Regression Tree and Multivariate Adaptive Regression Splines (MARS)

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July 28, 2020

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- To be discussed in next class

Regression Preblem

- Data: $(y_i, \mathbf{x}_i)$, $i = 1, \ldots, n$, where
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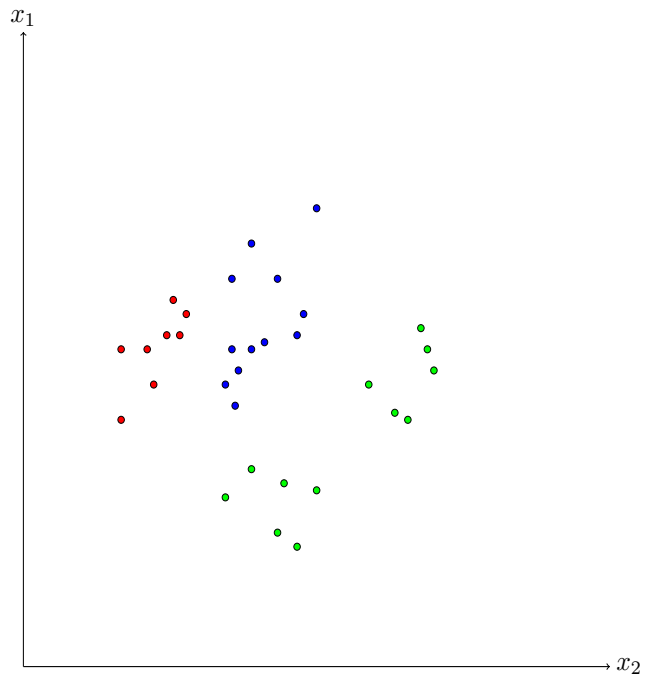

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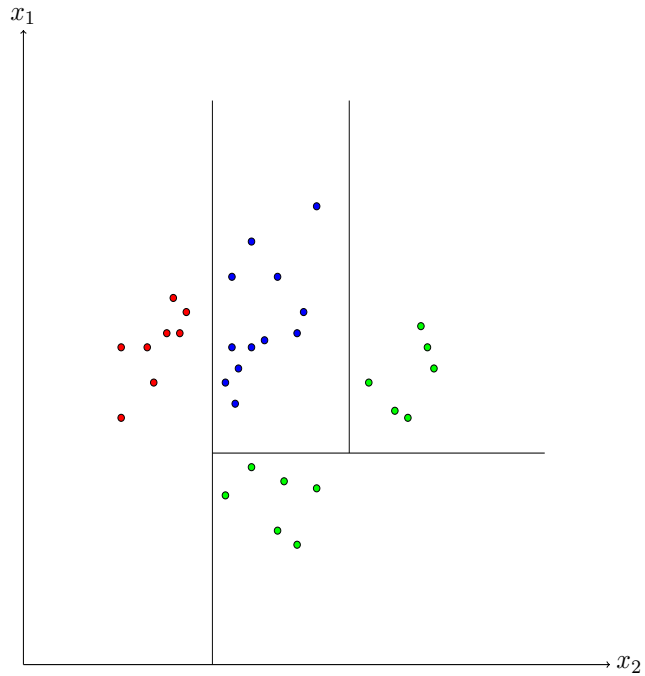

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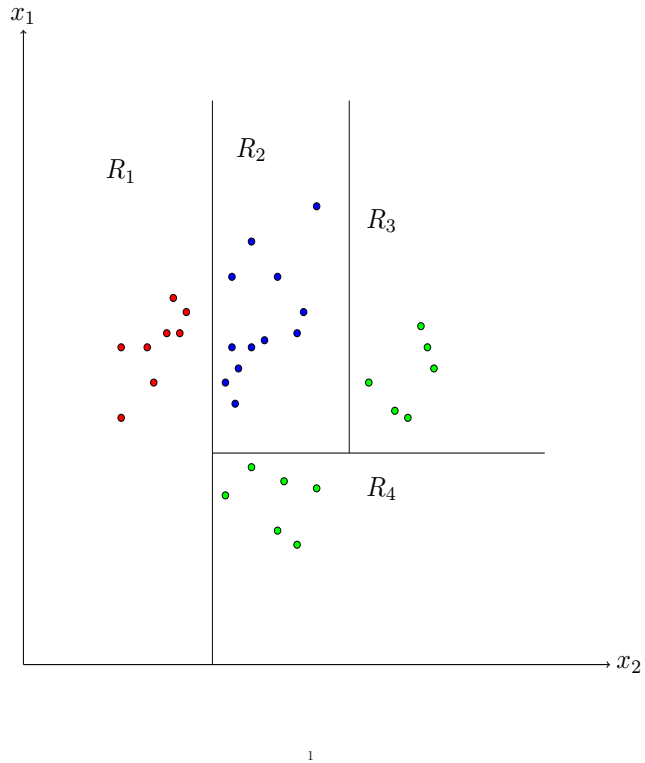
- The algorithm splits the x-space into partitions, say $\{R_1, R_2, \ldots, R_M\}$
- The regression tree model of response as $f(x) = \sum_{m=1}^{M} c_m \mathbf{I}(x \in R_m)$
- $\hat{c}_m = ave(y_i | \mathbf{x}_i \in R_m)$



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- Repeat the splitting process on each of the two regions.
- This process is continued until some stopping criterion is reached.

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- minimize$\{SSE + \alpha \times |T|\}$ where $|T|$ is the number of terminal nodes of the tree.
- Obtain the smallest pruned tree that has the lowest penalized error.

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  - Rattle is a free graphical user interface for Data Science, developed using R. R is a free software environment for statistical computing, graphics, machine learning and artificial intelligence. Together Rattle and R provide a sophisticated environment for data science, statistical analyses, and data visualisation.

Data Description

Body girth measurements and skeletal diameter measurements, as well as age, weight, height and gender, are given for 507 physically active individuals- 247 men and 260 women.

	Type	Column	Name of the Variable
2			Age (years)
3			Weight (kg)
4			Height (cm)
5			Gender (1 - male, 0 - female)
6	Skeletal Measurements	VAR1	Biacromial diameter (see Fig. 2)
7		VAR2	Biiliac diameter, or "pelvic breadth" (see Fig. 2)
8		VAR3	Bitrochanteric diameter (see Fig. 2)
9		VAR4	Chest depth between spine and sternum at nipple level,mid-expiration
10	Girth Measurements	VAR5	Chest diameter at nipple level, mid-expiration
11		VAR6	Elbow diameter, sum of two elbows
12		VAR7	Wrist diameter, sum of two wrists
13		VAR8	Knee diameter, sum of two knees
14		VAR9	Ankle diameter, sum of two ankles
15		VAR10	Shoulder girth over deltoid muscles
16		VAR11	Chest girth, nipple line in males and just above breast tissue in females, mid-expiration
17		VAR12	Waist girth, narrowest part of torso below the rib cage, average of contracted and relaxed position
18		VAR13	Navel (or "Abdominal") girth at umbilicus and iliac crest, iliac crest as a landmark
19		VAR14	Hip girth at level of bitrochanteric diameter
20		VAR15	Thigh girth below gluteal fold, average of right and left girths
21		VAR16	Bicep girth, flexed, average of right and left girths
22		VAR17	Forearm girth, extended, palm up, average of right and left girths
23		VAR18	Knee girth over patella, slightly flexed position, average of right and left girths
24		VAR19	Calf maximum girth, average of right and left girths
25		VAR20	Ankle minimum girth, average of right and left girths
26		VAR21	Wrist minimum girth, average of right and left girths
27	Other		
28			
29			

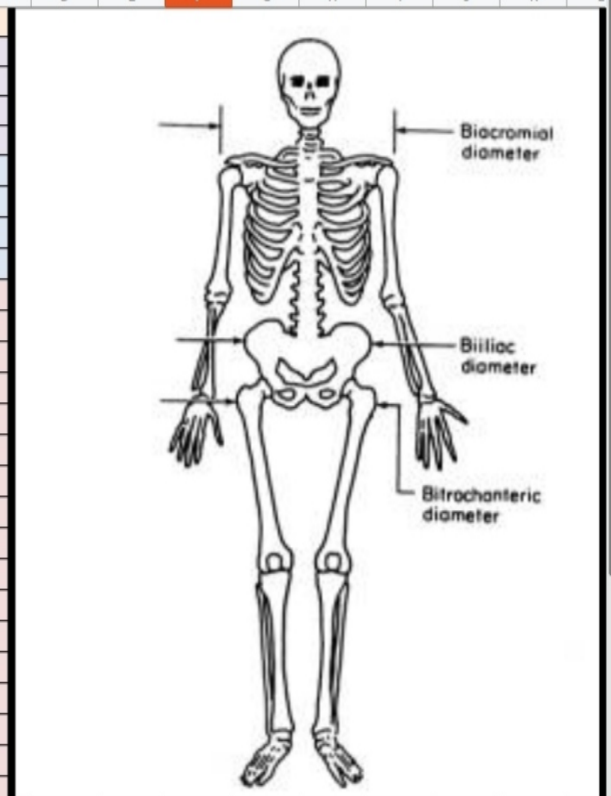
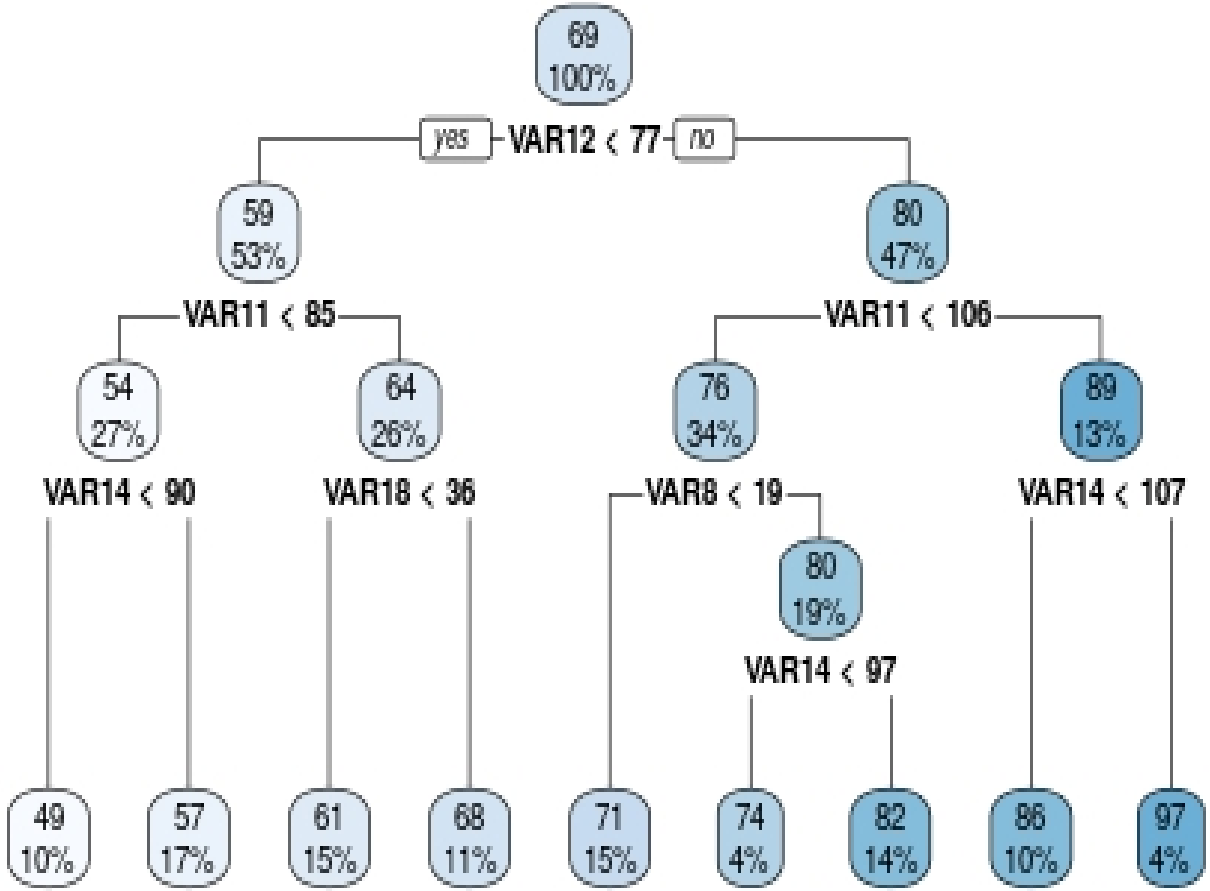


Figure: Body Dimension Data

Implementation using *rpart*

```
library(rsample)
library(rpart)
library(rpart.plot)
%library(Metrics) #rmse function

body_dimension_data_interest <- read.csv("~/Desktop/Workshop_stat_ml/
View(body_dimension_data_interest)

set.seed(123)
bodydim_split<-initial_split(body_dimension_data_interest, propo=.7)
bodydim_train<-training(bodydim_split)
bodydim_test<-testing(bodydim_split)

m1<-rpart(formula=weight~., data=bodydim_train, method="anova")
rpart.plot(m1) #to view the tree
plotcp(m1) #To check for the prunning

m2 <- rpart(formula=weight ~ .,data= bodydim_train, method= "anova",c
plotcp(m2)

pred <- predict(m1, newdata = bodydim_test)
obs<-bodydim_test$weight
rmse(pred, obs)
```

Ilustration using *rattle()*



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  - Lack of smoothness of the predictor surface (MARS alleviate)
  - Difficulty in modeling additive structure (MARS capture).

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- The spline functions used by MARS are:

$$(x-t)_+ = \begin{cases} x-t & x > t \\ 0 & \text{o/w} \end{cases} \quad \text{and} \quad (t-x)_+ = \begin{cases} t-x & t > x \\ 0 & \text{o/w} \end{cases}$$

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- Recall: Regression tree uses as basis functions: $I(X_j > c)$ and $I(X_j \leq c)$

Illustrative Example

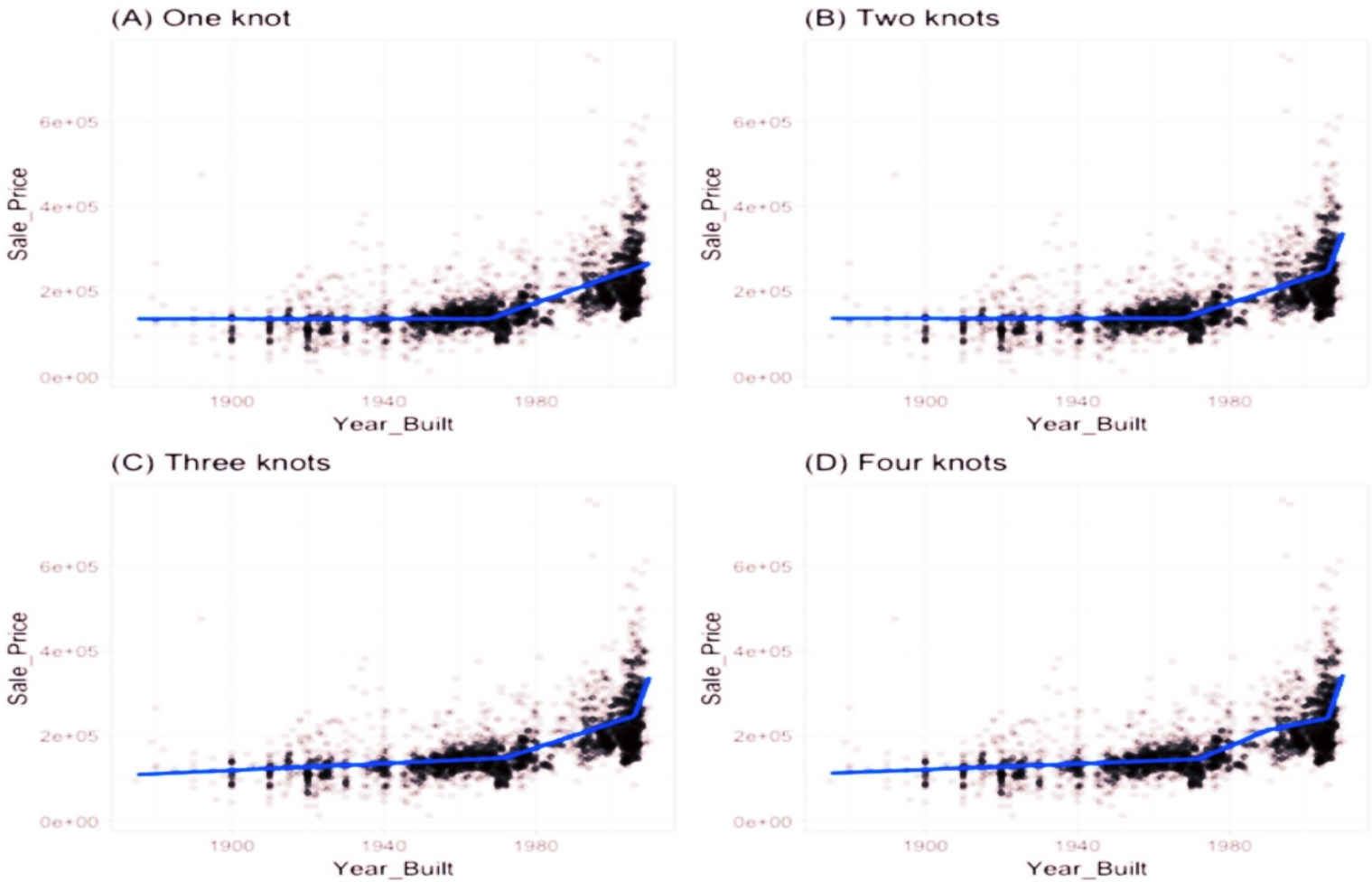

Figure 2: Examples of fitted regression splines of one (A), two (B), three (C), and four (D) knots.

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    - It prunes the model into its most effective part

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  Model: $\hat{f}^{(2)} = \hat{\beta}_0^{(2)} + \hat{\beta}_1^{(2)}(X_J - x_{kJ})_+ + \hat{\beta}_2^{(2)}(x_{kJ} - X_J)_+$,
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- ...
- Step m + 1: Add to the model a function of the form $b_{2m-1}h_l(X)(X_j - t)_+ + b_{2m}h_l(X)(t - X_j)_+$ with $h_l(X) \in \mathcal{M}^{(m)}$ that produces the largest decrease in training error. Say this is achieved by $j = J', t = x_{k'J'}$ and $l = L$.
  $\mathcal{M}^{(m+1)} = \mathcal{M}^{(m)} \cup \{h_{2m-1}(X), h_{2m}(X)\}$
  $h_{2m-1} = h_L(X)(X_{J'} - x_{k'J'})_+$ and $h_{2m} = h_L(X)(x_{k'J'} - X_{J'})_+$

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- $GCV(\lambda) = \frac{\sum_{i=1}^{n}(y_i - f_\lambda(x_i))^2}{(1-M(\lambda)/n)^2}$, where λ is size of the model and $M(\lambda)$ is the effective number of parameters in the model.

Implementation of MARS in R

```
#implementation of MARS
library(earth)
library(caret)

mars1 <- earth(weight ~ ., data = bodydim_train)
print(mars1) #for model summary
summary(mars1)
plot(mars1, which = 1)

mars2 <- earth(weight ~., data = bodydim_train, degree = 2)
summary(mars2)
plot(mars2,which=1,legend.pos=0)

#performance of the model in test dataset
summary(mars1,newdata=bodydim_test)
yhat=predict(mars1,newdata=bodydim_test)
yobs=bodydim_test$weight
rmse(yhat, yobs)
```

Advantage and Disadvantage of MARS

- Advantages:
  - Accurate if the local linear relationships are correct.
  - Quick computation.
  - Can work well even with large and small data sets.
  - Provides automated feature selection.
  - The non-linear relationship between the features and response are fairly intuitive.
  - Can be used for both regression and classification problems.
  - Does not require feature standardization.
- Disadvantages:
  - Not accurate if the local linear relationships are incorrect.
  - Typically not as accurate as more advanced non-linear algorithms (random forests, gradient boosting machines).
  - The earth package does not incorporate more advanced spline features (i.e. Piecewise cubic models).
  - Missing values must be pre-processed.

Thank You