Tutorial Worksheet 3 - Testing of Hypothesis (with Solutions)

Objective Questions

1. Consider a hypothesis H_0 where $\phi_0 = 5$ against H_1 where $\phi_1 > 5$. The test is?

   (a) Right tailed (b) Left tailed (c) Two tailed (d) Cross tailed

   Solution: Right tailed

2. Type 1 error occurs when?

   (a) We reject H_0 if it is True (b) We reject H_0 if it is False (c) We accept H_0 if it is True (d) We accept H_0 if it is False

   Solution: We reject H_0 if it is True

3. The probability of Type 1 error is referred as?

   (a) 1- α (b) β (c) α (d) Variance

   Solution: α

4. What is the assumption made for performing the hypothesis test with T-distribution?

   (a) the distribution is non-symmetric (b) the distribution has more than one modal class (c) the distribution has a constant variance (d) the distribution follows a normal distribution

   Solution: the distribution follows a normal distribution

5. If a hypothesis is rejected at 0.6 Level of Significance then

   (a) it will be rejected at any level (b) it must be rejected at 0.5 level (c) it may be rejected at 0.5 level (d) it cannot be rejected at 0.5 level

   Solution: it may be rejected at 0.5 level

6. In a two-tailed test when a Null Hypothesis is rejected for a True Alternative Hypothesis then it has

   (a) Type 1 error (b) Type 2 error (c) No error (d) Many errors

   Solution: No error

7. In a hypothesis test, what does the p value signify?

   (a) smallest level of significance for rejection of Null Hypothesis.
   (b) largest level of significance for rejection of Null Hypothesis.
   (c) smallest level of significance for acceptance of Null Hypothesis.
   (d) smallest level of significance for acceptance of Null Hypothesis.

   Solution: smallest level of significance for rejection of Null Hypothesis.

8. The choice of one-tailed test and two-tailed test depends upon

   (a) Null hypothesis (b) Alternative hypothesis (c) Composite hypothesis (d) None of these

   Solution: Alternative hypothesis

9. Arrange the following steps in the process of hypothesis testing in proper sequence:

(A) Select the level of significance
(B) Setup null and alternative hypothesis
(C) Establish the decision rule
(D) Performance computations
(E) Select test statistics
(F) Draw conclusion

Choose the correct answer from the options given below

(a) A, B, C, D, E, F (b) A, B, E, D, C, F (c) B, A, C, D, E, F (d) B, A, E, C, D, F

Solution: B, A, E, C, D, F

10. Match the items of List - II with the items of List - I and select the code of correct matching Some matchings from

List - I		List - II	
(a)	Chi-square Test	(i)	Testing the significance of the differences of the mean values among more than two sample groups.
(b)	ANOVA (F-test)	(ii)	Testing the goodness of fit of a distribution
(c)	Z-test	(iii)	Testing the significance of the difference of the mean values between two large sized samples
(d)	t - test	(iv)	Testing the significance of the difference of the mean values between two small sized samples when population standard deviation is not available

Column A and Column B are given below. Select the correct matching?

(A) (a) - (i), (b) - (ii), (c) - (iii), (d) - (iv) (B) (a) - (iv), (b) - (iii), (b) - (ii), (d) - (i) (C) (a) - (i), (b) - (ii), (c) - (iv), (d) - (iii) (D) (a) - (ii), (b) - (i), (c) - (iii), (d) - (iv)

Solution: (a) - (ii), (b) - (i), (c) - (iii), (d) - (iv)

Subjective Questions

Problem 1.

A drug company is testing a drug intended to increase heart rate. A sample of 100 yielded a mean increase of 1.4 beats per minute, with a standard deviation known to be 3.6. Since the company wants to avoid marketing an ineffective drug, it proposes a 0.001 significance level. Should it market the drug?

Solution:

In this problem we want to test, $H_0 : \mu = 0$ vs $H_1 : \mu > 0$, i.e., a right tailed test. Here, the significant level $\alpha = 0.001, n = 100, \sigma = 3.6$.
Under $H_0 : \mu = 0$, the test statistic $Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \sim N(0, 0)$.
Here, $Z = \frac{1.4-0}{\frac{3.6}{\sqrt{100}}} = 3.8889$.
Since, $Z_\alpha = z_{0.001} = 3.09$ we have $Z_{cal} > z_\alpha$. $\Rightarrow$ It falls in the rejection region, hence reject H_0.
At the $\alpha = 0.001$ level of significance, there exists enough evidence to conclude that the drug increases heart rate. Therefore, the company should market this drug.

Problem 2.

The accompanying data on the breakdown voltage of electrically stressed circuits was read from a normal probability plot that appeared in the article "Damage of Flexible Printed Wiring Boards Associated with Lightning-Induced Voltage Surges" (IEEE Transactions on Components, Hybrids, and Manuf. Tech., 1985: 214-220). The straightness of the plot gave strong support to the assumption that breakdown voltage is approximately normally distributed.

$$1470, 1510, 1690, 1740, 1900, 2000, 2030, 2100, 2190, 2200, 2290, 2380, 2390, 2480, 2500, 2580, 2700$$

(a) $P(\bar{X} \geq 2200)$

(b) $P(\bar{X} \leq 2000)$

(c) $P(1900 \leq \bar{X} \leq 2100)$

(d) Confidence Interval of the population mean at 95%

Solution: From the given data the calculated mean is 2126.471 and the standard deviation is 370.5729.

(a) $P(\bar{X} \geq 2200) = P(\frac{\bar{X}-2126.471}{370.5729/\sqrt{17}} \geq \frac{2200-2126.471}{370.5729/\sqrt{17}}) = P(Z \geq 0.81811) = 0.2061.$

(b) $P(\bar{X} \leq 2000) = P(\frac{\bar{X}-2126.471}{370.5729/\sqrt{17}} \leq \frac{2000-2126.471}{370.5729/\sqrt{17}}) = P(Z \leq -1.4072) = 0.0793.$

(c) $P(1900 \leq \bar{X} \leq 2100) = P(\frac{1900-2126.471}{370.5729/\sqrt{17}} \leq \frac{\bar{X}-2126.471}{370.5729/\sqrt{17}} \leq \frac{2100-2126.471}{370.5729/\sqrt{17}}) = P(-2.52 \leq Z \leq 0.29) = 0.38.$

(d) The confidence interval is given by

$$\bar{X} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 2126.471 \pm 1.96\frac{370.5729}{\sqrt{17}} = (1950.311,\ 2303.63)$$

Problem 3.

A company manufacturing automobile tyres finds that tyre-life is normally distributed with a mean of 40,000 km and a standard deviation of 3000 km. It is believed that a change in the production process will result in a better product and the company has developed a new tyre. A sample of 100 new tyres has been selected. The company has found that the mean life of these new tyres is 40,900 km. Can it be concluded that the new tyre is significantly better than the old one, using the significant level of 0.01?

Solution:

In this problem we want to test, $H_0 : \mu = 40000$ vs $H_1 : \mu > 40000$, i.e., a right tailed test. Here, the significant level $\alpha = 0.01, n = 100, \sigma = 3000$.
Under $H_0 : \mu = 140$, the test statistic $Z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} \sim N(0, 0)$.
Here, $Z = \frac{40{,}900-40{,}000}{\frac{3000}{\sqrt{100}}} = 3$.
Since, $Z_\alpha = z_{0.01} = 2.33$ we have $Z_{cal} > z_\alpha$. $\Rightarrow$ It falls in the rejection region, hence reject H_0.
Hence, we can conclude that the new tyre is significantly better than the old one.

Problem 4.

The Environmental Protection Department requires a mill to aerate its effluent so that the mean dissolved oxygen (DO) level is above 6.0 mg/L . To maintain the compliance, the Department collects air samples at 12 randomly selected dates. The data collected is given as

$$5.85, 6.28, 6.50, 6.21, 5.94, 6.12, 6.65, 6.14, 6.34, 6.19, 6.29, 6.40$$

The Department requires a strong evidence that the mean DO is high. It is required to test it with significance level, $\alpha = 1\%$. Answer the following questions. Assume that the population data follows normal distribution.

(a) What should be the null and alternate hypothesis?

(b) Which test statistics: z, t or χ^2 test is applicable in this case?

(c) According to your test statistics, calculate the critical value.

(d) From the sample data, calculate the test value.

(e) Decide if the null hypothesis is to be accepted or rejected.

Solution:

(a) The null hypothesis, H_0 and alternate hypothesis, H_1 are given below. $\mathrm{H}_0 : \mu = 6.0; \mathrm{H}_1 : \mu > 6.0$

(b) For the given data, the population mean is to be inferred and population variance is not known. Hence, it comes under t-test.

(c) With degree of freedom = 11 and $\alpha = 0.01$, from the t-distribution table, the critical t-value, $t_c = 2.7181$ for one tailed test.

(d) From the sample data, $\bar{x} = 6.2425, s^2 = 0.04957$ and n = 12. Hence t = $\frac{\bar{x}-\mu}{s\sqrt{n}} = 3.773$

(e) Since $t > t_c$, the null hypothesis is rejected. This means that the mill does not violate the Department's claim.

Problem 5.

It is claimed that an automobile is driven on average more than 20,000 kilometers per year. To test the claim, a random sample of 25 automobile owners is asked to keep a record of the kilometers they travel. The random sample showed an average of 23500 kilometers and a standard deviation of 3900 kilometers. It is planned to test the above with parametric-based hypothesis testing. Assume a 5% confidence level.

(a) What should be the null and alternate hypothesis?

(b) Which test statistics: z, t or χ^2 test is applicable in this case?

(c) According to your test statistics, calculate the critical value.

(d) From the sample data, calculate the test value.

(e) Decide if the null hypothesis is to be accepted or rejected.

Solution:

(a) The null hypothesis, H_0 and alternate hypothesis, H_1 are given below. $\mathrm{H}_0 : \mu = 20000; \mathrm{H}_1 : \mu > 20000$

(b) For the given data, the population mean is to be inferred and population variance is not known. Hence, it comes under t-test.

(c) With degree of freedom = 24 and $\alpha = 0.05$, from the t-distribution table, the critical t-value, $t_c = 1.711$ for one tailed test.

(d) The test value from the sample statistics is given below: $t = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{23500-20000}{3900/\sqrt{25}} = 4.48$

(e) Since $t > t_c$, the null hypothesis is rejected. The decision from the hypothesis testing is: H_0 is rejected that means an automobile is driven on average more than 20,000 kilometers per year.

Problem 6. A manufacturer of electric batteries claims that the average capacity of a certain type of battery that the company produces is at least 140 ampere-hours with a standard deviation of 2.66 ampere-hours. An independent sample of 20 batteries gave a mean of 138.47 ampere-hour. Test at 5% significance level $H_0 = 140$ against $H_1 < 140$. Can the manufacturer's claim be sustained on the basis of this sample?

Solution:

In this problem we want to test, $H_0 : \mu = 140$ vs $H_1 : \mu < 140$, i.e., a left tailed test. Here, the significant level is 5% i.e., $\alpha = 0.05, n = 20, \bar{x} = 138.47, s = 2.66$.

Under $H_0 : \mu = 40000$, the test statistic $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} \sim t_{n-1}$.

Here, $t = \frac{138.47-140}{\frac{2.66}{\sqrt{20}}} = -2.57$.

Since, $-t_{\alpha,n-1} = -t_{0.05,19} = -1.729$ we have $t_{cal} < -t_{\alpha,n-1}$. $\Rightarrow$ It falls in the rejection region, hence reject H_0.

Thus, we can conclude that the average capacity of a certain type of battery is less than 140 ampere-hours.

Problem 7. A manufacturer of car batteries claims that the life of the company's batteries is approximately normally distributed with a standard deviation equal to 0.9 per year. If a random sample of 10 of these batteries has a standard deviation of 1.2 years, do you think that $\sigma > 0.9$ year? Use a 0.05 level of significance.

Solution:

In this problem we want to test, $H_0 : \sigma^2 = 0.81$ vs $H_1 : \sigma^2 > 0.81$, i.e., a right tailed test. Here, the significant level is 5% i.e., $\alpha = 0.05, n = 10, s = 1.2$.

Under $H_0 : \sigma^2 = 0.81$, the test statistic $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} \sim \chi^2_{n-1}$, where $\sigma_0^2 = 0.81$.

Here, $\chi^2 = \frac{(10-1)\times 1.2^2}{0.81} = 16$.

Since, $\chi^2_{\alpha,n-1} = \chi^2_{0.05,9} = 16.919$ we have $\chi^2_{cal} < \chi^2_{\alpha,n_1+n_2-2}$. $\Rightarrow$ It do not fall in the rejection region, hence do not reject H_0.

Thus, we are unable to conclude that the standard deviation of battery life exceeds 0.9.

Problem 8.

The alkalinity, in milligrams per litre, of water in the upper reaches of rivers in a particular region is known to be normally distributed with a standard deviation of 10mg/l. Alkalinity readings in the lower reaches of rivers in the same region are also known to be normally distributed, but with a standard deviation of 25mg/l. Ten alkalinity readings are made in the upper reaches of a river in the region and fifteen in the lower reaches of the same river with the following results.

Upper reaches	91	75	91	88	94	63	86	77	71	69	-	-	-	-	-
Lower reaches	86	95	135	121	68	64	113	108	79	62	143	108	121	85	97

Investigate, at the 1% level of significance, the claim that the true mean alkalinity of water in the lower reaches of this river is greater than in the upper reaches.

Solution:

In this problem we want to test, $H_0 : \mu_1 = \mu_2$ vs $H_1 : \mu_1 > \mu_2$, i.e., a right tailed test. Here, the significant level $\alpha = 0.01, n_1 = 15, n_2 = 10, \sigma_1 = 25, \sigma_2 = 10, \bar{x}_1 = 1485/15 = 99.0, \bar{x}_2 = 805/10 = 80.5$.

Under $H_0 : \mu_1 = \mu_2$, the test statistic $Z = \frac{\bar{x}_1-\bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}} \sim N(0,1)$.

Here, $z = \frac{99.0-80.5}{\sqrt{\frac{25^2}{15}+\frac{10^2}{10}}} = 2.57$.

Since, $z_\alpha = z_{0.01} = 2.33$ we have $z_{cal} > z_\alpha$. $\Rightarrow$ It falls in the rejection region, hence reject H_0.

Thus there is evidence, at 1% level of significance, to suggest that the true mean alkalinity of water in the lower reaches of the river is greater than that in the upper reaches.

Problem 9. A chemical engineer claims that the population mean yield of a certain batch process is 500 grams per milliliter of raw material. To check this claim he samples 25 batches each month. If the computed t-value falls between $-t_{0.05}$ and $t_{0.05}$, he is satisfied with this claim. What conclusion should he draw from a sample that has a mean $\bar{x} = 518$ grams per milliliter and a sample standard deviation $s = 40$ grams? Assume the distribution of yields to be approximately normal.

Solution:

We want to test the null hypothesis $H_0 : \mu = 500$ against the alternative hypothesis $H_1 : \mu \neq 500$. We are given: $n = 25, \bar{x} = 9, s = 40$.
Under the null hypothesis, $H_0 : \sigma_x = \sigma_y$, the statistic $t = \frac{(\bar{x}-\mu)}{s/\sqrt{n}}$ follows t-distribution with $(n-1)$df. Therefore, $t = \frac{(518-500)}{40/\sqrt{25}} = 2.25$.
The tabulated significant values are: $t_{24}(0.05) = 1.711$ and $-t_{24}(0.05) = -1.711$. So if the t-statistic falls in the range $(-1.711, 1.711)$, the engineer is satisfied. Since the calculated value of $t = 2.25$ lies outside between -1.711 and 1.711, the engineer might conclude his process yield is different from 500 grams per milliliter.

Problem 10.
An experiment was performed to compare the abrasive wear of two different laminated materials. Twelve pieces of material 1 were tested by exposing each piece to a machine measuring wear. Ten pieces of material 2 were similarly tested. In each case, the depth of wear was observed. The samples of material 1 gave an average (coded) wear of 85 units with a sample standard deviation of 4, while the samples of material 2 gave an average of 81 with a sample standard deviation of 5. Can we conclude at the 0.05 level of significance that the abrasive wear of material 1 exceeds that of material 2 by more than 2 units? Assume the populations to be approximately normal with equal variances.
Solution:
In this problem we want to test, $H_0 : \mu_1 - \mu_2 = 2$ vs $H_1 : \mu_1 - \mu_2 > 2$, i.e., a right tailed test. Here, the significant level is 5% i.e., $\alpha = 0.05, n_1 = 12, n_2 = 10, s_1 = 4, s_2 = 5, \bar{x}_1 = 85, \bar{x}_2 = 81$.

Under $H_0 : \mu_1 - \mu_2 = 2(\mu_0)$, the test statistic $t = \frac{\bar{x}_1 - \bar{x}_2 - \mu_0}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_1}}} \sim t_{n_1+n_2-2}$, where $S_p = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}}$.
Here, $S_p = \sqrt{\frac{11\times 4^2+9\times 5^2}{12+10-2}} = 4.478$ and $t = \frac{(85-81)-2}{4.478\sqrt{\frac{1}{12}+\frac{1}{10}}} = 1.04$.
Since, $t_{\alpha, n_1+n_2-2} = t_{0.05,20} = 1.725$ we have $t_{cal} < t_{\alpha,n_1+n_2-2}$. $\Rightarrow$ It do not fall in the rejection region, hence do not reject H_0.
Thus, we are unable to conclude that the abrasive wear of material 1 exceeds that of material 2 by more than 2 units.

Problem 11. Pumpkins were grown under two experimental conditions. Two random samples of 11 and 9 pumpkins show the sample standard deviations of their weights as 0.8 and 0.5 respectively. Assuming that the weight distributions are normal, test the hypothesis that the true variances are equal, against the alternative that they are not, at the 10% level.
Solution:
We want to test the null hypothesis $H_0 : \sigma_x = \sigma_y$ against the alternative hypothesis $H_1 : \sigma_x \neq \sigma_y$.
We are given: $n_1 = 11, n_2 = 9, s_x = 0.8, s_y = 0.5$.
Under the null hypothesis, $H_0 : \sigma_x = \sigma_y$, the statistic $F = \frac{S_x^2}{S_y^2}$ follows F-distribution with $(n_1 - 1, n_2 - 1)$ df.
Now, $S_x^2 = \frac{n_1}{n_1-1}s_x^2 = \frac{11}{10}(0.8)^2 = 0.704$. Similarly, $S_y^2 = \frac{n_2}{n_2-1}s_y^2 = \frac{9}{8}(0.5)^2 = 0.28125$. Therefore, $F = \frac{0.704}{0.28125} = 2.5$
The significant values of F for two tailed test at level of significance $\alpha = 0.10$ are:

$$F > F_{10,8}\left(\frac{\alpha}{2}\right) = F_{10,8}(0.05) \text{ and } F < F_{10,8}\left(1 - \frac{\alpha}{2}\right) = F_{10,8}(0.95)$$

The tabulated significant values are: $F_{10,8}(0.05) = 3.35$ and $F_{10,8}(0.95) = 0.326$
Since the calculated value of $F(= 2.5)$ lies between 0.326 and 3.35, it is not significant and hence null hypothesis of equality of population variance can not be rejected at 10% level of significance.

Problem 12. In testing for the difference in the abrasive wear of the two materials in Problem 10. we assumed that the two unknown population variances are equal. Were we justified in making this assumption? Use a 0.10 level of significance.
Solution:
In this problem we want to test, $H_0 : \sigma_1^2 = \sigma_2^2$ vs $H_1 : \sigma_1^2 \neq \sigma_2^2$, i.e., a two tailed test. Here, the significant level is 10% i.e., $\alpha = 0.1, \frac{\alpha}{2} = 0.05, n_1 = 12, n_2 = 10, s_1 = 4, s_2 = 5$.

Under $H_0 : \sigma_1^2 = \sigma_2^2$, the test statistic $F = \frac{S_1^2}{S_2^2} \sim F_{n_1-1,n_2-1}$. Here, the test statistic is $F = \frac{\frac{n_1}{n_1-1}s_1^2}{\frac{n_2}{n_2-1}s_2^2} = 0.6284$ In this two sample test we reject H_0 when $F < F_{1-\alpha/2,n_1-1,n_2-1}$ or $F > F_{\alpha/2,n_1-1,n_2-1}$.
The critical values are $F_{0.05,11,9} = 3.11$ and $F_{0.95,11,9} = \frac{1}{F_{0.05,9,11}} = 0.34$. Since, we have $F_{cal} > F_{0.95,11,9}$ and $F_{cal} < F_{0.05,11,9}$. $\Rightarrow$ It do not fall in the rejection region, hence do not reject H_0.
Thus, we conclude that there is insufficient evidence that variances differ.

Problem 7. In a sale of 300 units of an item the following preferences for colors are observed for customers:

Color	Brown	Grey	Red	Blue	White	Total
Customers	88	65	52	40	55	300

Test the hypothesis that all colors are equally popular.

Solution:

Assuming that all the five colors are equally likely, hence, expected number of units of each colors $= \frac{300}{5} = 60$.
Thus the test statistic $\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i} = 21.633$.
Since the critical value at 0.05 level of significance is $\chi^2_{0.05,4} = 9.488$ and we have test statistic > critical value, hence we reject H_0 and conclude that all colors are not equally popular.

Problem 14.

A survey was conducted among 500 students who are studying either in "government funded collages" (GVT) or "privately funded colleges" (PVT). The objective of the survey is to see the choice of "classroom-based learning" (C) over "Internet-based learning" (I). It is proposed to apply the χ^2 test to verify if there exists any association between "colleges" and "learning".

Categories	C	I	Total
GVT	75	125	200
PVT	60	240	300

(a) Decide the null and alternate hypotheses in this case. Justify your answer.

(b) Calculate the χ^2-value from the sample data.

(c) Test the hypothesis with 5% confidence level.

Solution:

(a) The hypothesis of the χ^2-test is given below. H_0: There is no association between the attributes College and Learning vs H_1: There is an association between the attributes College and Learning.

(b) Here we have,

$$\chi^2 = \frac{(75-54)^2}{54} + \frac{(125-146)^2}{146} + \frac{(60-81)^2}{81} + \frac{(240-219)^2}{219}$$
$$= 8.16 + 3.02 + 5.44 + 2.01 = 18.63$$

Categories	C	I	Total
GVT	75 (54)	125 (146)	200
PVT	60 (81)	240 (219)	300
Total	135	365	500

(c) Critical value is = 3.841. As $|\chi^2| > 3.841$. Reject the null hypothesis i.e., there exists association between "colleges" and "learning".

Problem 15. Let X be the number of defects in printed circuit boards. A random sample of $n = 60$ printed circuit boards is taken and the number of defects is recorded. The results were as follows: Does the assumption of a Poisson distribution seem appropriate as a model for these data?

Solution:

To test $H_0 : X \sim \text{Poisson}$ vs $H_1 : X$ does not follow a Poisson distribution.
The mean of the (assumed) Poisson distribution is unknown so must be estimated from the data by the sample mean: $\hat{\mu} = 0.75$. Using the Poisson distribution with $\mu = 0.75$ we can compute p_i, the hypothesised probabilities associated with each class. From these we can calculate the expected frequencies (under the null hypothesis) as follows:

Number of Defects	Observed Frequency
0	32
1	15
2	9
3	4

$$p_0 = P(X=0) = \frac{e^{-0.75}(0.75)^0}{0!} = 0.472 \quad \Rightarrow \quad E_0 = 0.472 \times 60 = 28.32$$
$$p_1 = P(X=1) = \frac{e^{-0.75}(0.75)^1}{1!} = 0.354 \quad \Rightarrow \quad E_1 = 0.354 \times 60 = 21.24$$
$$p_2 = P(X=2) = \frac{e^{-0.75}(0.75)^2}{2!} = 0.133 \quad \Rightarrow \quad E_2 = 0.133 \times 60 = 7.98$$
$$p_3 = P(X \geq 3) = 1 - (p_0 + p_1 + p_2) = 0.041 \quad \Rightarrow \quad E_3 = 0.041 \times 60 = 2.46$$

Note: The chi-squared goodness of fit test is not valid if the expected frequencies are too small. There is no general agreement on the minimum expected frequency allowed, but values of 3,4, or 5 are often used. If an expected frequency is too small, two or more classes can be combined. In the above example the expected frequency in the last class is less than 3, so we should combine the last two classes to get:

Number of Defects	Observed Frequency	Expected Frequency
0	32	28.32
1	15	21.24
2 or more	13	10.44

The chi-squared statistic can now be calculated:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(32-28.32)^2}{28.32} + \frac{(15-21.24)^2}{21.24} + \frac{(13-10.44)^2}{10.44} = 2.94$$

The number of degrees of freedom is $k - p - 1$. Here we have $k = 3$ classes and we have $p = 1$ because we had to estimate one parameter (the mean, μ) from the data. So, our chi-squared statistic has $3 - 1 - 1 = 1$ df. If we look up 2.94 in tables of the chi-squared distribution with df = 1, we obtain a p-value of $0.05 < p < 0.1$. We conclude that there is no real evidence to suggest the data DO NOT follow a Poisson distribution, although the result is borderline.