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ISI DESCRIPTIVE TYPE QUESTIONS & SOLUTIONS

SET – 1

The test is of short answer type. It has altogether 10 questions. A candidate has to answer any 8 questions.

1. Find $\frac{dy}{dx}$ $if\ x^{cosy} + y^{cosx} = 1$

Ans:- u= x^{cosy}

Log u=cos y logx

$\frac{1}{u}.\frac{du}{dx} = cosy.\frac{1}{x} - logx.siny.\frac{dy}{dx}$

$\therefore \frac{du}{dx} = x^{cosy}(\frac{cosy}{x} - logx.siny.\frac{dy}{dx})$ …..(1)

v=y^{cosx} => $logv = cosxlogy$

$\frac{dv}{dx} = y^{cosx}\{-logy.sinx + cosx.\frac{1}{y}.\frac{dy}{dx}\}$ ………………(2)

Now, u+ v=1

$\frac{du}{dx}+\frac{dv}{dx} = 0.$

$\Rightarrow y^{cosx}.cosx.\frac{1}{y}.\frac{dy}{dx} - x^{cosy}.logxsiny\frac{dy}{dx} = y^{cosx}.logy.sinx - x^{cosy}.\frac{cosy}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y^{cosx}.logy.sinx - x^{cosy}.\frac{cosy}{x}}{y^{cosx}.cosx.\frac{1}{y} - x^{cosy}.logx.siny}.$

2. Find the inverse of the following matrix with $R_1 = (c_0, c_1, c_2, c_3)$; $R_2 = (c_2, c_3, c_0, c_1)$;

$R_3 = (c_3, -c_2, c_1, -c_0)$; $R_4 = (c_1, -c_0, c_3, -c_2)$,

where $c_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}, c_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}, c_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}, c_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}$

Ans:- Put ${c_0}^2 + {c_1}^2 + {c_2}^2 + {c_3}^2 = 1$

$c_0\ c_3 = -\frac{1}{16},\ c_2c_1 = \frac{3}{16},\ c_0c_2 = \frac{-3}{16},\ c_1c_3 = \frac{-3}{16}.$

Here $A^2 = I \Rightarrow A = A^{-1}$

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3. True/False: If f is a continuous function on ℝ ∋ f(x+ y)= f(x)+ f(y) ∀ x, y∈ ℝ.

Then f(0)= 0 ∀ x∈ℝ.

Ans:- False

f(x+ y)= f(x) .f(y)

Let f(x) = a^x , f(y)=a^y ∀ x, y∈ ℝ

f(x+ y)= a^x + y

f(0)=1 ≠0.

4. $\lim_{x \to 0} (\cos x)^{1/x^2} = ?$

Ans:- $(\cos x)^{1/x^2} = k, say$

∴ lnk= $\frac{1}{x^2} ln(\cos x)$

∴ $\lim_{x\to 0}(ln k) = \lim_{x\to 0}\frac{ln\cos x}{x^2}\left(\frac{0}{0}\right) = \lim_{x\to 0}\frac{-\tan x}{2x}\left(\frac{0}{0}\right) = \lim_{x\to 0}\frac{-cosec^2 x}{2} = -\frac{1}{2}$

∴ $\lim_{x\to 0} k = e^{-\frac{1}{2}}$

5. Maximize x+ y subject to the condition that $2x^2 + 3y^2 \le 1$.

Ans:- $\frac{x^2}{1/2} + \frac{y^2}{1/3} \le 1$

Let z= x+ y

Now, 4x + 6y$\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{3y}$

At the touching point. -$\frac{2x}{3y}$ = -1

⇒ 2x= 3y and $2x^2 + 3y^2$=1 ⇒ 2 $(\frac{3y}{2})^2 + (3y^2)$ = 1

⇒ $15y^2 = 2$ ⇒ y= $\pm\sqrt{\frac{2}{15}}$

∴ x= $\frac{3}{2}\left(\pm\sqrt{\frac{2}{15}}\right) = \pm\sqrt{\frac{3}{10}}$ ∴ Max (z)= $\sqrt{\frac{3}{10}} + \sqrt{\frac{2}{15}} = \frac{5}{\sqrt{30}}$.

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6. For any positive a, b prove that $(\boldsymbol{a}+\frac{\mathbf{1}}{\boldsymbol{a}})^{\mathbf{2}}+(\boldsymbol{b}+\frac{\mathbf{1}}{\boldsymbol{b}})^{\mathbf{2}} \geq \mathbf{8}$.

Ans: AM ≥ GM

$(a+\frac{1}{a})^2+(a+\frac{1}{a})^2 \geq 2\sqrt{(a+\frac{1}{a})^2+(b+\frac{1}{b})^2}$

≥ 2(ab+$\frac{1}{ab}+\frac{a}{b}+\frac{b}{a}$)

≥ 2(2+2) [∵ ab $+\frac{1}{ab} \geq 2$]

≥ 8.

7. Let A & B be two invertible n ×n real matrices. Assume that A + B is invertible.

Show that $\boldsymbol{A}^{-1}+\boldsymbol{B}^{-1}$ is also invertible.

Ans:- A, B are invertible

A + B is invertible.

|A||$A^{-1}+B^{-1}$||B| = |B+ A|≠0

⇒ |$A^{-1}+B^{-1}$| ≠0 as |A|, |B|≠ 0

⇒ $A^{-1}+B^{-1}$is invertible

8. Let A be n ×n orthogonal mtx where A is even and suppose |A| =-1. S.T. |I−A|= 0, where I denotes n ×n identity mtx.

Ans:- $A^{-1}=A^T$ |A| $=-1$

⇒ $\frac{1}{\lambda}=\lambda$ ⇒ $\prod_{i=1}^{n}\lambda_i=-1$ then at least one $\lambda_i=-1$

⇒ $\lambda=\pm 1$

∴ Characteristics Equation is $|\lambda I_n-A|$= 0

⇒$|I_n-A|$= 0 for $\lambda_i=+1$

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9. If f(x+ y)=f(x).f(y)for all x and y f(1)= 2 and $\alpha_n = f(n), n \in N$, then find equation of the circle having (α_1, α_2) and (α_3, α_4) as the ends of its one diameter?

Ans:- (a)

Given f(x+ y) = f(x)f(y), for all x, y ………………….(1)

f(1)= 2 ……………..(2)

Putting x = 1, y = 1 in (1), we get

f(2) = $(f(1))^2 = 2^2$

Putting x = 2, y = 1 in (1), we get

f (3) = f(2)f(1) = $2^2 . 2 = 2^3$

Similarly, f(n) = 2^n, n ∈ N

Given $\alpha_n = f(n)$

∴ $\alpha_n = 2^n$, n ∈ N ∴ $\alpha_1 = 2, \alpha_2 = 4, \alpha_3 = 8, \alpha_4 = 16$

Let P ≡ (2, 4), Q ≡ (8, 16)

∴ Equation of circle having PQ as a diameter is

(x - 2)(x - 8)+ (y - 4)(y - 16)= 0

10. If 0 < x < π, and f(x) = $\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2(1+\cos x)}}}}$ there being n number of 2's, then $\underset{n\to\infty}{\mathrm{Lt}} f(x) = \cdots$

Ans:- (b)

Let y= $\underset{n\to\infty}{\mathrm{Lt}} f(x)$

$when\ n \to \infty, y^2 = 2 + y \Longrightarrow y^2 - y - 2 = 0$

$\Longrightarrow$ y = 2, -1 $\Longrightarrow$ y = 2 (∵ y > 0)

Note:- Questions are collected from ISI Previous Year Sample Papers, Previous Question Papers & from Test of Mathematics at 10+2 level Book.

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