must set the same price in both markets. What is its optimal price? What is the reason behind the restriction that $a \leq 20$?
- (ii) Suppose the monopolist can charge different prices in the two markets. Compute the prices it will set in the two markets.
- (iii) Under what conditions does the monopolist benefit from the ability to charge different prices?
- (iv) Compute consumers' surplus in cases (i) and (ii). Who benefits from differential pricing and who does not relative to the case where the same price is charged in both markets?

[5]+[5]+[5]+[5]

6. Consider an industry with 3 firms, each having marginal cost equal to 0. The inverse demand curve facing this industry is $p = 120 - q$, where q is aggregate output.

- (i) If each firm behaves as in the Cournot model, what is firm 1's optimal output choice as a function of its beliefs about other firms' output choices?
- (ii) What output do the firms produce in equilibrium?
- (iii) Firms 2 and 3 decide to merge and form a single firm with marginal cost still equal to 0. What output do the two firms produce in equilibrium? Is firm 1 better off as a result? Are firms 2 and 3 better off post-merger? Would it be better for all the firms to form a cartel instead? Explain in each case.

[3]+[5]+[12]

7. Suppose the economy's production function is given by

$$Y_t = 0.5\sqrt{K_t}\sqrt{N_t} \tag{1}$$

Y_t denotes output, K_t denotes the aggregate capital stock in the economy, and N denotes the number of workers (which is fixed). The evolution of the capital stock is given by,

$$K_{t+1} = sY_t + (1-\delta)K_t \tag{2}$$

where the savings rate of the economy is denoted by, s, and the depreciation rate is given by, δ.

- (i) Using equation (2), show that the change in the capital stock per worker, $\frac{K_{t+1}-K_t}{N}$, is equal to savings per worker minus depreciation per worker.
- (ii) Derive the economy's steady state levels of $\frac{K}{N}$ and $\frac{Y}{N}$ in terms of the savings rate and the depreciation rate.
- (iii) Derive the equation for the steady state level of consumption per worker in terms of the savings rate and the depreciation rate.
- (iv) Is there a savings rate that is optimal, i.e., maximizes steady state consumption per worker ? If so, derive an expression for the optimal savings rate. Using words and graphs, discuss your answer.

[2]+[6]+[6]+[6]

8. Suppose there are 10 individuals in a society, 5 of whom are of high ability, and 5 of low ability. Individuals know their own abilities. Suppose that each individual lives for two periods and is deciding whether or not to go to college in period 1. When individuals make decisions in period 1, they choose that option which gives the highest lifetime payoff, i.e., the sum of earnings and expenses in both periods.

Education can only be acquired in period 1. In the absence of schooling, high and low ability individuals can earn y_H and y_L respectively in each period.

With education, period 2 earning increases to $(1 + a)y_H$ for high ability types and $(1 + a)y_L$ for low ability types. Earnings would equal 0 in period 1 if an individual decided to go to college in that period. Tuition fee for any individual is equal to T. Assume y_H and y_L are both positive, as is T.

- (i) Find the condition that determines whether each type of person will go to college in period 1. What is the minimum that a can be if it is to be feasible for any type of individual to acquire education?
- (ii) Suppose $y_H = 50$, $y_L = 40$, $a = 3$. For what values of T will a high ability person go to college? And a low ability person? Which type is more likely to acquire education?
- (iii) Now assume the government chooses to subsidise education by setting tuition equal to 60. What happens to educational attainment?
- (iv) Suppose now to pay for the education subsidy, the government decides to impose a $x\%$ tax on earnings in any period greater than 50. So if an individual earns 80 in a period, he would pay a tax in that period equal to $x\%$ of 30.The government wants all individuals to acquire education, and also wants to cover the cost of the education subsidy in period 1 through tax revenues collected in both periods. What value of x should the government set?

[5]+[5]+[2]+[8]

9. Consider the goods market with exogenous (constant) investment $\bar{I}$, exogenous government spending, $\bar{G}$ and constant taxes, T. The consumption equation is given by,

$$C = c_0 + c_1(Y - T),$$

where C denotes consumption, c_0 denotes autonomous consumption, and c_1 the marginal propensity to consume.

- (i) Solve for equilibrium output. What is the value of the multiplier ?
- (ii) Now let investment depend on Y and the interest rate, i

  $$I = b_0 + b_1 Y - b_2 i,$$

  where b_0 and b_1 are parameters. Solve for equilibrium output. At a given interest rate, is the effect of an increase in autonomous spending bigger than it was in part (i)? In answering this, assume that $c_1 + b_1 < 1$.
- (iii) Now, introduce the financial market equilibrium condition

  $$\frac{M}{P} = d_1 Y - d_2 i,$$

  where $\frac{M}{P}$ denotes the real money supply. Derive the multiplier. Assume that investment is given by the equation in part (ii).

- (iv) Is the multiplier you obtained in part (iii) smaller or larger than the multiplier you obtained in part (i). Explain how your answer depends on the behavioral equations for consumption, investment, and money demand.

[5]+[5]+[5]+[5]

10 (i) A college is trying to fill one remaining seat in its Masters programme. It judges the merit of any applicant by giving him an entrance test. It is known that there are two interested applicants who will apply sequentially. If the college admits the first applicant, it cannot admit the second. If it rejects the first applicant, it must admit the second. It is not possible to delay a decision on the first applicant till the second applicant is tested. At the time of admitting or rejecting the first applicant, the college thinks the second applicant's mark will be a continuous random variable drawn from the uniform distribution between 0 and 100. (Recall that a random variable x is uniformly distibuted on $[a, b]$ if the density function of x is given by $f(x) = \frac{1}{b-a}$ for $x \in [a, b]$). If the college wants to maximize the expected mark of its admitted student, what is the lowest mark for which it should admit the first applicant?

(ii) Now suppose there are three applicants who apply sequentially. Before an applicant is tested, it is known that his likely mark is an independent continuous random variable drawn from the uniform distribution between 0 and 100. What is the lowest mark for which the college should admit the first student? What is the lowest mark for which the college should admit the second student in case the first is rejected?
[8]+[12]

Test code: ME I/ME II, 2010

Syllabus for ME I, 2010

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Algebra: Binomial Theorem, AP, GP, HP, exponential, logarithmic series.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation and regression, Elementary probability theory, Probability distributions.

Sample Questions for ME I (Mathematics), 2010

1. The value of $100\left[\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\ldots+\frac{1}{99.100}\right]$
   (a) is 99,
   (b) is 100,
   (c) is 101,
   (d) is $\frac{(100)^2}{99}$.

2. The function $f(x) = x(\sqrt{x}+\sqrt{x+9})$ is
   (a) continuously differentiable at $x = 0$,
   (b) continuous but not differentiable at $x = 0$,
   (c) differentiable but the derivative is not continuous at $x = 0$,
   (d) not differentiable at $x = 0$.

3. Consider a GP series whose first term is 1 and the common ratio is a positive integer $r(> 1)$. Consider an AP series whose first term is 1 and whose $(r+2)^{\text{th}}$ term coincides with the third term of the GP series. Then the common difference of the AP series is
   (a) $r-1$,
   (b) r,
   (c) $r+1$,
   (d) $r+2$.

4. The first three terms of the binomial expansion $(1+x)^n$ are $1, -9, \frac{297}{8}$ respectively. What is the value of n?
   (a) 5
   (b) 8
   (c) 10
   (d) 12

5. Given $log_p x = \alpha$ and $log_q x = \beta$, the value of $log_{\frac{p}{q}} x$ equals
   (a) $\frac{\alpha\beta}{\beta-\alpha}$,
   (b) $\frac{\beta-\alpha}{\alpha\beta}$,
   (c) $\frac{\alpha-\beta}{\alpha\beta}$,
   (d) $\frac{\alpha\beta}{\alpha-\beta}$.

6. Let $P = \{1, 2, 3, 4, 5\}$ and $Q = \{1, 2\}$. The total number of subsets X of P such that $X \cap Q = \{2\}$ is

   (a) 6,
   (b) 7,
   (c) 8,
   (d) 9.

7. An unbiased coin is tossed until a head appears. The expected number of tosses required is

   (a) 1,
   (b) 2,
   (c) 4,
   (d) ∞.

8. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{c}{x^2} & \text{if } x \geq c \\ 0 & x < c. \end{cases}$$

   Then the expectation of X is

   (a) 0,
   (b) ∞,
   (c) $\frac{1}{c}$,
   (d) $\frac{1}{c^2}$.

9. The number of real solutions of the equation $x^2 - 5|x| + 4 = 0$ is

   (a) two,
   (b) three,
   (c) four.
   (d) None of these.

10. Range of the function $f(x) = \frac{x^2}{1+x^2}$ is

   (a) $[0, 1)$,
   (b) $(0, 1)$,
   (c) $[0, 1]$.
   (d) $(0, 1]$.

11. If a, b, c are in AP, then the value of the determinant

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

is

(a) $b^2 - 4ac$,

(b) $ab + bc + ca$,

(c) $2b - a - c$,

(d) $3b + a + c$.

12. If $a < b < c < d$, then the equation $(x-a)(x-b) + 2(x-c)(x-d) = 0$ has

(a) both the roots in the interval $[a, b]$,

(b) both the roots in the interval $[c, d]$,

(c) one root in the interval (a, b) and the other root in the interval (c, d),

(d) one root in the interval $[a, b]$ and the other root in the interval $[c, d]$.

13. Let f and g be two differentiable functions on $(0,1)$ such that $f(0) = 2$, $f(1) = 6$, $g(0) = 0$ and $g(1) = 2$. Then there exists $\theta \in (0,1)$ such that $f'(\theta)$ equals

(a) $\frac{1}{2}g'(\theta)$,

(b) $2g'(\theta)$,

(c) $6g'(\theta)$,

(d) $\frac{1}{6}g'(\theta)$.

14. The minimum value of $log_x a + log_a x$, for $1 < a < x$, is

(a) less than 1,

(b) greater than 2,

(c) greater than 1 but less than 2.

(d) None of these.

15. The value of $\int\limits_4^9 \frac{1}{2x(1+\sqrt{x})} dx$ equals

(a) $log_e 3 - log_e 2$,

(b) $2log_e 2 - log_e 3$,

(c) $2log_e 3 - 3log_e 2$,

(d) $3log_e 3 - 2log_e 2$.

16. The inverse of the function $f(x) = \frac{1}{1+x}$, $x > 0$, is

(a) $(1 + x)$,

(b) $\frac{1+x}{x}$,

(c) $\frac{1-x}{x}$,

(d) $\frac{x}{1+x}$.

17. Let X_i, $i = 1, 2, \ldots, n$ be identically distributed with variance σ^2. Let $cov(X_i, X_j) = \rho$ for all $i \neq j$. Define $\bar{X}_n = \frac{1}{n} \sum X_i$ and let $a_n = Var(\bar{X}_n)$. Then $\lim_{n\to\infty} a_n$ equals

(a) 0,

(b) ρ,

(c) $\sigma^2 + \rho$,

(d) $\sigma^2 + \rho^2$.

18. Let X be a Normally distributed random variable with mean 0 and variance 1. Let $\Phi(.)$ be the cumulative distribution function of the variable X. Then the expectation of $\Phi(X)$ is

(a) $-\frac{1}{2}$,

(b) 0,

(c) $\frac{1}{2}$,

(d) 1.

19. Consider any finite integer $r \geq 2$. Then $\lim\limits_{x\to 0} \left[\dfrac{log_e \left(\sum\limits_{k=0}^{r} x^k \right)}{\left(\sum\limits_{k=1}^{\infty} \frac{x^k}{k!} \right)} \right]$ equals

(a) 0,

(b) 1,

(c) e,

(d) $log_e 2$.

20. Consider 5 boxes, each containing 6 balls labelled $1, 2, 3, 4, 5, 6$. Suppose one ball is drawn from each of the boxes. Denote by b_i, the label of the ball drawn from the i-th box, $i = 1, 2, 3, 4, 5$. Then the number of ways in which the balls can be chosen such that $b_1 < b_2 < b_3 < b_4 < b_5$ is

(a) 1,

(b) 2,

(c) 5,

(d) 6.

21. The sum $\sum_{r=0}^{m} \binom{n+r}{r}$ equals

(a) $\binom{n+m+1}{n+m}$,

(b) $(n+m+1)\binom{n+m}{n+1}$,

(c) $\binom{n+m+1}{n}$,

(d) $\binom{n+m+1}{n+1}$.

22. Consider the following 2-variable linear regression where the error ϵ_i's are independently and identically distributed with mean 0 and variance 1;

$$y_i = \alpha + \beta(x_i - \bar{x}) + \epsilon_i, \qquad i = 1, 2, \ldots, n.$$

Let $\hat{\alpha}$ and $\hat{\beta}$ be ordinary least squares estimates of α and β respectively. Then the correlation coefficient between $\hat{\alpha}$ and $\hat{\beta}$ is

(a) 1,

(b) 0,

(c) -1,

(d) $\frac{1}{2}$.

23. Let f be a real valued continuous function on $[0, 3]$. Suppose that $f(x)$ takes only rational values and $f(1) = 1$. Then $f(2)$ equals

(a) 2,

(b) 4,

(c) 8.

(d) None of these.

24. Consider the function $f(x_1, x_2) = \int_0^{\sqrt{x_1^2+x_2^2}} e^{-(w^2/(x_1^2+x_2^2))} dw$ with the property that $f(0, 0) = 0$. Then the function $f(x_1, x_2)$ is

(a) homogeneous of degree -1,

(b) homogeneous of degree $\frac{1}{2}$,

(c) homogeneous of degree 1.

(d) None of these.

25. If $f(1) = 0$, $f'(x) > f(x)$ for all $x > 1$, then $f(x)$ is

(a) positive valued for all $x > 1$,

(b) negative valued for all $x > 1$,

(c) positive valued on $(1, 2)$ but negative valued on $[2, \infty)$.

(d) None of these.

26. Consider the constrained optimization problem

$$\max_{x\geq 0, y\geq 0} (ax+by) \text{ subject to } (cx+dy) \leq 100$$

where a, b, c, d are positive real numbers such that $\frac{d}{b} > \frac{(c+d)}{(a+b)}$. The unique solution (x^*, y^*) to this constrained optimization problem is

(a) $(x^* = \frac{100}{a}, y^* = 0)$,
(b) $(x^* = \frac{100}{c}, y^* = 0)$,
(c) $(x^* = 0, y^* = \frac{100}{b})$,
(d) $(x^* = 0, y^* = \frac{100}{d})$.

27. For any real number x, let $[x]$ be the largest integer not exceeding x. The domain of definition of the function $f(x) = \left(\sqrt{|[|x|-2]|-3}\right)^{-1}$ is

(a) $[-6, 6]$,
(b) $(-\infty, -6) \cup (+6, \infty)$,
(c) $(-\infty, -6] \cup [+6, \infty)$,
(d) None of these.

28. Let $f : \Re \to \Re$ and $g : \Re \to \Re$ be defined as

$$f(x) = \begin{cases} -1 & \text{if } x < -\frac{1}{2} \\ -\frac{1}{2} & \text{if } -\frac{1}{2} \leq x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0. \end{cases}$$

and $g(x) = 1 + x - [x]$, where $[x]$ is the largest integer not exceeding x. Then $f(g(x))$ equals

(a) -1,
(b) $-\frac{1}{2}$,
(c) 0,
(d) 1.

29. If f is a real valued function and $a_1 f(x) + a_2 f(-x) = b_1 - b_2 x$ for all x with $a_1 \neq a_2$ and $b_2 \neq 0$. Then $f(\frac{b_1}{b_2})$ equals

(a) 0,
(b) $-\left(\frac{2a_2b_1}{a_1^2 - a_2^2}\right)$,
(c) $\frac{2a_2b_1}{a_1^2 - a_2^2}$.
(d) More information is required to find the exact value of $f(\frac{b_1}{b_2})$.

30. For all $x, y \in (0, \infty)$, a function $f : (0, \infty) \to \Re$ satisfies the inequality

$$|f(x) - f(y)| \leq |x - y|^3.$$

Then f is

(a) an increasing function,

(b) a decreasing function,

(c) a constant function.

(d) None of these.

Syllabus for ME II (Economics), 2010

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2010

1.

(a) Imagine a closed economy in which tax is imposed only on income. The government spending (G) is required (by a balanced budget amendment to the relevant law) to be equal to the tax revenue; thus $G = tY$, where t is the tax rate and Y is income. Consumption expenditure (C) is proportional to disposable income and investment (I) is exogenously given.

(i) Explain why government spending is endogenous in this model.

(ii) Is the multiplier in this model larger or smaller than in the case in which government spending is exogenous?

(iii) When t increases, does Y decrease, increase or stay the same? Give an answer with intuitive explanation.

(b) Consider the following macroeconomic model with notation having usual meanings: $C = 100 + 1.3Y$ (Consumption function), $I = \frac{500}{r}$ (Investment function), $M^D = 150Y + 100 - 1500r$ (Demand for money function) and $M^S = 2100$ (Supply of money). Do you think that there exists an equilibrium? Justify your answer using the IS-LM model.

[3+8+4]+[5]

2. Consider a market with two firms. Let the cost function of each firm be $C(q) = mq$ where $q \geq 0$. Let the inverse demand functions of firms 1 and 2 be $P_1(q_1, q_2) = a - q_1 - sq_2$ and $P_2(q_1, q_2) = a - q_2 - sq_1$, respectively. Assume that $0 < s < 1$ and $a > m > 0$.

(a) Find the Cournot equilibrium quantities of the two firms.

(b) Using the inverse demand functions $P_1(q_1, q_2)$ and $P_2(q_1, q_2)$, derive direct demand functions $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$ of firms 1 and 2.

(c) Using the direct demand functions $D_1(p_1, p_2)$ and $D_2(p_1, p_2)$, find the Bertrand equilibrium prices.

[8]+[5]+[7]

3.

(a) A monopolist can sell his output in two geographically separated markets A and B. The total cost function is $TC = 5 + 3(Q_A + Q_B)$ where Q_A and Q_B are quantities sold in markets A and B respectively. The demand functions for the two markets are, respectively, $P_A = 15 - Q_A$ and $P_B = 25 - 2Q_B$. Calculate the firm's price, output, profit and the deadweight loss to the society if it can get involved in price discrimination.

(b) Suppose that you have the following information. Each month an airline sells 1500 business-class tickets at Rs. 200 per ticket and 6000 economy-class tickets at Rs. 80 per ticket. The airline treats business class and economy class as two separate markets. The airline knows the demand curves for the two markets and maximizes profit. It is also known that the demand curve of each of the two markets is linear and marginal cost associated with each ticket is Rs. 50.

(i) Use the above information to construct the demand curves for economy class and business class tickets.

(ii) What would be the equilibrium quantities and prices if the airline could not get involved in price discrimination?

[12]+[4+4]

4. Consider an economy producing two goods 1 and 2 using the following production functions: $X_1 = L_1^{\frac{1}{2}} K^{\frac{1}{2}}$ and $X_2 = L_2^{\frac{1}{2}} T^{\frac{1}{2}}$, where X_1 and X_2 are the outputs of good 1 and 2, respectively, K is capital used in production of good 1, T is land used in production of good 2 and L_1 and L_2 are amounts of labour used in production of good 1 and 2, respectively. Full employment of all factors is assumed implying the following: $K = \bar{K}, T = \bar{T}, L_1 + L_2 = \bar{L}$ where $\bar{K}$, $\bar{T}$ and $\bar{L}$ are total amounts of capital, land and labour available to the economy. Labour is assumed to be perfectly mobile between sectors 1 and 2. The underlying preference pattern of the economy generates the relative demand function, $\frac{D_1}{D_2} = \gamma \left(\frac{p_1}{p_2}\right)^{-2}$, where D_1 and D_2 are the demands and p_1 and p_2 prices of good 1 and 2 respectively. All markets (both commodities and factors) are competitive.

(a) Derive the relationship between $\frac{X_1}{X_2}$ and $\frac{p_1}{p_2}$.

(b) Suppose that γ goes up. What can you say about the new equilibrium relative price?

[15]+[5]

5. Consider the IS-LM representation of an economy with the following features:

(i) The economy is engaged in export and import of goods and services, but not in capital transactions with foreign countries.
(ii) Nominal exchange rate, that is, domestic currency per unit of foreign currency, e, is flexible.
(iii) Foreign price level (P^*) and domestic price level (P) are given exogenously.
(iv) There is no capital mobility and e has to be adjusted to balance trade in equilibrium. The trade balance (TB) equation (with an autonomous part $\bar{T} > 0$) is given by $TB = \bar{T} + \frac{\beta P^*}{P} - mY$, where Y is GDP and β and m are positive parameters, m being the marginal propensity to import.

(a) Taking into account trade balance equilibrium and commodity market equilibrium, derive the relationship between Y and the interest rate (r). Is it the same as in the IS curve for the closed economy? Explain. Draw also the LM curve on the (Y, r) plane.

(b) Suppose that the government spending is increased. Determine graphically the new equilibrium value of Y. How does the equilibrium value of e change?

(c) Suppose that P^* is increased. How does it affect the equilibrium values of Y and e?

[9]+[6]+[5]

6.

(a) A firm can produce its product with two alternative technologies given by $Y = \min\{\frac{K}{3}, \frac{L}{2}\}$ and $Y = \min\{\frac{K}{2}, \frac{L}{3}\}$. The factor markets are competitive and the marginal cost of production is $Rs.20$ with each of these two technologies. Find the equation of the expansion path of the firm if it uses a third production technology given by $Y = K^{\frac{2}{3}} L^{\frac{1}{3}}$.

(b) A utility maximizing consumer with a given money income consumes two commodities X and Y. He is a price taker in the market for X. For Y there are two alternatives: (A) He purchases Y from the market being a price taker, (B) The government supplies a fixed quantity of it through ration shops free of cost. Is the consumer necessarily better off in case (B)? Explain your answer with respect to the following cases:

   (i) Indifference curves are strictly convex to the origin.
   (ii) X and Y are perfect substitutes.
   (iii) X and Y are perfect complements.

[14]+[2+2+2]

7. Indicate, with adequate explanations, whether each of the following statements is TRUE or FALSE.

(a) If an increase in the price of a good leads a consumer to buy more of it, then an increase in his income will lead him to buy less of the good. By the same argument, if an increase in the price of the good leads him to buy less of it, then an increase in his income will lead him to buy more of the good.

(b) Suppose that a farmer, who receives all his income from the sale of his crop at a price beyond his control, consumes more of the crop as a result of the price increase. Then the crop is a normal good.

(c) If the non-wage income of a person increases then he chooses to work less at a given wage rate. Then he will choose to work more as his wage rate increases.

(d) The amount of stipends which Indian Statistical Institute pays to its students is a part of GDP.

[8]+[4]+[4]+[4]

8. Consider a Solow model with the production function $Y = K^{\frac{1}{2}}L^{\frac{1}{2}}$, where Y, K and L are levels of output, capital and labour, respectively. Suppose, 20% of income is saved and invested. Assume that the rate of growth of labour force, that is, $\left(\frac{\frac{dL}{dt}}{L}\right) = 0.05$.

(a) Find the capital-labour ratio, rate of growth of output, rate of growth of savings and the wage rate, in the steady state growth equilibrium.

(b) Suppose that the proportion of income saved goes up from 20% to 40%. What will be the new steady state growth rate of output?

(c) Is the rate of growth of output in the new steady state equilibrium different from that obtained just before attaining the new steady state (after deviating from the old steady state)? Explain.

[8]+[4]+[8]

9.

(a) Consider the utility function $U(x_1, x_2) = (x_1 - s_1)^{0.5}(x_2 - s_2)^{0.5}$, where $s_1 > 0$ and $s_2 > 0$ represent subsistence consumption and $x_1 \geq s_1$ and $x_2 \geq s_2$. Using the standard budget constraint, derive the budget share functions and demand functions of the utility maximizing consumer. Are they linear in prices? Justify your answer.

(b) Suppose that a consumer maximizes $U(x_1, x_2)$ subject to the budget constraint $p(x_1 + x_2) \leq M$ where $x_1 \geq 0$, $x_2 \geq 0$, $M > 0$ and $p > 0$. Moreover, assume that the utility function is symmetric, that is $U(x_1, x_2) = U(x_2, x_1)$ for all $x_1 \geq 0$ and $x_2 \geq 0$. If the solution (x_1^*, x_2^*) to the consumer's constrained optimization problem exists and is unique, then show that $x_1^* = x_2^*$.

[10]+[10]

10.

(a) Consider an economy with two persons (A and B) and two goods (1 and 2). Utility functions of the two persons are given by $U_A(x_{A1}, x_{A2}) = x_{A1}^{\alpha} + x_{A2}^{\alpha}$ with $0 < \alpha < 1$; and $U_B(x_{B1}, x_{B2}) = x_{B1} + x_{B2}$. Derive the equation of the contract curve and mention its properties.

(b) (i) A firm can produce a product at a constant average (marginal) cost of Rs. 4. The demand for the good is given by $x = 100 - 10p$. Assume that the firm owner requires a profit of Rs. 80. Determine the level of output and the price that yields maximum revenue if this profit constraint is to be fulfilled.

(ii) What will be the effects on price and output if the targeted profit is increased to Rs. 100?

(iii) Also find out the effects of the increase in marginal cost from Rs. 4 to Rs. 8 on price and output.

[8]+[6+3+3]

SYLLABUS & SAMPLE QUESTIONS FOR MS (QE)

2011

Syllabus for ME I, 2011

Matrix Algebra: Matrices and Vectors, Matrix Operations.

Permutation and Combination.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables, Unconstrained optimization, Definite and Indefinite Integrals: integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Algebra: Binomial Theorem, AP, GP, HP, exponential, logarithmic series.

Theory of Polynomial Equations (up to third degree).

Elementary Statistics: Measures of central tendency; dispersion, correlation and regression, Elementary probability theory, Probability distributions.

Sample Questions for ME I (Mathematics), 2011

1. The expression $\sqrt{13+3\sqrt{23/3}}+\sqrt{13-3\sqrt{23/3}}$ is

   (a) A natural number,
   (b) A rational number but not a natural number,
   (c) An irrational number not exceeding 6,
   (d) An irrational number exceeding 6.

2. The domain of definition of the function $f(x)=\dfrac{\sqrt{(x+3)}}{(x^2+5x+4)}$ is

   (a) $(-\infty,\infty)\setminus\{-1,-4\}$,
   (b) $(-0,\infty)\setminus\{-1,-4\}$
   (c) $(-1,\infty)\setminus\{-4\}$
   (d) None of these.

3. The value of

$$\log_4 2 - \log_8 2 + \log_{16} 2 - \ldots\ldots$$

(a) $\log_e 2$, (b) $1 - \log_e 2$,
(c) $\log_e 2 - 1$, (d) None of these.

4. The function $\max\{1, x, x^2\}$, where x is any real number, has

(a) Discontinuity at one point only,
(b) Discontinuity at two points only,
(c) Discontinuity at three points only,
(d) No point of discontinuity.

5. If $x, y, z > 0$ are in HP, then $\dfrac{x-y}{y-z}$ equals

(a) $\dfrac{x}{y}$, (b) $\dfrac{y}{z}$, (c) $\dfrac{x}{z}$, (d) None of these.

6. The function $f(x) = \dfrac{x}{1+|x|}$, where x is any real number is,

(a) Everywhere differentiable but the derivative has a point of discontinuity.
(b) Everywhere differentiable except at 0.
(c) Everywhere continuously differentiable.
(d) Everywhere differentiable but the derivative has 2 points of discontinuity.

7. Let the function $f : R_{++} \to R_{++}$ be such that $f(1) = 3$ and $f'(1) = 9$, where R_{++} is the positive part of the real line. Then $\lim_{x \to 0} \left(\dfrac{f(1+x)}{f(1)} \right)^{1/x}$ equals

(a) 3, (b) e^2, (c) 2, (d) e^3.

8. Let $f, g : [0, \infty) \to [0, \infty)$ be decreasing and increasing respectively. Define $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(1)$ is

(a) Nonpositive for $x \geq 1$, positive otherwise, (b) Always negative,
(c) Always positive, (d) Positive for $x \geq 1$, nonpositive otherwise.

9. A committee consisting of 3 men and 2 women is to be formed out of 6 men and 4 women. In how many ways this can be done if Mr. X and Mrs. Y are not to be included together?

   (a) 120, (b) 140, (c) 90, (d) 60.

10. The number of continuous functions f satisfying $x\, f(y)+y\, f(x)=(x+y)\, f(x) \quad f(y)$, where x and y are any real numbers, is

   (a) 1, (b) 2, (c), 3,
   (d) None of these.

11. If the positive numbers $x_1, \ldots, x_n$ are in AP, then

$$\frac{1}{\sqrt{x_1}+\sqrt{x_2}}+\frac{1}{\sqrt{x_2}+\sqrt{x_3}}+........+\frac{1}{\sqrt{x_{n-1}}+\sqrt{x_n}}$$ equals

   (a) $\dfrac{n}{\sqrt{x_1}+\sqrt{x_n}}$, (b) $\dfrac{1}{\sqrt{x_1}+\sqrt{x_n}}$,
   (b) $\dfrac{2n}{\sqrt{x_1}+\sqrt{x_n}}$, (d) None of these.

12. If x, y, z are any real numbers, then which of the following is always true?

   (a) $\max\{x,\ y\} < \max\{x,\ y,\ z\}$,
   (b) $\max\{x,\ y\} > \max\{x,\ y,\ z\}$,

   (c) $\max\{x,\ y\} = \dfrac{x+y+|x-y|}{2}$
   (d) None of these.

13. If $x_1, x_2, x_3, x_4 > 0$ and $\sum_{i=1}^{4} x_i = 2$, then $P = (x_1 + x_2)\,(x_3 + x_4)$ is

(a) Bounded between zero and one,
(b) Bounded between one and two,
(c) Bounded between two and three,
(d) Bounded between three and four.

14. Everybody in a room shakes hand with everybody else. Total number of handshakes is 91. Then the number of persons in the room is

(a) 11, (b) 12, (c) 13,
(d) 14.

15. The number of ways in which 6 pencils can be distributed between two boys such that each boy gets at least one pencil is

(a) 30, (b) 60, (c) 62, (d) 64.

16. Number of continuous functions characterized by the equation $x\,f(x) + 2\,f(-x) = -1$, where x is any real number, is

(a) 1, (b) 2, (c) 3, (d) None of these.

17. The value of the function $f(x) = x + \int_0^1 (xy^2 + x^2 y)\, f(y) dy$ is

$px + qx^2$, where

(a) p = 80, q = 180,
(b) p = 40, q = 140
(c) p = 50, q = 150,
(d) None of these.

18. If x and y are real numbers such that $x^2 + y^2 = 1$, then the maximum value of $|x| + |y|$ is

(a) $\frac{1}{2}$, (b) $\sqrt{2}$, (c) $\frac{1}{\sqrt{2}}$, (d) 2.

19. The number of onto functions from $A=\{p,q,r,s\}$ to $B=\{p,r\}$ is

(a) 16, (b) 2, (c) 8, (d) 14.

20. If the coefficients of $(2r+5)$ th and $(r-6)$ th terms in the expansion of $(1+x)^{39}$ are equal, then ${}^{r}C_{12}$ equals

(a) 45, (b) 91, (c) 63, (d) None of these.

21. If $X=\begin{bmatrix} C & 2 \\ 1 & C \end{bmatrix}$ and $|X^7|=128$, then the value of C is

(a) ± 5, (b) ± 1, (c) ± 2, (d) None of these.

22. Let $f(x)=Ax^2+Bx+C$, where A, B, C re real numbers. If $f(x)$ is an integer whenever x is an integer, then

(a) 2 A and A + B are integers, but C is not an integer.
(b) A + B and C are integers, but 2 A is not an integer.
(c) 2 A, A + B and C are all integers.
(d) None of these.

23. Four persons board a lift on the ground floor of a seven-storey building. The number of ways in which they leave the lift, such that each of them gets down at different floors, is

(a) 360, (b) 60, (c) 120, (d) 240.

24. The number of vectors (x,x_1,x_2), where $x,x_1,x_2>0$, for which

$$|\log(xx_1)|+|\log(xx_2)|+\left|\log\left(\frac{x}{x_1}\right)\right|+\left|\log\left(\frac{x}{x_2}\right)\right|$$

$= |\log x_1 + \log x_2|$ holds, is

(a) One, (b) Two, (c) Three, (d) None of these.

25. In a sample of households actually invaded by small pox, 70% of the inhabitants are attacked and 85% had been vaccinated. The minimum percentage of households (among those vaccinated) that

must have been attacked [Numbers expressed as nearest integer value] is

(a) 55, (b) 65, (c) 30, (d) 15.

26. In an analysis of bivariate data (X and Y) the following results were obtained.

Variance of $X\,(\sigma_x^2)=9$, product of the regression coefficient of Y on X and X on Y is 0.36, and the regression coefficient from the regression of Y on X (β_{yx}) is 0.8.

The variance of Y is

(a) 16, (b) 4, (c) 1.69 ,d) 3.

27. For comparing the wear and tear quality of two brands of automobile tyres, two samples of 50 customers using two types of tyres under similar conditions were selected. The number of kilometers x_1 and x_2 until the tyres became worn out, was obtained from each of them for the tyres used by them. The sample results were as follows : $\bar{x}_1 = 13{,}200$ km, $\bar{x}_2 = 13{,}650$ km, S_{x1} = 300 km, S_{x2} = 400 km. What would you conclude about the two brands of tyres (at 5% level of significance) as far as the wear and tear quality is concerned?

(a) The two brands are alike ,
(b) The two brands are not the same,
(c) Nothing can be concluded,
(d) The given data are inadequate to perform a test.

28. A continuous random variable x has the following probability density function:

$$f(x) = \frac{\alpha}{x_0}\left(\frac{x_0}{x}\right)^{\alpha+1} \quad \text{for } x > x_0,\ \alpha > 1.$$

The distribution function and the mean of x are given respectively by

(a) $1-\left(\dfrac{x}{x_0}\right)^{\alpha}, \dfrac{\alpha-1}{\alpha}x_0$,

(b) $1-\left(\dfrac{x}{x_0}\right)^{-\alpha}$, $\dfrac{\alpha-1}{\alpha}\, x_0$,

(c) $1-\left(\dfrac{x}{x_0}\right)^{-\alpha}$, $\dfrac{\alpha x_0}{\alpha-1}$,

(d) $1-\left(\dfrac{x}{x_0}\right)^{\alpha}$, $\dfrac{\alpha x_0}{\alpha-1}$

29. Suppose a discrete random variable X takes on the values $0, 1, 2, \dots, n$ with frequencies proportional to binomial coefficients $\binom{n}{0}$, $\binom{n}{1}$, $\dots$, $\binom{n}{n}$ respectively. Then the mean (μ) and the variance (σ^2) of the distribution are

(A) $\mu = \dfrac{n}{2}$ and $\sigma^2 = \dfrac{n}{2}$;

(B) $\mu = \dfrac{n}{4}$ and $\sigma^2 = \dfrac{n}{4}$;

(C) $\mu = \dfrac{n}{2}$ and $\sigma^2 = \dfrac{n}{4}$;

(D) $\mu = \dfrac{n}{4}$ and $\sigma^2 = \dfrac{n}{2}$.

30. Let $\{X_i\}$ be a sequence of *i.i.d* random variables such that

$$X_i = 1 \text{ with probability } p$$
$$= 0 \text{ with probability } 1 - p$$

Define $y = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} X_i = 100 \\ 0 & \text{otherwise} \end{cases}$

Then $E(y^2)$ is

(a) ∞, (b) $\binom{n}{100} p^{100}(1-p)^{n-100}$, (c) $n\,p$, (d) $(n\,p)^2$.

Syllabus for ME II (Economics), 2011

Microeconomics: Theory of consumer behaviour, Theory of Production, Market Structures under Perfect Competition, Monopoly, Price Discrimination, Duopoly with Cournot and Bertrand Competition (elementary problems) and Welfare economics.

Macroeconomics: National Income Accounting, Simple Keynesian Model of Income Determination and the Multiplier, IS-LM Model, Model of Aggregate Demand and Aggregate Supply, Harrod-Domar and Solow Models of Growth, Money, Banking and Inflation.

Sample questions for ME II (Economics), 2011

1. A monopolist sells two products, X and Y. There are three consumers with asymmetric preferences. Each consumer buys either one unit of a product or does not buy the product at all. The per-unit maximum willingness to pay of the consumers is given in the table below.

Consumer No.	X	Y
1	4	0
2	3	3
3	0	4

The monopolist who wants to maximize total payoffs has three alternative marketing strategies: (i) sell each commodity separately and so charge a uniform unit price for each commodity separately (simple monopoly pricing); (ii) offer the two commodities for sale only in a package comprising of one unit of each, and hence charge a price for the whole bundle (pure bundling strategy), and (iii) offer each commodity separately as well as a package of both, that is, offer unit price for each commodity as well as charge a bundle price (mixed bundling strategy). However, the monopolist cannot price discriminate between the consumers. Given the above data, find out the monopolist's optimal strategy and the corresponding prices of the products.

[30]

2. Consider two consumers with identical income M and utility function $U = xy$ where x is the amount of restaurant good consumed and y is the amount of any other good consumed. The unit prices of the goods are given. The consumers have two alternative plans to meet the restaurant bill. Plan A: they eat together at the restaurant and each pays his own bill. Plan B: they eat together at the restaurant but each pays one-half of the total restaurant bill. Find equilibrium consumption under plan A.
   (a) Find equilibrium consumption under plan B.
   (b) Explain your answer if the equilibrium outcome in case (b) differs from that in case (a).

[6+18+6=30]

3. Consider a community having a fixed stock X of an exhaustible resource (like oil) and choosing, over an infinite horizon, how much of this resource is to be used up each period. While doing so, the community maximizes an intertemporal utility function $U = \sum_{t=0}^{\infty} \delta^t \ln C_t$ where C_t represents consumption or use of the resource at period *t* and δ ($0 < \delta < 1$) is the discount factor.

(a) Set up the utility maximization problem of the community and interpret the first order condition.

(b) Express the optimal consumption C_t for any period *t* in terms of the parameters δ and *X*.

(c) If an unanticipated discovery of an additional stock of X' occurs at the beginning of period T ($0 < T < \infty$), what will be the new level of consumption at each period from *T* onwards?

[7+16+7=30]

4. A consumer, with a given money income *M*, consumes *n* goods $x_1, x_2, ..., x_n$ with given prices $p_1, p_2, ..., p_n$.

(a) Suppose his utility function is $U(x_1, x_2, ..., x_n) = Max(x_1, x_2, ..., x_n)$. Find the Marshallian demand function for good x_i and draw it in a graph.

(b) Suppose his utility function is $U(x_1, x_2, ..., x_n) = Min(x_1, x_2, ..., x_n)$. Find the income and the own price elasticities of demand for good x_i.

[15+15=30]

5. An economy, consisting of *m* individuals, is endowed with quantities $\omega_1, \omega_2, ...\omega_n$ of *n* goods. The *i*th individual has a utility function $U(C_1^i, C_2^i, ...C_n^i) = C_1^i C_2^i ... C_n^i$, where C_j^i is consumption of good *j* of individual *i*.
   (a) Define an *allocation*, a *Pareto inferior allocation* and a *Pareto optimal allocation* for this economy.
   (b) Find an allocation which is *Pareto inferior* and an allocation which is *Pareto optimal*.
   (c) Consider an allocation where $C_j^i = \lambda^i \omega_j \forall j$, $\sum_i \lambda^i = 1$. Is this allocation *Pareto optimal*?

[6+18+6=30]

6. Suppose that a monopolist operates in a domestic market facing a demand curve $p = 5 - \frac{3}{2} q_h$, where p is the domestic price and q_h is the quantity sold in the domestic market. This monopolist also has the option of selling the product in the foreign market at a constant price of 3. The monopolist has a cost function given by $C(q) = q^2$, where q is the total quantity that the monopolist produces. Now, answer the following questions.
   (a) How much will the monopolist sell in the domestic market and how much will it sell in the foreign market?
   (b) Suppose, the home government imposes a restriction on the amount that the monopolist can sell in the foreign market. In particular, the monopolist is not allowed to sell more than 1/6 units

of the good in the foreign market. Now find out the amount the monopolist sells in the domestic market and in the foreign market.

[6+24=30]

7. An economy produces two goods, food (F) and manufacturing (M).

Food is produced by the production function $F = (L_F)^{\frac{1}{2}}(T)^{\frac{1}{2}}$, where L_F is the labour employed, T is the amount of land used and F is the amount of food produced. Manufacturing is produced by the production function $M = (L_M)^{\frac{1}{2}}(K)^{\frac{1}{2}}$, where L_M is the labour employed, K is the amount of capital used and M is the amount of manufacturing production. Labour is perfectly mobile between the sectors (i.e. food and manufacturing production) and the total amount of labour in this economy is denoted by L. All the factors of production are fully employed. Land is owned by the landlords and capital is owned by the capitalists. You are also provided with the following data: $K = 36$, $T = 49$, and $L = 100$. Also assume that the price of food and that of manufacturing are the same and is equal to unity.

(a) Find out the equilibrium levels of labour employment in the food sector and the manufacturing sector (i.e. L_F and L_M respectively)

(b) Next, we introduce a small change in the description of the economy given above. Assume, everything remains the same except for the fact that land is owned by none; land comes for free! How much labour would now be employed in the food and the manufacturing sectors?

(c) Suggest a measure of welfare for the economy as a whole.

(d) Using the above given data and your measure of welfare, determine whether the scenario given in problem (b), where land is owned by none, better or worse for the economy as a whole, compared to the scenario given in problem (a), where land is owned by the landlords?

(e) What do you think is the source of the difference in welfare levels (if any) under case (a) and case (b).

[6+10+4+6+4=30]

8. An economy produces a single homogeneous good in a perfectly competitive set up, using the production function $Y = AF(L, K)$, where Y is the output of the good, L and K are the amount of labour and capital respectively and A is the technological productivity parameter. Further, assume that F is homogeneous of degree one in L and K. Labour and capital in this economy remains fully employed. It has also been observed that the total wage earning of this economy is equal to the total earnings of capital in the economy at all points in time.

Answer the following questions.

(a) It is observed that over a given period the labour force grew by 4%, the capital stock grew by 3%, and output grew by 9%. What then was the growth rate of the technological productivity parameter (A) over that period?

(b) Over another period the wage rate of labour in this economy exhibited a growth of 30% , rental rate of capital grew by 10% and the price of the good over the same period grew by 5%. Find out the growth rate of the technological productivity parameter (A) over this period.

(c) Over yet another period, it was observed that there was no growth in the technological productivity, and the wages grew by 30% and rental rate grew by 10%. Infer from this, the growth rate of the price of the good over the period.

[4+20+6=30]

9. An economy produces two goods - *m* and *g*. Capitalists earn a total income, R (R_m from sector *m* plus R_g from sector *g*), but consumes only good *m*, spending a fixed proportion (c) of their income on it. Workers do *not* save. Workers in sector *m* spend a fixed proportion α of their income (W) on good *g* and the rest on good *m*. [However, whatever wages are paid in sector *g* are spent entirely for the consumption of good *g* only so that we ignore wages in this sector for computing both income generation therein and the expenditure made on its output.] The categories of income and expenditure in the two sectors are shown in detail in the chart below.

Sector m Sector g

Income generated	Expenditure on good m	Income (net of wages) generated	Expenditure (net of that by own workers) on good g
• Capitalists' income (R_m) • Wages (W)	• Capitalists' consumption (C = c. R) • Consumption of workers of sector m: ($\{1-\alpha\}.W$) • Investment: (I)	Capitalists' Income (R_g)	• Consumption of workers of sector m ($\alpha .W$)

Further, *investment* expenditure (I), made exclusively on m-good, is *autonomous* and income distribution in sector m is exogenously given:

$R_m = \theta . W$ (θ given).

(a) Equating aggregate income with aggregate expenditure for the economy, show that capitalists' income (R) is determined exclusively by their *own* expenditure (C and I). Is there any multiplier effect of I on R ? Give arguments.

(b) Show that I (along with c, α and θ) also determines W.

[15 +15= 30]

10. Consider two countries – a domestic country (with excess capacity and unlimited supply of labour) and a benevolent foreign country. The domestic country produces a single good at a fixed price of Re.1 per

unit and is in equilibrium initially (i.e. in year 0) with income at Rs. 100 and consumption, investment and savings at Rs. 50 each. Investment expenditure is autonomous. Final expenditure in any year t shows up as income in year t (Y_t), but consumption expenditure in year t (C_t) is given by: $C_t = 0.5\ Y_{t-1}$. The foreign country agrees to give a loan of Rs.100 to the domestic country in year 1 at *zero* interest rate, but on conditions that it be (i) used for investment only and (ii) repaid in full at the beginning of the next year. The loan may be renewed every year, but on the same conditions as above. Find out income, consumption, investment and savings of the domestic country in year 1, year 2 and in final equilibrium in each of the following two alternative cases:

(a) The country takes the loan in year 1 only.

(b) The country takes the loan in year 1 and renews it every year.

[15 + 15=30]

SYLLABUS AND SAMPLE QUESTIONS FOR MS(QE)

2012

Syllabus for ME I (Mathematics), 2012

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency; dispersion, correlation and regression, probability distributions, standard distributions–Binomial and Normal.

Sample Questions for MEI (Mathematics), 2012

1. Kupamonduk, the frog, lives in a well 14 feet deep. One fine morning she has an urge to see the world, and starts to climb out of her well. Every day she climbs up by 5 feet when there is light, but slides back by 3 feet in the dark. How many days will she take to climb out of the well?
(A) 3,
(B) 8,
(C) 6,
(D) None of the above.

2. The derivative of $f(x) = |x|^2$ at $x = 0$ is,
(A) -1,
(B) Non-existent,
(C) 0,
(D) 1/2.

3. Let $\mathcal{N} = \{1, 2, 3, \ldots\}$ be the set of natural numbers. For each $n \in \mathcal{N}$, define $A_n = \{(n+1)k : k \in \mathcal{N}\}$. Then $A_1 \cap A_2$ equals
(A) A_3,
(B) A_4,
(C) A_5,
(D) A_6.

4. Let $S = \{a, b, c\}$ be a set such that a, b and c are distinct real numbers. Then $\min\{\max\{a, b\}, \max\{b, c\}, \max\{c, a\}\}$ is always
(A) the highest number in S,
(B) the second highest number in S,
(C) the lowest number in S,
(D) the arithmetic mean of the three numbers in S.

5. The sequence $< -4^{-n} >$, $n = 1, 2, \cdots$, is
(A) Unbounded and monotone increasing,
(B) Unbounded and monotone decreasing,
(C) Bounded and convergent,
(D) Bounded but not convergent.

6. $\int \frac{x}{7x^2+2} dx$ equals

(A) $\frac{1}{14} ln(7x^2 + 2) +$ constant,
(B) $7x^2 + 2$,
(C) $ln\ x +$ constant,
(D) None of the above.

7. The number of real roots of the equation

$$2(x-1)^2 = (x-3)^2 + (x+1)^2 - 8$$

is
(A) Zero,
(B) One,
(C) Two,
(D) None of the above.

8. The three vectors $[0, 1]$, $[1, 0]$ and $[1000, 1000]$ are
(A) Dependent,
(B) Independent,
(C) Pairwise orthogonal,
(D) None of the above.

9. The function $f(.)$ is increasing over $[a, b]$. Then $[f(.)]^n$, where n is an odd integer greater than 1, is necessarily
(A) Increasing over $[a, b]$,
(B) Decreasing over $[a, b]$,
(C) Increasing over $[a, b]$ if and only if $f(.)$ is positive over $[a, b]$,
(D) None of the above.

10. The determinant of the matrix $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ is
(A) 21,
(B) -16,
(C) 0,
(D) 14.

11. In what ratio should a given line be divided into two parts, so that the area of the rectangle formed by the two parts as the sides is the maximum possible?
(A) 1 is to 1,
(B) 1 is to 4,
(C) 3 is to 2,
(D) None of the above.

12. Suppose (x^*, y^*) solves:

$$Minimize\ \ ax + by,$$

subject to

$$x^\alpha + y^\alpha = M,$$

and $x, y \geq 0$, where $a > b > 0$, $M > 0$ and $\alpha > 1$. Then, the solution is,

(A) $\frac{x*^{\alpha-1}}{y*^{\alpha-1}} = \frac{a}{b}$,
(B) $x^* = 0,\ y^* = M^{\frac{1}{\alpha}}$,
(C) $y^* = 0,\ x^* = M^{\frac{1}{\alpha}}$,
(D) None of the above.

13. Three boys and two girls are to be seated in a row for a photograph. It is desired that no two girls sit together. The number of ways in which they can be so arranged is
(A) $4_{P_2} \times 3!$,
(B) $3_{P_2} \times 2!$
(C) $2! \times 3!$
(D) None of the above.

14. The domain of x for which $\sqrt{x} + \sqrt{3-x} + \sqrt{x^2 - 4x}$ is real is,
(A) [0,3],
(B) (0,3),
(C) $\{0\}$,
(D) None of the above.

15. P (x) is a quadratic polynomial such that P (1) = P (-1). Then
(A) The two roots sum to zero,
(B) The two roots sum to 1,
(C) One root is twice the other,
(D) None of the above.

16. The expression $\sqrt{11+6\sqrt{2}} + \sqrt{11-6\sqrt{2}}$ is
(A) Positive and an even integer,
(B) Positive and an odd integer,
(C) Positive and irrational,
(D) None of the above.

17. What is the maximum value of $a(1-a)b(1-b)c(1-c)$, where a, b, c vary over all positive fractional values?

A 1,

B $\frac{1}{8}$,

C $\frac{1}{27}$,

D $\frac{1}{64}$.

18. There are four modes of transportation in Delhi: (A) Auto-rickshaw, (B) Bus, (C) Car, and (D) Delhi-Metro. The probability of using transports A, B, C, D by an individual is $\frac{1}{9}$, $\frac{2}{9}$, $\frac{4}{9}$, $\frac{2}{9}$ respectively. The probability that he arrives late at work if he uses transportation A, B, C, D is $\frac{5}{7}$, $\frac{4}{7}$, $\frac{6}{7}$, and $\frac{6}{7}$ respectively. What is the probability that he used transport A if he reached office on time?

A $\frac{1}{9}$,

B $\frac{1}{7}$,

C $\frac{3}{7}$,

D $\frac{2}{9}$.

19. What is the least (strictly) positive value of the expression $a^3+b^3+c^3-3abc$, where a, b, c vary over all strictly positive integers? (You may use the identity $a^3+b^3+c^3-3abc = \frac{1}{2}(a+b+c)\big((a-b)^2+(b-c)^2+(c-a)^2\big)$.)

A 2,

B 3,

C 4,

D 8.

20. If $a^2+b^2+c^2=1$, then $ab+bc+ca$ is,
(A) -0.75,
(B) Belongs to the interval $[-1,\ -0.5]$,
(C) Belongs to the interval $[0.5,\ 1]$,
(D) None of the above.

21. Consider the following linear programming problem:
Maximize $a+b$ subject to
$a+2b \leq 4$,
$a+6b \leq 6$,

$a - 2b \leq 2$,
$a, b \geq 0$.
An optimal solution is:
(A) a=4, b=0,
(B) a=0, b=1,
(C) a=3,b=1/2,
(D) None of the above.

22. The value of $\int_{-4}^{-1} \frac{1}{x} dx$ equals,
(A) $ln\ 4$,
(B) Undefined,
(C) $ln(-4) - ln(-1)$,
(D) None of the above.

23. Given $x \geq y \geq z$, and $x + y + z = 9$, the maximum value of x + 3y + 5z is
(A) 27,
(B) 42,
(C) 21,
(D) 18.

24. A car with six sparkplugs is known to have two malfunctioning ones. If two plugs are pulled out at random, what is the probability of getting at least one malfunctioning plug.
(A) 1/15,
(B) 7/15,
(C) 8/15,
(D) 9/15.

25. Suppose there is a multiple choice test which has 20 questions. Each question has two possible responses - true or false. Moreover, only one of them is correct. Suppose a student answers each of them randomly.Which one of the following statements is correct?
(A) The probability of getting 15 correct answers is less than the probability of getting 5 correct answers,
(B) The probability of getting 15 correct answers is more than the

probability of getting 5 correct answers,
(C) The probability of getting 15 correct answers is equal to the probability of getting 5 correct answers,
(D) The answer depends on such things as the order of the questions.

26. From a group of 6 men and 5 women, how many different committees consisting of three men and two women can be formed when it is known that 2 of the men do not want to be on the committee together?
(A) 160,
(B) 80,
(C) 120,
(D) 200.

27. Consider any two consecutive integers a and b that are both greater than 1. The sum $(a^2 + b^2)$ is
(A) Always even,
(B) Always a prime number,
(C) Never a prime number,
(D) None of the above statements is correct.

28. The number of real non-negative roots of the equation

$$x^2 - 3|x| - 10 = 0$$

is,
(A) 2,
(B) 1,
(C) 0,
(D) 3.

29. Let $< a^n >$ and $< b^n >$, $n = 1, 2, \cdots$, be two different sequences, where $< a^n >$ is convergent and $< b^n >$ is divergent. Then the sequence $< a^n + b^n >$ is,
(A) Convergent,
(B) Divergent,
(C) Undefined,
(D) None of the above.

30. Consider the function

$$f(x) = \frac{|x|}{1+|x|}.$$

This function is,
(A) Increasing in x when $x \geq 0$,
(B) Decreasing in x,
(C) Increasing in x for all real x,
(D) None of the above.

Syllabus for ME II (Economics), 2012

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample Questions for ME II (Economics), 2012

1. A price taking firm makes machine tools Y using labour and capital according to the following production function

$$Y = L^{0.25}K^{0.25}.$$

Labour can be hired at the beginning of every week, while capital can be hired only at the beginning of every month. It is given that the wage rate = rental rate of capital = 10. Show that the short run (week) cost function is $10Y^4/K^*$ where the amount of capital is fixed at K^* and the long run (month) cost function is $20Y^2$.

2. Consider the following IS-LM model
$C = 200 + 0.25Y_D$,
$I = 150 + 0.25Y - 1000i$,

$G = 250,$
$T = 200,$
$(m/p)^d = 2Y - 8000i,$
$(m/p) = 1600,$

where C = aggregate consumption, I = investment, G = government expenditures, T = taxes, $(m/p)^d$ = money demand, (m/p) = money supply, Y_D = disposable income $(Y - T)$. Solve for the equilibrium values of all variables. How is the solution altered when money supply is increased to $(m/p) = 1840$? Explain intuitively the effect of expansionary monetary policy on investment in the short run.

3. Suppose that a price-taking consumer A maximizes the utility function $U(x, y) = x^\alpha + y^\alpha$ with $\alpha > 0$ subject to a budget constraint. Assume prices of both goods, x and y, are equal. Derive the demand function for both goods. What would your answer be if the price of x is twice that of the price of y?

4. Assume the production function for the economy is given by

$$Y = L^{0.5}K^{0.5}$$

where Y denotes output, K denotes the capital stock and L denotes labour. The evolution of the capital stock is given by

$$K_{t+1} = (1 - \delta)K_t + I_t$$

where δ lies between 0 and 1 and is the rate of depreciation of capital. I represents investment, given by $I_t = sY_t$, where s is the savings rate. Derive the expression of steady state consumption and find out the savings rate that maximizes steady state consumption.

5. There are two goods x and y. Individual A has endowments of 25 units of good x and 15 units of good y. Individual B has endowments of 15 units of good x and 30 units of good y. The price of good y is Re. 1, no matter whether the individual buys or sells the good. The price of good x is Re. 1 if the individual wishes to sell it. It is, however, Rs. 3 if

the individual wishes to buy it. Let C_x and C_y denote the consumption of these goods. Suppose that individual B chooses to consume 20 units of good x and individual A does not buy or sell any of the goods and chooses to consume her endowment. Could A and B have the same preferences?

6. A monopolist has cost function $c(y) = y$ so that its marginal cost is constant at Re. 1 per unit. It faces the following demand curve
$$D(p) = \begin{cases} 0, \text{ if} & p > 20 \\ \dfrac{100}{p}, \text{ if} & p \leq 20. \end{cases}$$
Find the profit maximizing level of output if the government imposes a per unit tax of Re. 1 per unit, and also the dead-weight loss from the tax.

7. A library has to be located on the interval $[0, 1]$. There are three consumers A, B and C located on the interval at locations 0.3, 0.4 and 0.6, respectively. If the library is located at x, then A, B and C's utilities are given by $-|x - 0.3|$, $-|x - 0.4|$ and $-|x - 0.6|$, respectively. Define a Pareto-optimal location and examine whether the locations $x = 0.1, x = 0.3$ and $x = 0.6$ are Pareto-optimal or not.

8. Consider an economy where the agents live for only two periods and where there is only one good. The life-time utility of an agent is given by $U = u(c) + \beta v(d)$, where u and v are the first and second period utilities, c and d are the first and second period consumptions and β is the discount factor. β lies between 0 and 1. Assume that both u and v are strictly increasing and concave functions. In the first period, income is w and in the second period, income is zero. The interest rate on savings carried from period 1 to period 2 is r. There is a government that taxes first period income. A proportion τ of income is taken away by the government as taxes. This is then returned in the second period to the agent as a lump sum transfer T. The government's budget is balanced i.e., $T = \tau w$. Set up the agent's optimization problem and write the first order condition assuming an interior solution. For given

values of r, β, w, show that increasing T will reduce consumer utility if the interest rate is strictly positive.

9. A monopolist sells two products, X and Y . There are three consumers with asymmetric preferences. Each consumer buys either one unit of a product or does not buy the product at all. The per-unit maximum willingness to pay of the consumers is given in the table below.

Consumer No.	X	Y
1	4	0
2	3	3
3	0	4.

The monopolist who wants to maximize total payoffs has three alternative marketing strategies: (i) sell each commodity separately and so charge a uniform unit price for each commodity separately (simple monopoly pricing);(ii) offer the two commodities for sale only in a package comprising of one unit of each, and hence charge a price for the whole bundle (pure bundling strategy), and (iii) offer each commodity separately as well as a package of both, that is, offer unit price for each commodity as well as charge a bundle price (mixed bundling strategy). However, the monopolist cannot price discriminate between the consumers. Given the above data, find out the monopolist's optimal strategy and the corresponding prices of the products.

10. Consider two consumers with identical income M and utility function $U = xy$ where x is the amount of restaurant good consumed and y is the amount of any other good consumed. The unit prices of the goods are given. The consumers have two alternative plans to meet the restaurant bill. Plan A: they eat together at the restaurant and each pays his own bill. Plan B: they eat together at the restaurant but each pays one-half of the total restaurant bill. Find equilibrium consumption under plan B.

11. Consider a community having a fixed stock X of an exhaustible resource (like oil) and choosing, over an infinite horizon, how much of this resource is to be used up each period. While doing so, the com-

munity maximizes an intertemporal utility function $U = \sum \delta^t ln(C_t)$ where C_t represents consumption or use of the resource at period t and $\delta (0 < \delta < 1)$ is the discount factor. Express the optimal consumption C_t for any period t in terms of the parameter δ and X.

12. A consumer, with a given money income M, consumes 2 goods x_1 and x_2 with given prices p_1 and p_2. Suppose that his utility function is $U(x_1, x_2) = Max(x_1, x_2)$. Find the Marshallian demand function for goods x_1, x_2 and draw it in a graph. Further, suppose that his utility function is $U(x_1, x_2) = Min(x_1, x_2)$. Find the income and the own price elasticities of demand for goods x_1 and x_2.

13. An economy, consisting of m individuals, is endowed with quantities $\omega_1, \omega_2, ..., \omega_n$ of n goods. The ith individual has a utility function $U(C_1^i, C_2^i, ...C_n^i) = C_1^i C_2^i ... C_n^i$, where C_j^i is consumption of good j of individual i.
(a) Define an allocation,a Pareto inferior allocation and a Pareto optimal allocation for this economy.
(b) Consider an allocation where $C_j^i = \lambda^i \omega_j$ for all j ,$\sum_i \lambda^i = 1$. Is this allocation Pareto optimal?

14. Suppose that a monopolist operates in a domestic market facing a demand curve $p = 5 - (\frac{3}{2})q_h$, where p is the domestic price and q_h is the quantity sold in the domestic market. This monopolist also has the option of selling the product in the foreign market at a constant price of 3. The monopolist has a cost function given by $C(q) = q^2$, where q is the total quantity that the monopolist produces. Suppose, that the monopolist is not allowed to sell more than 1/6 units of the good in the foreign market. Now find out the amount the monopolist sells in the domestic market and in the foreign market.

15. An economy produces two goods, food (F) and manufacturing (M). Food is produced by the production function $F = (L_F)^{1/2}(T)^{1/2}$, where L_F is the labour employed, T is the amount of land used and F is the amount of food produced. Manufacturing is produced by the production function $M = (L_M)^{1/2}(K)^{1/2}$, where L_M is the labour employed,

K is the amount of capital used and M is the amount of manufacturing production. Labour is perfectly mobile between the sectors (i.e. food and manufacturing production) and the total amount of labour in this economy is denoted by L. All the factors of production are fully employed. Land is owned by the landlords and capital is owned by the capitalists. You are also provided with the following data: $K = 36, T = 49$, and $L = 100$. Also assume that the price of food and that of manufacturing are the same and is equal to unity.

(a) Find the equilibrium levels of labour employment in the food sector and the manufacturing sector (i.e. L_F and L_M respectively)

(b)Next, we introduce a small change in the description of the economy given above. Assume that everything remains the same except for the fact that land is owned by none; land comes for free! How much labour would now be employed in the food and the manufacturing sectors?

16. Consider two countries - a domestic country (with excess capacity and unlimited supply of labour) and a benevolent foreign country. The domestic country produces a single good at a fixed price of Re.1 per unit and is in equilibrium initially (i.e., in year 0) with income at Rs. 100 and consumption, investment and savings at Rs. 50 each. Investment expenditure is autonomous. Final expenditure in any year t shows up as income in year t (say, Y_t) , but consumption expenditure in year t (say, C_t) is given by: $C_t = 0.5Y_{t-1}$.

    The foreign country agrees to give a loan of Rs.100 to the domestic country in year 1 at zero interest rate, but on conditions that it be (i) used for investment only and (ii) repaid in full at the beginning of the next year. The loan may be renewed every year, but on the same conditions as above. Find the income, consumption, investment and savings of the domestic country in year 1, year 2 and in final equilibrium when the country takes the loan in year 1 only.

SYLLABUS AND SAMPLE QUESTIONS FOR MSQE
(Program Code: MQEK and MQED)

2013

Syllabus for PEA (Mathematics), 2013

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency; dispersion, correlation and regression, probability distributions, standard distributions - Binomial and Normal.

Sample Questions for PEA (Mathematics), 2013

1. Let $f(x) = \frac{1-x}{1+x}, x \neq -1$. Then $f(f(\frac{1}{x})), x \neq 0$ and $x \neq -1$, is
(A) 1,
(B) x,
(C) x^2,
(D) $\frac{1}{x}$.

2. The limiting value of $\frac{1.2+2.3+\ldots+n(n+1)}{n^3}$ as $n \to \infty$ is,
(A) 0,
(B) 1,
(C) 1/3,
(D) 1/2.

3. Suppose $a_1, a_2, \ldots, a_n$ are n positive real numbers with $a_1 a_2 \ldots a_n = 1$. Then the minimum value of $(1 + a_1)(1 + a_2) \ldots (1 + a_n)$ is
(A) 2^n,
(B) 2^{2n},
(C) 1,
(D)None of the above.

4. Let the random variable X follow a Binomial distribution with parameters n and p where $n(> 1)$ is an integer and $0 < p < 1$. Suppose further that the probability of $X = 0$ is the same as the probability of $X = 1$. Then the value of p is
(A) $\frac{1}{n}$,
(B) $\frac{1}{n+1}$,
(C) $\frac{n}{n+1}$,
(D) $\frac{n-1}{n+1}$.

5. Let X be a random variable such that $E(X^2) = E(X) = 1$. Then $E(X^{100})$ is
(A) 1,
(B) 2^{100},

(C) 0,
(D) None of the above.

6. If α and β are the roots of the equation $x^2 - ax + b = 0$, then the quadratic equation whose roots are $\alpha + \beta + \alpha\beta$ and $\alpha\beta - \alpha - \beta$ is
(A) $x^2 - 2ax + a^2 - b^2 = 0$,
(B) $x^2 - 2ax - a^2 + b^2 = 0$,
(C) $x^2 - 2bx - a^2 + b^2 = 0$,
(D) $x^2 - 2bx + a^2 - b^2 = 0$.

7. Suppose $f(x) = 2(x^2 + \frac{1}{x^2}) - 3(x + \frac{1}{x}) - 1$ where x is real and $x \neq 0$. Then the solutions of $f(x) = 0$ are such that their product is
(A) 1,
(B) 2,
(C) -1,
(D) -2.

8. Toss a fair coin 43 times. What is the number of cases where number of 'Head'> number of 'Tail'?
(A) 2^{43},
(B) $2^{43} - 43$,
(C) 2^{42},
(D) None of the above.

9. The minimum number of real roots of $f(x) = |x|^3 + a|x|^2 + b|x| + c$, where a, b and c are real, is
(A) 0,
(B) 2,
(C) 3,
(D) 6.

10. Suppose $f(x, y)$ where x and y are real, is a differentiable function satisfying the following properties:

(i) $f(x+k, y) = f(x, y) + ky$;

(ii) $f(x, y+k) = f(x, y) + kx$; and

(iii) $f(x, 0) = m$, where m is a constant.

Then $f(x, y)$ is given by

(A) $m + xy$,

(B) $m + x + y$,

(C) mxy,

(D) None of the above.

11. Let $I = \int\limits_{2}^{343} \{x - [x]\}^2 dx$ where $[x]$ denotes the largest integer less than or equal to x. Then the value of I is

(A) $\frac{343}{3}$,

(B) $\frac{343}{2}$,

(C) $\frac{341}{3}$,

(D) None of the above.

12. The coefficients of three consecutive terms in the expression of $(1+x)^n$ are 165, 330 and 462. Then the value of n is

(A) 10,

(B) 11,

(C) 12,

(D) 13.

13. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in

(A) $[\frac{1}{2}, 1]$,

(B) $[-1, 1]$,

(C) $[-\frac{1}{2}, \frac{1}{2}]$,

(D) $[-\frac{1}{2}, 1]$.

14. Let the function $f(x)$ be defined as $f(x) = |x-4| + |x-5|$. Then which of the following statements is true?

(A) $f(x)$ is differentiable at all points,

(B) $f(x)$ is differentiable at $x = 4$, but not at $x = 5$,
(C) $f(x)$ is differentiable at $x = 5$ but not at $x = 4$,
(D) None of the above.

15. The value of the integral $\int\limits_0^1 \int\limits_0^x x^2 e^{xy} dx dy$ is

(A) e,
(B) $\frac{e}{2}$,
(C) $\frac{1}{2}(e-1)$,
(D) $\frac{1}{2}(e-2)$.

16. Let $\mathcal{N} = \{1, 2, \ldots\}$ be a set of natural numbers. For each $x \in \mathcal{N}$, define $A_n = \{(n+1)k, k \in \mathcal{N}\}$. Then $A_1 \cap A_2$ equals

(A) A_2,
(B) A_4,
(C) A_5,
(D) A_6.

17. $\lim\limits_{x \to 0} \{\frac{1}{x}(\sqrt{1+x+x^2} - 1)\}$ is

(A) 0,
(B) 1,
(C) $\frac{1}{2}$,
(D) Non-existent.

18. The value of $\binom{n}{0} + 2\binom{n}{1} + 3\binom{n}{2} + \ldots + (n+1)\binom{n}{n}$ equals
(A) $2^n + n2^{n-1}$,
(B) $2^n - n2^{n-1}$,
(C) 2^n,
(D) 2^{n+2}.

19. The rank of the matrix $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$ is

(A) 1,
(B) 2,
(C) 3,
(D) 4.

20. Suppose an odd positive integer $2n+1$ is written as a sum of two integers so that their product is maximum. Then the integers are

(A) $2n$ and 1,
(B) $n+2$ and $n-1$,
(c) $2n-1$ and 2,
(D) None of the above.

21. If $|a| < 1, |b| < 1$, then the series $a(a+b)+a^2(a^2+b^2)+a^3(a^3+b^3)+\ldots\ldots$ converges to
(A) $\frac{a^2}{1-a^2} + \frac{b^2}{1-b^2}$,
(B) $\frac{a(a+b)}{1-a(a+b)}$,
(C) $\frac{a^2}{1-a^2} + \frac{ab}{1-ab}$,
(D) $\frac{a^2}{1-a^2} - \frac{ab}{1-ab}$.

22. Suppose $f(x) = x^3 - 6x^2 + 24x$. Then which of the following statements is true?
(A) $f(x)$ has a maxima but no minima,
(B) $f(x)$ has a minima but no maxima,
(C) $f(x)$ has a maxima and a minima,
(D) $f(x)$ has neither a maxima nor a minima.

23. An urn contains 5 red balls, 4 black balls and 2 white balls. A player draws 2 balls one after another with replacement. Then the probability of getting at least one red ball or at least one white ball is

(A) $\frac{105}{121}$,

(B) $\frac{67}{121}$,

(C) $\frac{20}{121}$,

(D) None of the above.

24. If $log_t x = \frac{1}{t-1}$ and $log_t y = \frac{t}{t-1}$, where $log_t x$ stands for logarithm of x to the base t. Then the relation between x and y is

(A) $y^x = x^{1/y}$,
(B) $x^{1/y} = y^{1/x}$,
(C) $x^y = y^x$,
(D) $x^y = y^{1/x}$.

25. Suppose $\frac{f''(x)}{f'(x)} = 1$ for all x. Also, $f(0) = e^2$ and $f(1) = e^3$. Then $\int\limits_{-2}^{2} f(x)dx$ equals
(A) $2e^2$,
(B) $e^2 - e^{-2}$,
(C) $e^4 - 1$,
(D) None of the above.

26. The minimum value of the objective function $z = 5x + 7y$, where $x \geq 0$ and $y \geq 0$, subject to the constraints

$2x + 3y \geq 6, \;\; 3x - y \leq 15, \;\; -x + y \leq 4$, and $2x + 5y \leq 27$
is

(A) 14,
(B) 15,
(C) 25,
(D) 28.

27. Suppose A is a 2×2 matrix given as $\begin{pmatrix} 2 & 5 \\ 3 & 1 \end{pmatrix}$.
Then the matrix $A^2 - 3A - 13I$, where I is the 2×2 identity matrix, equals

(A) I,
(B) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$,
(C) $\begin{pmatrix} 1 & 5 \\ 3 & 0 \end{pmatrix}$,

(D) None of the above.

28. The number of permutations of the letters $a, \; b, \; c,$ and d such that b does not follow a, c does not follow b, and d does not follow c is
(A) 14,
(B) 13,
(C) 12,
(D) 11.

29. Given n observations $x_1, \; x_2, \; \ldots, x_n$, which of the following statements is true ?
(A) The mean deviation about arithmetic mean can exceed the standard deviation,
(B) The mean deviation about arithmetic mean cannot exceed the standard deviation,

(C) The root mean square deviation about a point A is least when A is the median,
(D) The mean deviation about a point A is minimum when A is the arithmetic mean.

30. Consider the following classical linear regression of y on x,

$$y_i = \beta x_i + u_i, i = 1, 2, \ldots, \ldots, n$$

where $E(u_i) = 0,\ V(u_i) = \sigma^2$ for all i, and $u_i's$ are homoscedastic and non-autocorrelated. Now, let $\hat{u}_i$ be the ordinary least square estimate of u_i. Then which of the following statements is true?

(A) $\sum_{i=1}^{n} \hat{u}_i = 0$,
(B) $\sum_{i=1}^{n} \hat{u}_i = 0$, and $\sum_{i=1}^{n} x_i\hat{u}_i = 0$,
(C) $\sum_{i=1}^{n} \hat{u}_i = 0$, and $\sum_{i=1}^{n} x_i\hat{u}_i \neq 0$,
(D) $\sum_{i=1}^{n} x_i\hat{u}_i = 0$.

Syllabus for PEB (Economics), 2013

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample Questions for PEB (Economics), 2013

1. An agent earns w units of wage while young, and earns nothing while old. The agent lives for two periods and consumes in both the periods. The utility function for the agent is given by $u = \log c_1 + \log c_2$, where c_i is the consumption in period $i = 1, 2$. The agent faces a constant rate of interest r (net interest rate) at which it can freely lend or borrow,

   (a) Find out the level of saving of the agent while young.

   (b) What would be the consequence of a rise in the interest rate, r, on the savings of the agent?

2. Consider a city that has a number of fast food stalls selling Masala Dosa (MD). All vendors have a marginal cost of Rs. 15/- per MD, and can sell at most 100 MD a day.

   (a) If the price of an MD is Rs. 20/-, how much does each vendor want to sell?

   (b) If demand for MD be $d(p) = 4400 - 120p$, where p denotes price per MD, and each vendor sells exactly 100 units of MD, then how many vendors selling MD are there in the market?

   (c) Suppose that the city authorities decide to restrict the number of vendors to 20. What would be the market price of MD in that case?

   (d) If the city authorities decide to issue permits to the vendors keeping the number unchanged at 20, what is the maximum that a vendor will be willing to pay for obtaining such a permit?

3. A firm is deciding whether to hire a worker for a day at a daily wage of Rs. 20/-. If hired, the worker can work for a maximum of 10 hours during the day. The worker can be used to produce two intermediate inputs, 1 and 2, which can then be combined to produce a final good. If

the worker produces only 1, then he can produce 10 units of input 1 in an hour. However, if the worker produces only 2, then he can produce 20 units of input 2 in an hour. Denoting the levels of production of the amount produced of the intermediate goods by k_1 and k_2 , the production function of the final good is given by $\sqrt{k_1 k_2}$. Let the final product be sold at the end of the day at a per unit price of Rs. 1/-. Solve for the firms optimal hiring, production and sale decision.

4. A monopolist has contracted with the government to sell as much of its output as it likes to the government at Rs. 100/- per unit. Its sales to the government are positive, and it also sells its output to buyers at Rs. 150/- per unit. What is the price elasticity of demand for the monopolists services in the private market?

5. Consider the following production function with usual notations.

$$Y = K^{\alpha} L^{1-\alpha} - \beta K + \theta L \text{ with } 0 < \alpha < 1, \beta > 0, \theta > 0.$$

   Examine the validity of the following statements.

   (a) Production function satisfies constant returns to scale.

   (b) The demand function for labour is defined for all non-negative wage rates.

   (c) The demand function for capital is undefined when price of capital service is zero.

6. Suppose that due to technological progress labour requirement per unit of output is halved in a Simple Keynesian model where output is proportional to the level of employment. What happens to the equilibrium level of output and the equilibrium level of employment in this case? Consider a modified Keynesian model where consumption expenditure is proportional to labour income and wage-rate is given. Does technological progress produce a different effect on the equilibrium level of output in this case?

7. A positive investment multiplier does not exist in an open economy simple Keynesian model when the entire amount of investment goods is supplied from import. Examine the validity of this statement.

8. A consumer consumes two goods, x_1 and x_2, with the following utility function

$$U(x_1, x_2) = U_1(x_1) + U_2(x_2).$$

   Suppose that the income elasticity is positive. It is claimed that in the above set-up all goods are normal. Prove or disprove this claim.

9. A consumer derives his market demand, say x, for the product X as $x = 10 + \frac{m}{10p_x}$, where $m > 0$ is his money income and p_x is the price per unit of X. Suppose that initially he has money income $m = 120$, and the price of the product is $p_x = 3$. Further, the price of the product is now changed to $p'_x = 2$. Find the price effect. Then decompose price effect into substitution effect and income effect.

10. Consider an otherwise identical Solow model of economic growth where the entire income is consumed.

    (a) Analyse how wage and rental rate on capital would change over time.

    (b) Can the economy attain steady state equilibrium?

SYLLABUS AND SAMPLE QUESTIONS FOR MSQE
(Program Code: MQEK and MQED)
2014

Syllabus for PEA (Mathematics), 2014

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations; (up to third degree).
Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.
Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.
Elementary Statistics: Elementary probability theory, measures of central tendency, dispersion, correlation and regression, probability distributions, standard distributions - Binomial and Normal.

Sample questions for PEA (Mathematics), 2014

1. Let $f(x) = \frac{1-x}{1+x}$, $x \neq -1$. Then $f(f(\frac{1}{x}))$, $x \neq 0$ and $x \neq -1$, is

   (a) 1,

   (b) x,

   (c) x^2,

   (d) $\frac{1}{x}$.

2. What is the value of the following definite integral?

$$2\int_0^{\frac{\pi}{2}} e^x \cos(x)dx.$$

   (a) $e^{\frac{\pi}{2}}$.

   (b) $e^{\frac{\pi}{2}} - 1$.

   (c) 0.

(d) $e^{\frac{\pi}{2}} + 1$.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as follows:

$$f(x) = |x - 1| + (x - 1).$$

Which of the following is not true for f?

(a) $f(x) = f(x')$ for all $x, x' < 1$.
(b) $f(x) = 2f(1)$ for all $x > 1$.
(c) f is not differentiable at 1.
(d) The derivative of f at $x = 2$ is 2.

4. Population of a city is 40 % male and 60 % female. Suppose also that 50 % of males and 30 % of females in the city smoke. The probability that a smoker in the city is male is closest to

(a) 0.5.
(b) 0.46.
(c) 0.53.
(d) 0.7.

5. A blue and a red die are thrown simultaneously. We define three events as follows:

- Event E: the sum of the numbers on the two dice is 7.
- Event F: the number on the blue die equals 4.
- Event G: the number on the red die equals 3.

Which of the following statements is true?

(a) E and F are disjoint events.
(b) E and F are independent events.

(c) E and F are not independent events.

(d) Probability of E is more than the probability of F.

6. Let $p > 2$ be a prime number. Consider the following set containing 2×2 matrices of integers:

$$T_p = \left\{ A = \begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} : a, b \in \{0, 1, \ldots, p-1\} \right\}.$$

A matrix $A \in T_p$ is p-special if determinant of A is not divisible by p. How many matrices in T_p are p-special?

(a) $(p-1)^2$.

(b) $2p - 1$.

(c) p^2.

(d) $p^2 - p + 1$.

7. A "good" word is any seven letter word consisting of letters from $\{A, B, C\}$ (some letters may be absent and some letter can be present more than once), with the restriction that A cannot be followed by B, B cannot be followed by C, and C cannot be followed by A. How many good words are there?

(a) 192.

(b) 128.

(c) 96.

(d) 64.

8. Let n be a positive integer and $0 < a < b < \infty$. The total number of real roots of the equation $(x-a)^{2n+1} + (x-b)^{2n+1} = 0$ is

(a) 1.

(b) 3.

(c) $2n - 1$.

(d) $2n + 1$.

9. Consider the optimization problem below:

$$\begin{aligned} \max_{x,y}\ & x + y \\ \text{subject to } & 2x + y \leq 14 \\ & -x + 2y \leq 8 \\ & 2x - y \leq 10 \\ & x, y \geq 0. \end{aligned}$$

The value of the objective function at optimal solution of this optimization problem:

(a) does not exist.

(b) is 8.

(c) is 10.

(d) is unbounded.

10. A random variable X is distributed in $[0, 1]$. Mr. Fox believes that X follows a distribution with cumulative density function (cdf) $F : [0, 1] \to [0, 1]$ and Mr. Goat believes that X follows a distribution with cdf $G : [0, 1] \to [0, 1]$. Assume F and G are differentiable, $F \neq G$ and $F(x) \leq G(x)$ for all $x \in [0, 1]$. Let $\mathbb{E}_F[X]$ and $\mathbb{E}_G[X]$ be the expected values of X for Mr. Fox and Mr. Goat respectively. Which of the following is true?

(a) $\mathbb{E}_F[X] \leq \mathbb{E}_G[X]$.

(b) $\mathbb{E}_F[X] \geq \mathbb{E}_G[X]$.

(c) $\mathbb{E}_F[X] = \mathbb{E}_G[X]$.

(d) None of the above.

11. Let $f : [0, 2] \to [0, 1]$ be a function defined as follows:

$$f(x) = \begin{cases} x & \text{if } x \leq \alpha \\ \frac{1}{2} & \text{if } x \in (\alpha, 2]. \end{cases}$$

where $\alpha \in (0, 2)$. Suppose X is a random variable distributed in $[0, 2]$ with probability density function f. What is the probability that the realized value of X is greater than 1?

(a) 1.

(b) 0.

(c) $\frac{1}{2}$.

(d) $\frac{3}{4}$.

12. The value of the expression

$$\sum_{k=1}^{100} \int_0^1 \frac{x^k}{k} dx$$

is

(a) $\frac{100}{101}$.

(b) $\frac{1}{99}$.

(c) 1.

(d) $\frac{99}{100}$.

13. Consider the following system of inequalities.

$$\begin{aligned} x_1 - x_2 &\leq 3 \\ x_2 - x_3 &\leq -2 \\ x_3 - x_4 &\leq 10 \\ x_4 - x_2 &\leq \alpha \\ x_4 - x_3 &\leq -4, \end{aligned}$$

where α is a real number. A value of α for which this system has a solution is

(a) -16.
(b) -12.
(c) -10.
(d) None of the above.

14. A fair coin is tossed infinite number of times. The probability that a head turns up for the first time after even number of tosses is

(a) $\frac{1}{3}$.
(b) $\frac{1}{2}$.
(c) $\frac{2}{3}$.
(d) $\frac{3}{4}$.

15. An entrance examination has 10 "true-false" questions. A student answers all the questions randomly and his probability of choosing the correct answer is 0.5. Each correct answer fetches a score of 1 to the student, while each incorrect answer fetches a score of zero. What is the probability that the student gets the mean score?

(a) $\frac{1}{4}$.
(b) $\frac{63}{256}$.
(c) $\frac{1}{2}$.
(d) $\frac{1}{8}$.

16. For any positive integer k, let S_k denote the sum of the infinite geometric progression whose first term is $\frac{(k-1)}{k!}$ and common ratio is $\frac{1}{k}$. The value of the expression $\sum_{k=1}^{\infty} S_k$ is

(a) e.
(b) $1+e$.
(c) $2+e$.
(d) e^2.

17. Let $G(x) = \int_0^x te^t dt$ for all non-negative real number x. What is the value of $\lim_{x\to 0} \frac{1}{x} G'(x)$, where $G'(x)$ is the derivative of G at x.

   (a) 0.
   (b) 1.
   (c) e.
   (d) None of the above.

18. Let $\alpha \in (0,1)$ and $f(x) = x^\alpha + (1-x)^\alpha$ for all $x \in [0,1]$. Then the maximum value of f is

   (a) 1.
   (b) greater than 2.
   (c) in $(1,2)$.
   (d) 2.

19. Let n be a positive integer. What is the value of the expression

$$\sum_{k=1}^{n} kC(n,k),$$

   where $C(n,k)$ denotes the number of ways to choose k out of n objects?

   (a) $n2^{n-1}$.
   (b) $n2^{n-2}$.
   (c) 2^n.
   (d) $n2^n$.

20. The first term of an arithmetic progression is a and common difference is $d \in (0,1)$. Suppose the k-th term of this arithmetic progression equals the sum of the infinite geometric progression whose first term is a and common ratio is d. If $a > 2$ is a prime number, then which of the following is a possible value of d?

(a) $\frac{1}{2}$.
(b) $\frac{1}{3}$.
(c) $\frac{1}{5}$.
(d) $\frac{1}{9}$.

21. In period 1, a chicken gives birth to 2 chickens (so, there are three chickens after period 1). In period 2, each chicken born in period 1 either gives birth to 2 chickens or does not give birth to any chicken. If a chicken does not give birth to any chicken in a period, it does not give birth in any other subsequent periods. Continuing in this manner, in period $(k + 1)$, a chicken born in period k either gives birth to 2 chickens or does not give birth to any chicken. This process is repeated for T periods - assume no chicken dies. After T periods, there are in total 31 chickens. The maximum and the minimum possible values of T are respectively

(a) 12 and 4.
(b) 15 and 4.
(c) 15 and 5.
(d) 12 and 5.

22. Let a and p be positive integers. Consider the following matrix

$$A = \begin{bmatrix} p & 1 & 1 \\ 0 & p & a \\ 0 & a & 2 \end{bmatrix}$$

If determinant of A is 0, then a possible value of p is

(a) 1.
(b) 2.
(c) 4.
(d) None of the above.

23. For what value of α does the equation $(x-1)(x^2-7x+\alpha)=0$ have exactly two unique roots?

    (a) 6.

    (b) 10.

    (c) 12.

    (d) None of the above.

24. What is the value of the following infinite series?

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k^2} \log_e 3^k.$$

    (a) $\log_e 2$.

    (b) $\log_e 2 \log_e 3$.

    (c) $\log_e 6$.

    (d) $\log_e 5$.

25. There are 20 persons at a party. Each person shakes hands with some of the persons at the party. Let K be the number of persons who shook hands with odd number of persons. What is a possible value of K?

    (a) 19.

    (b) 1.

    (c) 20.

    (d) All of the above.

26. Two independent random variables X and Y are uniformly distributed in the interval $[0,1]$. For a $z \in [0,1]$, we are told that probability that $\max(X,Y) \leq z$ is equal to the probability that $\min(X,Y) \leq (1-z)$. What is the value of z?

    (a) $\frac{1}{2}$.

(b) $\frac{1}{\sqrt{2}}$.

(c) any value in $[\frac{1}{2}, 1]$.

(d) None of the above.

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a function that satisfies for all $x, y \in \mathbb{R}$

$$f(x+y)f(x-y) = \big(f(x) + f(y)\big)^2 - 4x^2 f(y).$$

Which of the following is not possible for f?

(a) $f(0) = 0$.

(b) $f(3) = 9$.

(c) $f(5) = 0$.

(d) $f(2) = 2$.

28. Consider the following function $f : \mathbb{R} \to \mathbb{Z}$, where $\mathbb{R}$ is the set of all real numbers and $\mathbb{Z}$ is the set of all integers.

$$f(x) = \lceil x \rceil,$$

where $\lceil x \rceil$ is the smallest integer that is larger than x. Now, define a new function g as follows. For any $x \in \mathbb{R}$, $g(x) = |f(x)| - f(|x|)$, where $|x|$ gives the absolute value of x. What is the range of g?

(a) $\{0, 1\}$.

(b) $[-1, 1]$.

(c) $\{-1, 0, 1\}$.

(d) $\{-1, 0\}$.

29. The value of $\lim_{x \to -1} \frac{x+1}{|x+1|}$ is.

(a) 1.

(b) -1.

(c) 0.

(d) None of the above.

30. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(x) = 2$ if $x \leq 2$ and $f(x) = a^2 - 3a$ if $x > 2$, where a is a positive integer. Which of the following is true?

(a) f is continuous everywhere for some value of a.

(b) f is not continuous.

(c) f is differentiable at $x = 2$.

(d) None of the above.

Syllabus for PEB (Economics), 2014

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM Model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample questions for PEB (Economics), 2014

1. Consider a firm that can sell in the domestic market where it is a monopolist, and/or in the export market. The domestic demand is given by $p = 10 - q$, and export price is 5. Suppose the firm has a constant marginal cost of 4 and a capacity constraint on output of 100.

    (a) Solve for the optimal production plan of the firm. [**15 marks**]

    (b) Solve for the optimal production plan of the firm if its constant marginal cost is 6. [**5 marks**]

2. (a) Consider a consumer who can consume either A or B, with the quantities being denoted by a and b respectively. If the utility function of the consumer is given by

    $$-\left[(10-a)^2 + (10-b)^2\right].$$

    Suppose prices of both the goods are equal to 1.

    i. Solve for the optimal consumption of the consumer when his income is 40. [**10 marks**]

    ii. What happens to his optimal consumption when his income goes down to 10. [**5 marks**]

    (b) A monopolist faces the demand curve $q = 60 - p$ where p is measured in rupees per unit and q in thousands of units. The monopolist's total cost of production is given by $C = \frac{1}{2}q^2$.

i. What is the deadweight loss due to monopoly? [**3 marks**]

ii. Suppose a government could set a price ceiling (maximum price) that the monopolist can charge. Find the price ceiling that the government should set to minimize the deadweight loss. [**2 marks**]

3. (a) A cinema hall has a capacity of 150 seats. The owner can offer students a discount on the price when they show their student IDs. The demand for tickets from students is

$$D_s = 220 - 40P_s,$$

where P_s is the price of tickets for students after the discount. The demand for tickets for non-students is

$$D_n = 140 - 20P_n,$$

where P_n is the price of tickets for non-students.

i. What is the maximum profit the owner can make? [**8 marks**]

ii. What is the maximum profit he could make if the demand functions of students and non-students were interchanged? [**4 marks**]

(b) There are 11 traders and 6 identical (indivisible) chickens. Each trader wants to consume at most one chicken. There is also a (divisible) good called "money". Let D_i equal to 1 indicate that trader i consumes a chicken; 0 if he does not. Trader $i's$ utility function is given by $u_i D_i + m_i$, where u_i is the value he attaches to consuming a chicken, m_i is the units of money that the trader has. The valuations for the 11 traders are:

$u_1 = 10$; $u_2 = 8$; $u_3 = 7$; $u_4 = 4$; $u_5 = 3$; $u_6 = 1$; $u_7 = u_8 = 3$; $u_9 = 5$; $u_{10} = 6$; $u_{11} = 8$.

Initially each trader is endowed with 25 units of money. Traders $6, 7, 8, 9, 10, 11$ are endowed with one chicken each.

i. What is a possible equilibirum market price (units of money per chicken) in a competitive market? [**4 marks**]

ii. Is the equilibrium unique? [**4 marks**]

4. (a) Consider a monopolist who faces a market demand for his product:

$$p(q) = 20 - q,$$

where p is the price and q is the quantity. He has a production function given by

$$q = \min\left\{\frac{L}{2}, \frac{K}{3}\right\},$$

where L denotes labour and K denotes capital. There is a physical restriction on the availability of capital, that is, $\overline{K}$. Let both wage rates (w) and rental rates (r) be equal to 1. Find the monopoloy equilibrium quantity and price when (i) when $\overline{K} = 24$; (ii) $\overline{K} = 18$. [**12 marks**]

(b) Define Samuelson's Weak Axiom of Revealed Preference (WARP). [**2 marks**]

(c) Prove that WARP implies non-positivity of the own-price substitution effect and the demand theorem. [**6 marks**]

5. Consider two firms: 1 and 2, with their output levels denoted by q_1 and q_2. Suppose both have identical and linear cost functions, $C(q_i) = q_i$. Let the market demand function be $q = 10 - p$, where q denotes aggregate output and p the market price.

(a) Suppose the firms simultaneously decide on their output levels. Define the equilibrium in this market. Solve for the reaction functions of the two fims. Using these, find the equilibrium. [**10 marks**]

(b) Suppose the firms still compete over quantities, but both have a capacity constraint at an output level of 2. Find these reaction functions and the equilibrium in this case. [**10 marks**]

6. (a) Suppose the government subsidizes housing expenditures of low-income families by providing them a rupee-for-rupee subsidy for

their expenditure. The Lal family qualifies for this subsidy. They spend Rs. 250 on housing, and receive Rs. 250 as subsidy from the government.

Recently, a new policy has been proposed to replace the earlier policy. The new policy proposes to provide each low income family with a lump-sum transfer of Rs. 250, which can be used for housing or other goods.

i. Explain graphically if the Lal family would prefer the current program over the proposed program. [**6 marks**]

ii. Can they be indifferent between the two programs? [**3 marks**]

iii. Does the optimal consumption of housing and other goods change compared to the subsidy scheme? If yes, how? [**3 marks**]

(b) A drug company company is a monopoly supplier of Drug X which is protected by a patent. The demand for the drug is

$$p = 100 - X$$

and the monopolist's cost function is

$$C = 25 + X^2$$

i. Determine the profit maximizing price and quantity of the monopolist. [**2 marks**]

ii. Suppose the patent expires at a certain point in time, and after that any new drug company can enter the market and produce Drug X, facing the same cost function. What will be the competitive equilibrium industry output and price? How many firms will be there in the market? [**6 marks**]

7. Assume that an economy's production function is given by

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}$$

where Y_t is output at time t, K_t is the capital stock at time t and N is the *fixed* level of employment (number of workers), $\alpha \in (0, 1)$ is the share of output paid to capital. The evolution of the capital stock is given by

$$K_{t+1} = (1 - \delta) K_t + I_t$$

where I_t is investment at time t and $\delta \in [0, 1]$ is the depreciation rate.

(a) Derive an expression for $\frac{Y}{N}$. [**5 marks**]

(b) How large is the effect of an increase in the savings rate on the steady state level of output per worker. [**10 marks**]

(c) What is the savings rate that would maximize steady state consumption per worker? [**5 marks**]

8. In an IS-LM model, graphically compare the effect of an expansionary monetary policy with an expansionary fiscal policy on investment (I) in (1) the short-run and (2) the medium run (where the aggregate supply and aggregate demand curves adjust). Assume that

$$I = I(i, Y),$$

where i is the interest rate and Y is the output. Also, $\frac{\partial I}{\partial i} < 0$ and $\frac{\partial I}{\partial Y} > 0$. [**15 marks**]

Under which policy (expansionary monetary or fiscal), is the investment higher in the medium run? [**5 marks**]

9. Suppose the economy is characterized by the following equations:

$$\begin{aligned} C &= c_0 + c_1 Y_D \\ Y_D &= Y - T \\ I &= b_0 + b_1 Y, \end{aligned}$$

where C is consumption, Y is the income, Y_D is the disposable income, T is tax, I is investment, and c_0, c_1, b_0, b_1 are positive constants with $c_1 < 1, b_1 < 1$. Government spending is constant.

(a) Solve for equilibrium output. [**5 marks**]

(b) What is the value of the multiplier? For the multiplier to be positive, what condition must $c_1 + b_1$ satisfy? [**5 marks**]

(c) How will equilibrium output be affected when b_0 is changed? What will happen to saving? [**5 marks**]

(d) Instead of fixed T, suppose $T = t_0 + t_1 Y$, where $t_0 > 0$ and $t_1 \in (0, 1)$. What is the effect of increase in b_0 on equilibrium Y? Is it larger or smaller than the case where taxes are autonomous? [**5 marks**]

10. Consider an economy where a representitive agent lives for three periods. In the first period, she is young - this is the time when she gets education. In the second period, she is middle-aged and with the level of education acquired in the first period, she generates income. More specifically, if she has h units of education in the first period, she can earn $\overline{w}h$, in the second period, where $\overline{w}$ is the exogenously given wage rate.

    The agent borrows funds for her education when she is young and repays with interest when she is middle aged. If in the first period, the agent borrows e, then the human capital h at the beginning of the second period becomes $h(e)$, where $\frac{dh}{de} > 0$ along with $\frac{d^2h}{de^2} < 0$.

    In the third period of her life, she consumes out of her savings made in the second period, that is, when she was middle aged. Assume that the exogenous rate of interest (gross) on saving or borrowing is $\overline{R}$. For simplicity, assume that an agent does not consume when she is young and, thus, the life time utility is $u(c^M) + \beta u(c^O)$, where c^M and c^O are the level of consumption when they are middle-aged and old respectively and $\beta \in (0, 1)$ is the discount factor.

    (a) Write down the utility maximization problem of the agent and the first order conditions. [**10 marks**]

    (b) How does the optimal level of education vary with the wage rate and the rate of interest? [**10 marks**]

SYLLABUS AND SAMPLE QUESTIONS FOR MSQE
(Program Code: MQEK and MQED)
2015
Syllabus for PEA (Mathematics), 2015

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Polynomial Equations; (up to third degree).

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants. Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution, Constrained optimization of functions of not more than two variables.

Elementary Statistics: Elementary probability theory, measures of central tendency, dispersion, correlation and regression, probability distributions, standard distributions-Binomial and Normal.

Sample questions for PEA (Mathematics), 2015

1. $\lim_{x\to 0+} \frac{\sin\{\sqrt{x}\}}{\{\sqrt{x}\}}$, where $\{x\}$ = decimalpart of x, is

   (a) 0 (b) 1 (c) non-existent (d)none of these

2. $f:[0,\ 1]\to[0,\ 1]$ is continuous.Then it is true that

   (a) $f(0)=0,\ f(1)=1$
   (b) f is differentiable only at $x=\frac{1}{2}$
   (c) $f'(x)$ is constant for all $x \in (0,1)$
   (d) $f(x) = x$ for at least one $x \in [0,1]$

3. $f(x)=|x-2|+|x-4|$.Then f is
   (a) continuously differentiable at x = 2
   (b) differentiable but not continuously differentiable at $x = 2$
   (c) f has both left and right derivatives at $x = 2$
   (d) none of these

4. In an examination of 100 students, 70 passed in Mathematics, 65 passed in Physics and 55 passed in Chemistry. Out of these students, 35 passed in all the three subjects, 50 passed in Mathematics and Physics, 45 passed in Mathematics and Chemistry and 40 passed in Physics and Chemistry.Then the number of students who passed in exactly one subject is

(a) 30 (b) 25 (c) 10 (d) none of these

5. The square matrix of the matrix $\begin{vmatrix} a & b \\ c & 0 \end{vmatrix}$ is a null matrix if and only if

(a) $a = b = c = 0$
(b) $a = c = 0$, b is any non-zero real number
(c) $a = b = 0$, c is any non-zero real number
(d) $a = 0$ and either $b = 0$ or $c = 0$

6. If the positive numbers x, y, z are in harmonic progression, then $\log(x+z) + \log(x-2y+z)$ equals

(a) $4\log(x-z)$ (b) $3\log(x-z)$
(c) $2\log(x-z)$ (d) $\log(x-z)$

7. If f (x+2y, x-2y)= xy, then f (x,y) equals

(a) $\frac{x^2-y^2}{8}$ (b) $\frac{x^2-y^2}{4}$ (c) $\frac{x^2+y^2}{4}$ (d) none of these

8. The domain of the function $f(x) = \sqrt{x^2-1} - \log\left(\sqrt{1-x}\right)$,x≥ 0,is

(a) $(-\infty, -1)$ (b) $(-1, 0)$ (c) null set (d) none of these

9. The graph of the function $y = \log(1-2x+x^2)$ intersects the x axis at

(a) 0, 2 (b) 0,- 2 (c) 2 (d) 0

10. The sum of two positive integers is 100.The minimum value of the sum of their reciprocals is

(a) $\frac{3}{25}$ (b) $\frac{6}{25}$ (c) $\frac{1}{25}$ (d) none of these

11. The range of the function $f(x) = 4^x + 2^x + 4^{-x} + 2^{-x} + 3$, where $x \in (-\infty, \infty)$, is

(a) $\left(\frac{3}{4}, \infty\right)$ (b) $\left[\frac{3}{4}, \infty\right)$ (c) $(7, \infty)$ (d) $[7, \infty)$

12. The function $f : R \to R$ satisfies $f(x+y) = f(x) + f(y) \forall x, y \in R$, where R is the real line, and $f(1) = 7$. Then $\sum_{r=1}^{n} f(r)$ equals

(a) $\frac{7n}{2}$ (b) $\frac{7(n+1)}{2}$ (c) $\frac{7n(n+1)}{2}$ (d) $7n(n+1)$

13. Let f and g be differentiable functions for $0 < x < 1$ and $f(0) = g(0) = 0, f(1) = 6$ .Suppose that for all $x \in (0, 1)$, the equality $f'(x) = 2g'(x)$ holds. Then $g(1)$ equals

(a) 1 (b) 3 (c) -2 (d) -1

14. Consider the real valued function $f(x) = ax^2 + bx + c$ defined on [1, 2].Then it is always possible to get a $k \in (1, 2)$ such that

(a) $k = 2a + b$ (b) $k = a + 2b$ (c) $k = 3a + b$ (d) none of these

15. In a sequence the first term is $\frac{1}{3}$.The second term equals the first term divided by 1 more than the first term. The third term equals the second term divided by 1 more than the second term, and so on. Then the 500thterm is

(a) $\frac{1}{503}$ (b) $\frac{1}{501}$ (c) $\frac{1}{502}$ (d) none of these

16. In how many ways can three persons, each throwing a single die once, make a score of 10?

(a) 6 (b) 18 (c) 27 (d) 36

17. Let a be a positive integer greater than 2.The number of values of x for which $\int_a^x (x+y)dy = 0$ holds is

(a) 1 (b) 2 (c) a (d) a + 1

18. Let (x^*, y^*) be a solution to any optimization problem $max_{(x,y)\in\Re^2} f(x,y)$ subject to $g_1(x,y) \le c_1$. Let (x', y') be a solution to the same optimization problem $max_{(x,y)\in\Re^2} f(x,y)$ subject to $g_1(x,y) \le c_1$ with an added constraint that $g_2(x,y) \le c_2$.Then which one of the following statements is always true?

(a) $f(x^*, y^*) \ge f(x', y')$ (b) $f(x^*, y^*) \le f(x', y')$
(c) $|f(x^*, y^*)| \ge |f(x', y')|$ (d) $|f(x^*, y^*)| \le |f(x', y')|$

19. Let (x^*, y^*) be a real solution to: $max_{(x,y)\in\Re^2} \sqrt{x} + y$ subject to $px + y \le m$, where $m > 0, p > 0$ and $y^* \in (0, m)$.Then which one of the following statements is true?

(a) x^*depends only on p (b) x^* depends only on m
(c) x^* depends on both p and m (d) x^* is independent of both p and m.

20. Let $0 < a_1 < a_2 < 1$ and let $f(x; a_1, a_2) = -|x - a_1| - |x - a_2|$.Let X be the set of all values of x for which$f(x; a_1, a_2)$ achieves its maximum. Then

(a) $X = \{x | x \in \left\{\frac{a_1}{2}, \frac{1+a_2}{2}\right\}\}$ (b) $X = \{x | x \in \{a_1, a_2\}\}$
(c) $X = \{x | x \in \left\{0, \frac{a_1+a_2}{2}, 1\right\}\}$ (d) $X = \{x | x \in [a_1, a_2]\}$.

21. Let A and B be two events with positive probability each, defined on the same sample space. Find the correct answer:

(a) P(A/B) > P(A) always (b) P(A/B) < P(A) always
(c) P(A/B) > P(B) always (d) None of the above

22. Let A and B be two mutually exclusive events with positive probability each, defined on the same sample space. Find the correct answer:

(b) A and B are necessarily independent
(c) A and B are necessarily dependent
(d) A and B are necessarily equally likely
(e) None of the above

23. The salaries of 16 players of a football club are given below (units are in thousands of rupees).

100, 100, 111, 114, 165, 210, 225, 225, 230,
575, 1200, 1900, 2100, 2100, 2650, 3300

Now suppose each player received an extra Rs. 200,000 as bonus. Find the correct answer:

(a) Mean will increase by Rs. 200,000 but the median will not change
(b) Both mean and median will be increased by Rs. 200,000
(c) Mean and standard deviation will both be changed
(d) Standard deviation will be increased but the median will be unchanged

24. Let Pr(X=2)= 1. Define $\mu_{2n} = E(X-\mu)^{2n}, \mu = E(X)$. Then:

(a) μ_{2n}=2 (b) μ_{2n}=0 (c) $\mu_{2n} > 0$ (d) None of the above

25. Consider a positively skewed distribution. Find the correct answer on the position of the mean and the median:

(a) Mean is greater than median (b) Mean is smaller than median
(c) Mean and median are same (d) None of the above

26. Puja and Priya play a fair game (i.e. winning probability is ½ for both players) repeatedly for one rupee per game. If originally Puja has *a* rupees and Priya has *b* rupees (where a>b), what is Puja's chances of winning all of Priya's money, assuming the play goes on until one person has lost all her money?

(a) 1 (b) 0 (c) b/(a+b) (d) a/(a+b)

27. An urn contains *w* white balls and *b* black balls (*w>0*) and (*b>0*).The balls are thoroughly mixed and two are drawn, one after the other, *without* replacement. Let *Wi* denote the outcome 'white on the *i*-th draw' for *i*=1,2.Which one of the following is true?

(a) P(W2) = P(W1) = w/(w+b)
(b) P(W2) = P(W1) = (w-1)/(w+b-1)
(c) P(W1)= w/(w+b), P(W2)= (w-1)/(w+b-1)
(d) P(W1)= w/(w+b), P(W2) = {w(w-1)}/{(w-b)(w+b-1)}

28. A bag contains four pieces of paper, each labeled with one of the digits 1, 2, 3, 4, with no repeats. Three of these pieces are drawn, one at a time without replacement, to construct a three-digit number. What is the probability that the three-digit number is a multiple of 3?

(a) ¾ (b) ½ (c) ¼ (d) 9/24

29. Consider two random variables X and Y where X takes values -2,-1,0,1,2 each with probability 1/5 and Y=|X|. Which of the following is true?
   (a) The variables X and Yare independent and Pearson's correlation coefficient between X and Y is 0.
   (b) The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 0.
   (c) The variables X and Y are independent and Pearson's correlation coefficient between X and Y is 1.
   (d) The variables X and Y are dependent and Pearson's correlation coefficient between X and Y is 1.
30. Two friends who take the metro to their jobs from the same station arrive to the station uniformly randomly between 7 and 7:20 in the morning. They are willing to wait for one another for 5 minutes, after which they take a train whether together or alone. What is the probability of their meeting at the station?

   (a) 5/20   (b) 25/400   (c) 10/20   (d) 7/16

Syllabus for PEB (Economics), 2015

Microeconomics: Theory of consumer behaviour, theory of production, market structure under perfect competition, monopoly, price discrimination, duopoly with Cournot and Bertrand competition (elementary problems) and welfare economics.

Macroeconomics: National income accounting, simple Keynesian Model of income determination and the multiplier, IS-LM model, models of aggregate demand and aggregate supply, Harrod-Domar and Solow models of growth, money, banking and inflation.

Sample questions for PEB (Economics), 2015

1. Consider an agent in an economy with two goods X_1 and X_2.Suppose she has income 20.Suppose also that when she consumes amounts x_1 and x_2 of the two goods respectively, she gets utility

$$u(x_1, x_2) = 2x_1 + 32x_2 - 3{x_2}^2.$$

   (a) Suppose the prices of X_1 and X_2 are each 1.What is the agent's optimal consumption bundle? [**5 marks**]
   (b) Suppose the price of X_2 increases to 4, all else remaining the same.Which consumption bundle does the agent choose now? [**5 marks**]
   (c) How much extra income must the agent be given to compensate her for the increase in price of X_2? [**10 marks**]

2. Suppose a government agency has a monopoly in the provision of internet connections. The marginal cost of providing internet connections is ½, whereas the inverse demand function is given by: $p = 1 - q$. The official charge per connection is set at 0; thus, the state provides a subsidy of ½ per connection. However, the state can only provide budgetary support for the supply of 0.4 units, which it raises through taxes on consumers. Bureaucrats in charge of sanctioning internet connections are in a position to ask for bribes, and consumers are willing to pay them in order to get connections. Bureaucrats cannot, however, increase supply beyond 0.4 units.

(a) Find the equilibrium bribe rate per connection and the social surplus. [**5 marks**]

(b) Now suppose the government agency is privatized and the market is deregulated; however, due large fixed costs of entry relative to demand, the privatized company continues to maintain its monopoly. Find the new equilibrium price, bribe rate and social surplus, specifying whether privatization increases or reduces them. [**10 marks**]

(c) Suppose now a technological innovation becomes available to the privatized monopoly, which reduces its marginal cost of providing an internet connection to *c*, $0 < c < ½$.Find the range of values of c for which privatization increases consumers' surplus. [**5 marks**]

3. Suppose the borders of a state, B, coincide with the circumference of a circle of radius r > 0, and its population is distributed uniformly within its borders (so that the proportion of the population living within some region of B is simply the proportion of the state's total land mass contained in that region), with total population normalized to 1.For any resident of B, the cost of travelling a distance d is kd, with $k>0$.Every resident of B is endowed with an income of 10, and is willing to spend up to this amount to consume one unit of a good, G, which is imported from outside the state at zero transport cost. The Finance Minister of B has imposed an entry tax at the rate $100t\%$ on shipments of G brought into B. Thus, a unit of G costs $p(1+t)$ inside the borders of B, but can be purchased for just p outside; $p(1+t) < 10$.Individual residents of B have to decide whether to travel beyond its borders to consume the good or to purchase it inside the state. Individuals can travel anywhere to shop and consume, but have to return to their place of origin afterwards.

(a) Find the proportion of the population of B which will evade the entry tax by shopping outside the state. [**5 marks**]

(b) Find the social welfare-maximizing tax rate. Also find the necessary and sufficient conditions for it to be identical to the revenue-maximizing tax rate. [**5 marks**]

(c) Assume that the revenue-maximizing tax rate is initially positive. Find the elasticity of tax revenue with respect to the external price of G, supposing that the Finance Minister always chooses the revenue-maximizing tax rate. [**10 marks**]

4. Suppose there are two firms, 1 and 2, each producing chocolate, at 0 marginal cost. However, one firm's product is not identical to the product of the other. The inverse demand functions are as follows:

$$p_1 = A_1 - b_{11}q_1 - b_{12}q_2, p_2 = A_2 - b_{21}q_1 - b_{22}q_2;$$

where p_1 and q_1 are respectively price obtained and quantity produced by firm 1 and p_2 and $q_2$are respectively price obtained and quantity produced by firm 2.$A_1, A_2, b_{11}, b_{12}, b_{21}, b_{22}$are all positive.Assume the firms choose independently how much to produce.

(a) How much do the two firms produce, assuming both produce positive quantities? [**10 marks**]

(b) What conditions on the parameters $A_1, A_2, b_{11}, b_{12}, b_{21}, b_{22}$ are together both necessary and sufficient to ensure that both firms produce positive quantities? [**5 marks**]

(c) Under what set of conditions on these parameters does this model reduce to the standard Cournot model? [**5 marks**]

5. Suppose a firm manufactures a good with labor as the only input. Its production function is $Q = L$, where Q is output and L is total labor input employed. Suppose further that the firm is a monopolist in the product market and a monopsonist in the labor market. Workers may be male (M) or female (F); thus, $L = L_M + L_F$.Let the inverse demand function for output and the supply functions for gender-specific labor be respectively $p = A - \frac{Q}{2}$; $L_i = w_i^{\varepsilon_i}$, $\varepsilon_i > 0$; where p is the price received per unit of the good and w_i is the wage the firm pays to each unit of labor of gender i, $i \in \{M, F\}$.Let $\varepsilon_M \varepsilon_F = 1$.Suppose, in equilibrium, the firm is observed to hire both M and F workers, but pay M workers double the wage rate that it pays F workers.

(a) Derive the exact numerical value of the elasticity of supply of male labor. [**10 marks**]

(b) What happens to total *male* labor income as a proportion of total labor income when the output demand parameter A increases? Prove your claim. [**10 marks**]

6. An economy comprises of a consolidated household sector, a firm sector and the government. The household supplies labour (L) to the firm.The firm produces a single good (Y) by means of a production function$Y = F(L); F' > 0, F'' < 0$, and maximizes profits $\Pi = PY - WL$,where P is the price of Y and W is the wage rate.The household, besides earning wages, is also entitled to the profits of the firm. The household maximizes utility (U), given by:

$$U = \frac{1}{2}\ln C + \frac{1}{2}\ln\frac{M}{P} - d(L);$$

where C is consumption of the good and $\frac{M}{P}$ is real balance holding. The term $d(L)$ denotes the disutility from supplying labour; with $d' > 0, d'' > 0$.The household's budget constraint is given by:

$$PC + M = WL + \Pi + \overline{M} - PT;$$

where $\overline{M}$ is the money holding the household begins with, M is the holding they end up with and T is the real taxes levied by the government. The government's demand for the good is given by G.The government's budget constraint is given by:

$$M - \overline{M} = PG - PT.$$

Goods market clearing implies: $Y = C + G$.

(a) Prove that $\frac{dY}{dG} \in (0,1)$, and that government expenditure crowds out private consumption (i.e., $\frac{dC}{dG} < 0$). [**15 marks**]

(b) Show that everything else remaining the same, a rise in $\overline{M}$ leads to an equi-proportionate rise in P. [**5 marks**]

7. Consider the Solow growth model in continuous time, where the exogenous rate of technological progress, g, is zero. Consider an intensive form production function given by:

$$f(k) = k^4 - 6k^3 + 11k^2 - 6k; \qquad (1)$$

where $k = \frac{K}{L}$ (the capital labour ratio).

(a) Specify the assumptions made with regard to the underlying extensive form production function $F(K, L)$ in the Solow growth model, and explain which ones among these assumptions are violated by (1). [**10 marks**]

(b) Graphically show that, with a suitable value of $(n + \delta)$, where n is the population growth rate, and $\delta \in [0,1]$ is the depreciation rate on capital, there exist three steady state equilibria. [**5 marks**]

(c) Explain which of these steady state equilibria are locally unstable, and which are locally stable. Also explain whether any of these equilibria can be globally stable. [**5 marks**]

8. Consider a standard Solow model in discrete time, with the law of motion of capital is given by

$$K(t+1) = (1-\delta)K(t) + I(t),$$

where $I(t)$ is investment at time *t* and$K(t)$ is the capital stock at time *t*; the capital stock depreciates at the rate $\delta \in [0,1]$.Suppose output, $Y(t)$, is augmented by government spending, $G(t)$, in every period, and that the economy is closed; thus:

$$Y(t) = C(t) + I(t) + G(t),$$

where $C(t)$ is consumption at time *t*. Imagine that government spending is given by:

$$G(t) = \sigma Y(t),$$

where $\sigma \in [0,1]$.

(a) Suppose that: $C(t) = (\emptyset - \lambda\sigma)Y(t)$; where $\lambda \in [0,1]$.Derive the effect of higher government spending (in the form of higher σ) on the steady state equilibrium. [**10 marks**]

(b) Does a higher σ lead to a lower value of the capital stock in every period (i.e, along the entire transition path)? Prove your claim. [**10 marks**]