MS (QMS) : 2014
INDIAN STATISTICAL INSTITUTE

INSTRUCTIONS

The test is divided into two sessions (i) Forenoon session and (ii) Afternoon session. Each session is for two hours. For the forenoon session question paper, the test code is QMA and for the afternoon session question paper, the test code is QMB. Candidates appearing for MS(QMS) should verify and ensure that they are answering the right question paper.

The test QMA is multiple choice type. For each question, exactly one of the four choices is correct. You get four marks for each correct answer, one mark for each unanswered question, and zero mark for each incorrect answer.

The test QMB is of short answer type. It has altogether 10 questions. A candidate has to answer any 8 questions.

Questions will be set on the following and related topics.

Syllabus for QMA (Mathematics) and QMB (Mathematics)

Algebra: Binomial Theorem, AP, GP, HP, Exponential, Logarithmic Series, Sequence, Permutations and Combinations, Theory of Equations.

Matrix Algebra: Vectors and Matrices, Matrix Operations, Determinants.

Calculus: Functions, Limits, Continuity, Differentiation of functions of one or more variables. Unconstrained Optimization, Definite and Indefinite Integrals: Integration by parts and integration by substitution.

Sample questions for QMA (Mathematics) (2014)

1. If $x = \frac{p}{(q-r)}, y = \frac{q}{(r-p)}, z = \frac{r}{(p-q)}$ with $p \neq q \neq r$, then the value of $yz + zx + xy$ is

   a) 1
   b) 0
   c) -1
   d) 2

2. The value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$, when $u = tan^{-1}\left(\frac{x}{y}\right)$ is

   a) -1
   b) 0
   c) 1
   d) -2

3. The value of integral $\int\int xy\, dx\, dy$, over the region A, where A is the domain bounded by x-axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$ is

   a) $\frac{a^3}{2}$
   b) $\frac{a^4}{3}$
   c) $\frac{a^2}{4}$
   d) 1

4. $\int \frac{1}{x + xlogx} dx$ equals to

   a) $log|x + logx|$ + Constant
   b) $log|1 + xlogx|$ + Constant
   c) $log|logx|$ + Constant
   d) $log|1 + logx|$ + Constant

5. The value of $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \cdots$ equals to

a) e
b) $\frac{1}{e^2}$
c) $1 - \frac{1}{e}$
d) $\frac{1}{e}$

6. The sales of a firm is promoted by advertisement campaign according to the rule $\frac{dS}{dt} = 100e^{-0.25t}$ where S denotes the sales (in millions of rupees) and t is the number of years since the close of campaign. The sales promoted during the fifth year equals to

a) 30.56 millions of rupees
b) 31.44 millions of rupees
c) 32.32 millions of rupees
d) 32.56 millions of rupees

7. The Indian cricket team consists of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways can a cricket team of eleven be selected if we have to select 1 wicket keeper and at least 4 bowlers?

a) 1200
b) 1056
c) 1072
d) 1092

8. Let $\begin{vmatrix} a & a^2 & a^3 - 1 \\ b & b^2 & b^3 - 1 \\ c & c^2 & c^3 - 1 \end{vmatrix} = 0$, in which a, b, c are all different. Then the value of abc is

a) 10
b) 1
c) 0.1
d) 0.01

9. The product of the first 100 positive integers ends with:

a) 21 zeroes
b) 22 zeroes
c) 23 zeroes
d) 24 zeroes

10. The function $f(x) = |x| + sin\, x + cos^3 x$ is:

a) continuous but not differentiable at $x = 0$
b) differentiable at $x = 0$
c) a bounded function which is not continuous at $x = 0$
d) a bounded function which is continuous at $x = 0$

11. Consider a cubical box of 1 m side which has one corner placed at (0,0,0) and the opposite corner placed at (1,1,1). The least distance that an ant crawling from the point (0,0,0) to the point (1,1,1) must travel is:

a) $\sqrt{6}\, m$
b) $\sqrt{5}\, m$
c) $2\sqrt{3}\, m$
d) $(1 + \sqrt{3})m$

12. The roots of the equation $x^4 + x^2 = 1$ are

a) all real and positive
b) never real
c) 2 positive and 2 negative
d) 1 positive, 1 negative and 2 non-real

13. I sold 2 books for Rs.30 each. My profit on one was 25 per cent and the loss on the other was 25 per cent. Then, on the whole, I

a) lost Rs. 5
b) lost Rs. 4
c) gained Rs. 4
d) neither gained nor lost

14. If a, b, c are real numbers so that $x^3 \ + ax^2 \ + bx + c = (x^2 + 1)g(x)$ for some polynomial g, then

a) $b \ = \ 1, a \ = \ c$
b) $b \ = \ 0 \ = \ c$
c) $a \ = \ 0$
d) none of the above

15. If $E = \frac{1}{4} \cdot \frac{2}{6} \cdot \frac{3}{8} \cdot \frac{4}{10} \cdots \frac{30}{62} \cdot \frac{31}{64} = 8^x$ then the value of x is

a) -7
b) -9
c) -10
d) -12

16. If A and B are two square matrices such that $B = -A^{-1}BA$, then $(A + B)^2$ is equal to

a) O
b) $A^2 \ + B^2$
c) $A^2 \ + 2AB + B^2$
d) $A \ + B$

Sample questions for QMB (Mathematics) (2014)

1. (a) A set contains $(2n + 1)$ elements. Find the number of subsets of the set which contain at most n elements.

   (b) If $A = \begin{bmatrix} cos\alpha & sin\alpha \\ -sin\alpha & cos\alpha \end{bmatrix}$ then show that $A^n = \begin{bmatrix} cosn\alpha & sinn\alpha \\ -sinn\alpha & cosn\alpha \end{bmatrix}$.

2. (a) Let $f: R \rightarrow R$ such that for $x\ , y \in R,\ |f(x) - f(y)| \leq (x - y)^2$. Show that f is constant.

   (b) If $\alpha_1, \alpha_2, \cdots, \alpha_n$ are the roots of the equation $x^n + nax - b = 0$ then show that $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3) \cdots (\alpha_1 - \alpha_n) = n(\alpha_1^{n-1} + a)$.

3. (a) Suppose integers are formed by taking one or more digits from the following: 2, 2, 3, 3, 4, 5, 5, 5, 6, 7. For example, 355 is a possible choice while 44 is not. Find the number of distinct integers that can be formed in which
   (i) the digits are non-decreasing;
   (ii) the digits are strictly increasing.

   (b) Show that $\begin{vmatrix} x & l & m & 1 \\ \alpha & x & n & 1 \\ \alpha & \beta & x & 1 \\ \alpha & \beta & \gamma & 1 \end{vmatrix} = (x - \alpha)(x - \beta)(x - \gamma)$.

4. (a) Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dy\ dx$.
   (b) Let $A = ((a_{ij}))$ be an $n \times n$ matrix such that $a_{ij} = 0$ whenever $i \geq j$. Prove that A^n is a zero matrix.

5. (a) Two functions f and g are defined as follows for all non-negative real numbers. $f(x) = \exp(-x)$ and g is the piece-wise linear function defined by connecting $f(0), f(1), f(2), \cdots$. That is, for any non-negative integer k, the function g is obtained on the interval $[k, k+1)$ by connecting the points $(k, f(k))$ and $(k+1, f(k+1))$ with a line segment. Find the total area of the region which lies between the curves of f and g.

(b) For real numbers α and β, consider the function

$$f(x) = (\alpha - |x|)^2 e^{(\beta - |x|)^2}, -\infty < x < \infty.$$

(i) For what values of α and β is f continuous at all x?

(ii) For what values of α and β is f differentiable at all x?