Statistical Techniques For Business Analytics (STBA – 2017)

Hands-on-Session with R-Studio

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Course Outline

Chapter	Topic	Chapter	Topic
1	Introduction to RStudio	8	Analysis of Variance
2	Matrix Algebra	9	Regression Analysis
3	Fundamentals of Statistics	10	Binary Logistic Regression
4	Descriptive Statistics	11	Classification & Regression Tree
5	Data Processing	12	Principal Component Analysis
6	Normality Tests	13	Cluster Analysis
7	Test of Hypothesis	14	Modelling Nonlinear Relations

INTRODUCTION TO RSTUDIO

INSTALLATION

1.Download R software from http://cran.r-project.org/bin/windows/base/
2.Run the R set up (exe) file and follow instructions
3.Double click on the R icon in the desktop and R window will open
4.Download RStudio from http://www.rstudio.com/
5.Run R studio set up file and follow instructions
6.Click on R studio icon, R Studio IDE Studio will load
7.Tools – Global Options – Appearances – Change Color Size Theme
(if you wish to change the backround, not a mandatory step)
4.Go to R-Script (Ctrl + Shift + N)
5. Write "Hello World !"
6.Save & Run (Ctrl + Enter)

It will print "Hello World !"
Congrats ! You have written your very first R-Program

MATRIX ALGEBRA

MATRIX ALGEBRA

Matrix multiplication – Code

Read the matrix A and B

```
A = matrix(c(21,57,89,31,7,98), nrow =2, ncol=3, byrow = TRUE)
```

```
B = matrix(c(24, 35, 15, 34, 56,25), nrow = 3, ncol = 2, byrow = TRUE)
```

Multiplication of matrices

```
C = A%*%B
```

```
C
```

Determinant – R Code

```
A = matrix(c(51, 10, 23, 64), nrow = 2, ncol =2, byrow =TRUE)
```

```
det(A)
```

MATRIX ALGEBRA

Matrix Inverse – R code

```
A = matrix(c(51, 10, 23, 64), nrow = 2, ncol =2, byrow =TRUE)
solve(A)
```

Eigen values and Eigen vectors – R Code

```
A = matrix(c(1, -2, 3, -4), nrow = 2, ncol = 2, byrow = TRUE)
eigen(A)
```

FUNDAMENTALS OF STATISTICS

Statistics as a Key Technology……

"Statistics is the universal tool of inductive inference, research in natural and social sciences, and technological applications.

Statistics, therefore, must always have purpose, either in the pursuit of knowledge or in the promotion of human welfare."

– Professor Prasanta Chandra Mahalanobis

A Roadmap…

Collection of Data – Summarization of Data – Analysis of Data – Interpretation of Data towards a VALID DECISION.

Applications of Statistics in real world problems

- Finance – correlation and regression, index numbers, time series analysis, volatility modelling.
- Marketing – hypothesis testing, chi-square tests, nonparametric statistics.
- Development Studies – Gender Inequality, Child Health, Poverty : Econometrics & Applied Statistics.
- Demonetization – Text Mining Study (on twitter data)
- Forecasting – Call Centre Call Forecasting using TS.
- Self-Driving Cars, Robotics, AI, Image & Video Processing etc. as well uses STATISTICS extensively.

Solving these Problems Statistically

- What is Statistical Thinking ?
- How it is useful to solve Social Science Problems ?
- Consider a Problem.
- A 5 Phase Approach to solve.
- 5 Steps :

DEFINE A PROBLEM

- Identify the Problem.
- Define Scope.
- Data Requirement

I. **Voice of Customers** (Call Centre Data, Demonetization Data, Customer Service Records)

II. Survey Method based Data (Sample Survey (?), Sample Size (?), Sampling Techniques (?))

III. Secondary Data (NSSO CSO & other available Data)

- Create a Project Charter at the end.

MEASURING A PROBLEM

- Collect the data (economical, demographic, development related issues, etc.).
- Data Visualization (Tools !)
- Data Scrutiny (Mistakes, Missing data)
- Basic Statistics Check

ANALYSE THE DATA

- Finding Root cause of the problem.
- Prioritize root causes
- Relationship between Ys & Xs.
- Regression (?), Parametric, Non-parametric, Estimation (?), Testing (?), Time Series (?) & Panel Data Analysis (?), Econometric Modelling (?), Health Statistics, Vital Statistics, Data Mining.
- Classification & Clustering Problems.
- Supervised (LR, MLR, Decision Tree, kNN, SVM, ANN) Vs Unsupervised Approaches (Clustering & ARM (Famous Walmart Example) to model the data.

CHECKLIST BEFORE DATA ANALYSIS

- Basic Descriptive Analysis :

I. Descriptive Statistics Values (C.T., Dispersion, Shape, Relationship Plots)

II. Graphical Display of Data (Line, Bar, Pie, Histogram, Box-Plot, etc.)

- Outlier Tests, Normality Tests & Missing Values Check
- Multicollinearity, Autocorrelation, etc check.
- Variable Selection (PCA, FA, IV, Correlation Matrix, etc)
- Fitting Model & Validation (Adj. R-Square, p-value, MAPE, Confusion Matrix, etc)
- Interpret, Use

SHORTCUT FOR YOU

Dependent Variable Type (Ys)	Independent Variable Type (Xs)	Modelling Technique
Numerical	Numerical	1. Linear Regression (Best Subset Regression) 2. Non-linear Regression, Regression Splines
Numerical	Categorical + Numerical	1. Linear Regression with Dummy Variables 2. Polynomial Regression with Dummy Variables
Categorical	Numerical	1. Logistics Regression 2. Regression Trees
Categorical	Categorical + Numerical	1. Logistic Regression with Dummy Variables 2. Classification Trees

IMPLEMENT THE FINDINGS

- Design of Experiments
- Feedback Data
- Analysis of Improvement results
- Decision Criteria

CONTROLLING THE PROBLEM

- Monitoring the problem.
- Adjust accordingly
- Write report.
- Publish the paper (☺).
- Sign off.

DESCRIPTIVE STATISTICS

DESCRIPTIVE STATISTICS

Exercise 1: The monthly credit card expenses of an individual in 1000 rupees is given in the file Credit_Card_Expenses.csv.

a.Read the dataset to R studio

b.Compute mean, median minimum, maximum, range, variance, standard deviation, skewness, kurtosis and quantiles of Credit Card Expenses

c.Compute default summary of Credit Card Expenses

d.Draw Histogram of Credit Card Expenses

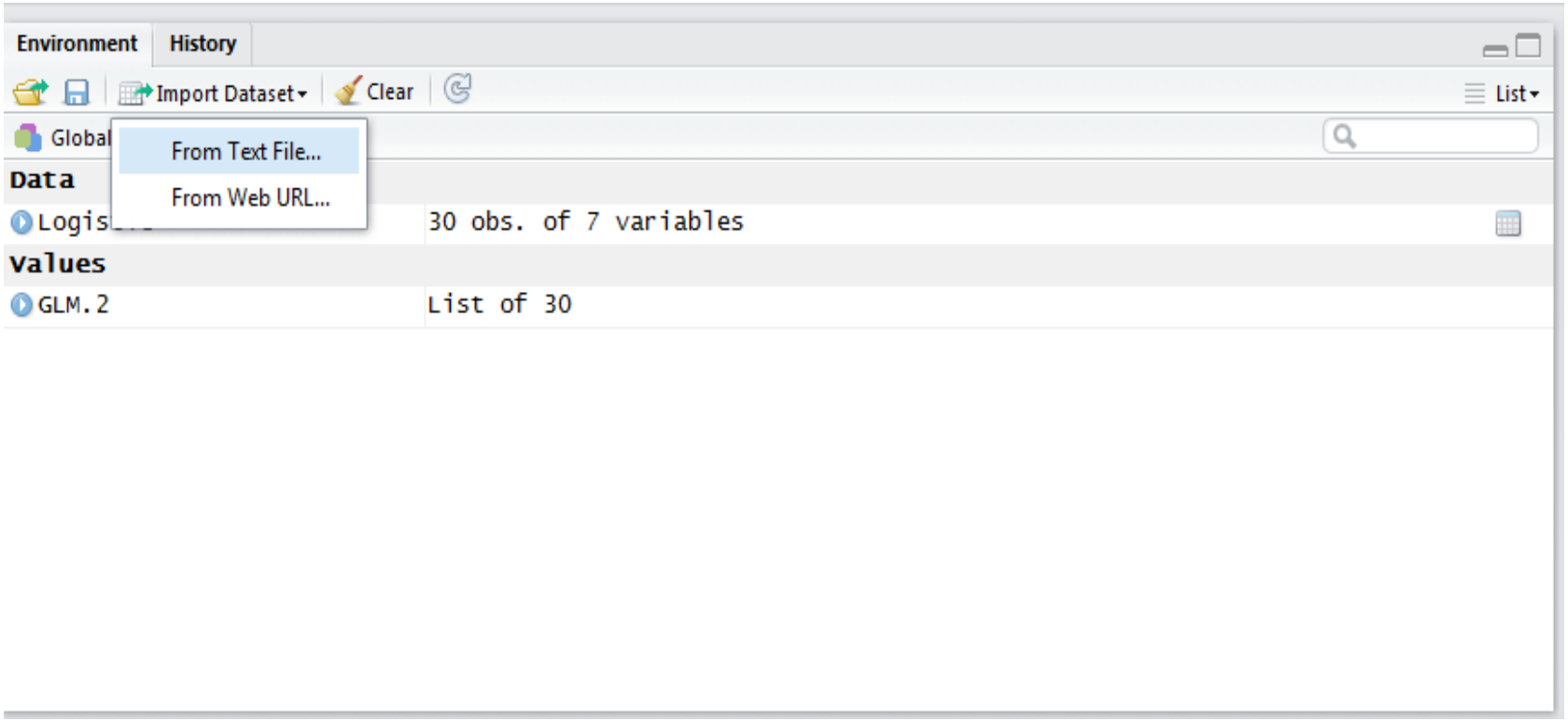
DESCRIPTIVE STATISTICS

The monthly credit card expenses of an individual in 1000 rupees is given below. Kindly summarize the data

Month	Credit Card Expenses	Month	Credit Card Expenses
1	55	11	63
2	65	12	55
3	59	13	61
4	59	14	61
5	57	15	57
6	61	16	59
7	53	17	61
8	63	18	57
9	59	19	59
10	57	20	63

DESCRIPTIVE STATISTICS

Reading a csv file to R Studio

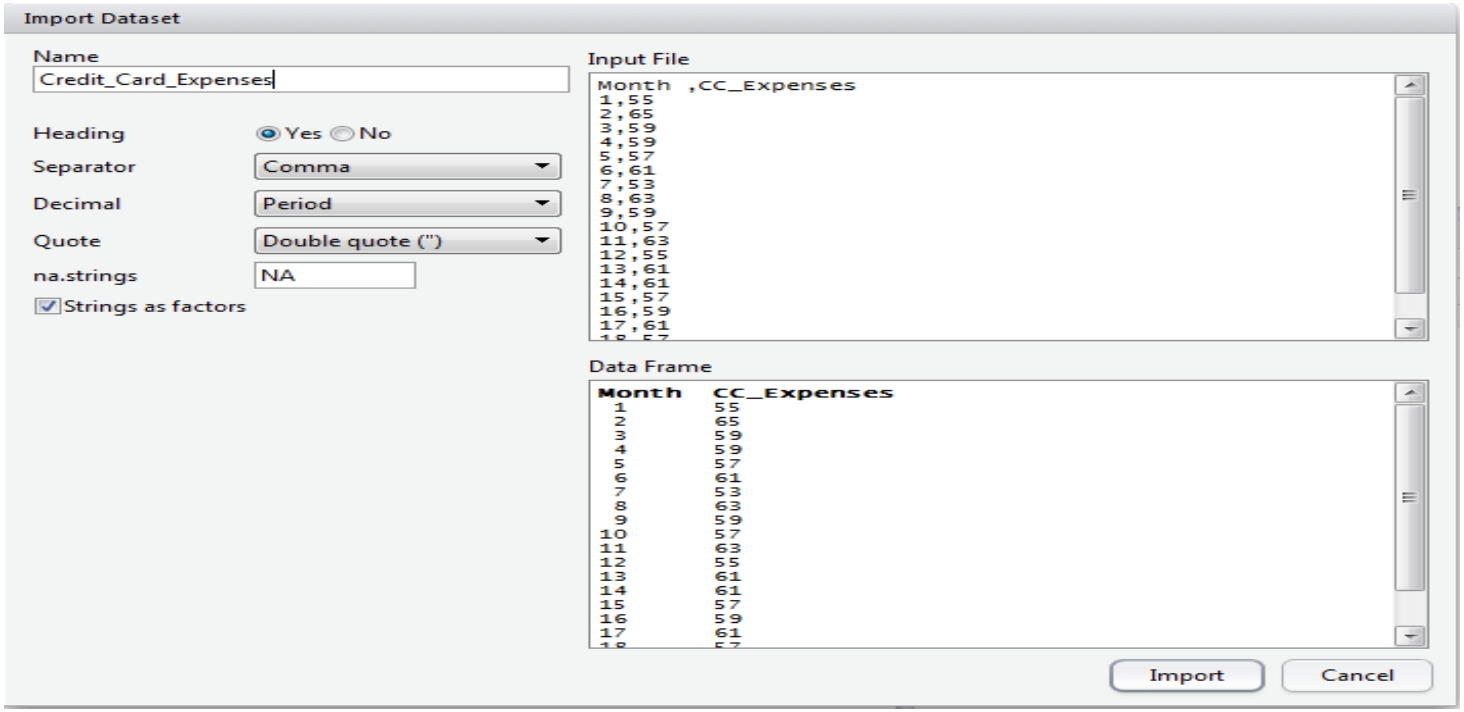


The file open dialog box will pop up

Browse to the file

DESCRIPTIVE STATISTICS

Reading a csv file to R Studio



Click Import button

R studio will read the data set to a data frame with specified name

DESCRIPTIVE STATISTICS

Reading a csv file to R Studio : Source code

➢Credit_Card_Expenses <- read.csv("D:/Infosys/DataSets/Credit_Card_Expenses.csv")

To change the name of the data set to : mydata

> mydata = Credit_Card_Expenses

To display the contents of the data set

> print(mydata)

To read a particular column or variable of data set to a ne variable Example: Read CC_Expenses to CC

>CC = mydata$CC_Expenses

DESCRIPTIVE STATISTICS

Reading data from MS Excel formats to R Studio

Format	Code
Excel	library(xlsx) mydata <- read.xlsx("c:/myexcel.xlsx", “Sheet1”)

Reading data from databases to R Studio

Function	Description
odbcConnect(*dsn*, uid="", pwd="")	Open a connection to an ODBC database
sqlFetch(*channel*, *sqtable*)	Read a table from an ODBC database into a data frame
sqlQuery(*channel*, *query*)	Submit a query to an ODBC database and return the results
sqlSave(*channel*, *mydf*, tablename = *sqtable*, append = *FALSE*)	Write or update (append=True) a data frame to a table in the ODBC database
sqlDrop(*channel*, *sqtable*)	Remove a table from the ODBC database
close(*channel*)	Close the connection

DESCRIPTIVE STATISTICS

Operators - Arithmetic

Operator	Description
+	addition
-	subtraction
*	multiplication
/	division
^ or **	exponentiation
x %% y	modulus (x mod y) 5%%2 is 1
x %/% y	integer division 5%/%2

DESCRIPTIVE STATISTICS

Operators - Logical

Operator	Description
<	less than
<=	less than or equal to
>	greater than
>=	greater than or equal to
==	exactly equal to
! =	not equal to
!x	Not x
x | y	x OR y
x & y	x AND y
isTRUE(x)	test if X is TRUE

DESCRIPTIVE STATISTICS

Descriptive Statistics

Computation of descriptive statistics for variable CC

Function	Code	Value
Mean	> mean(CC)	59.2
Median	> median(CC)	59
Standard deviation	> sd(CC)	3.105174
Variance	> var(CC)	9.642105
Minimum	> min(CC)	53
Maximum	> max(CC)	65
Range	> range(CC)	53 65

DESCRIPTIVE STATISTICS

Descriptive Statistics

Function	Code
Quantile	> quantile(CC)

Output					
Quantile	0%	25%	50%	75%	100%
Value	53	57	59	61	65

Function	Code
Summary	>summary(CC)

Output					
Minimum	Q1	Median	Mean	Q3	Maximum
53	57	59	59.2	61	65

DESCRIPTIVE STATISTICS

Descriptive Statistics

Function	Code
describe	> libray(psych) > describe(CC)

Output	
Statistics	Values
N	20
mean	59.2
sd	3.11
median	59
Trimmed	59.25
mad	2.97
min	53
Max	65
Range	12
Skew	-0.08
Kurtosis	-0.85
se	0.69

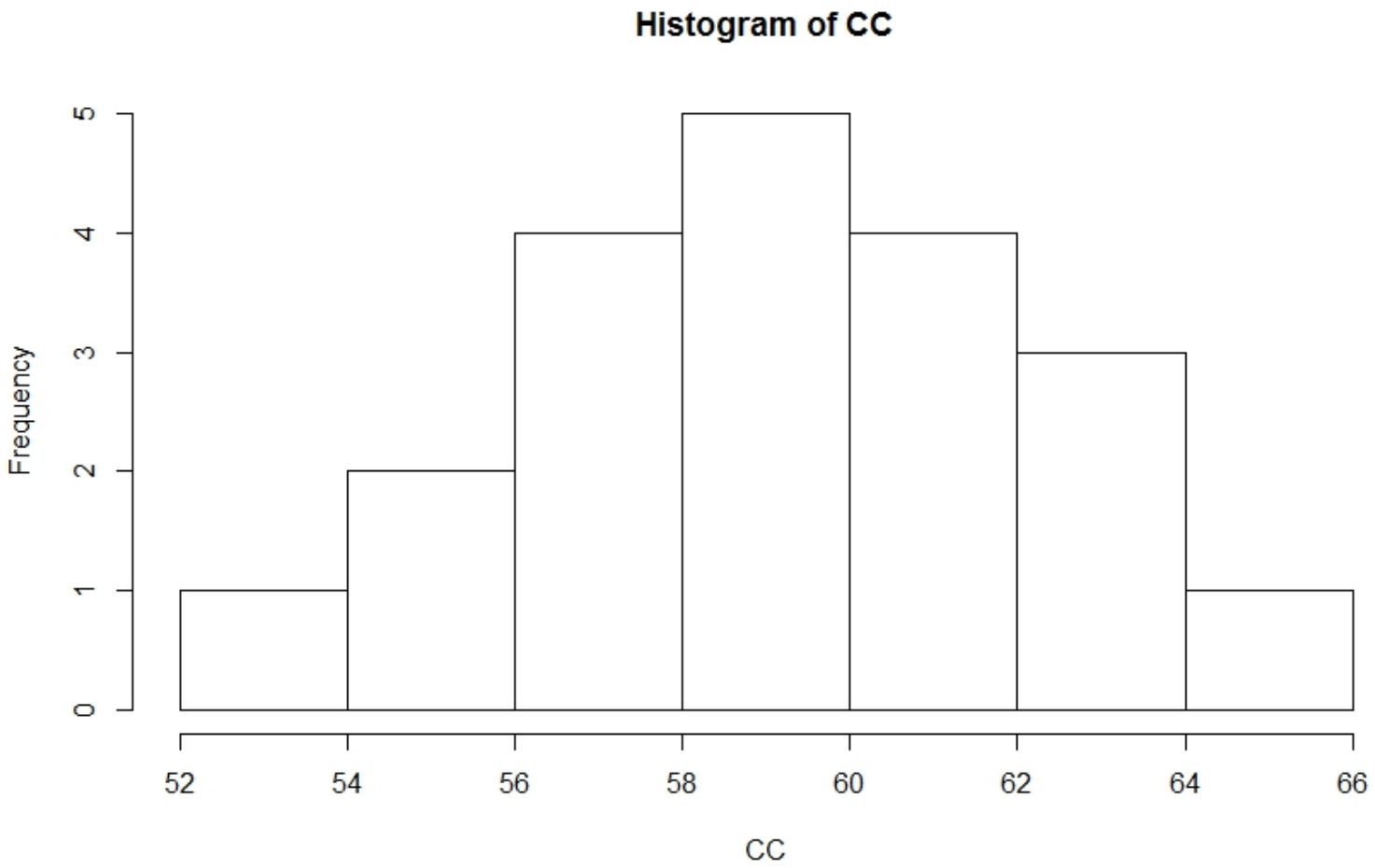
DESCRIPTIVE STATISTICS

Graphs

Graph	Code
Histogram	> hist(CC)
Histogram colour (“Blue”)	> hist(CC,col="blue")
Dot plot	> dotchart(CC)
Box plot	> boxplot(CC)
Box plot colour	> boxplot(CC, col="dark green")

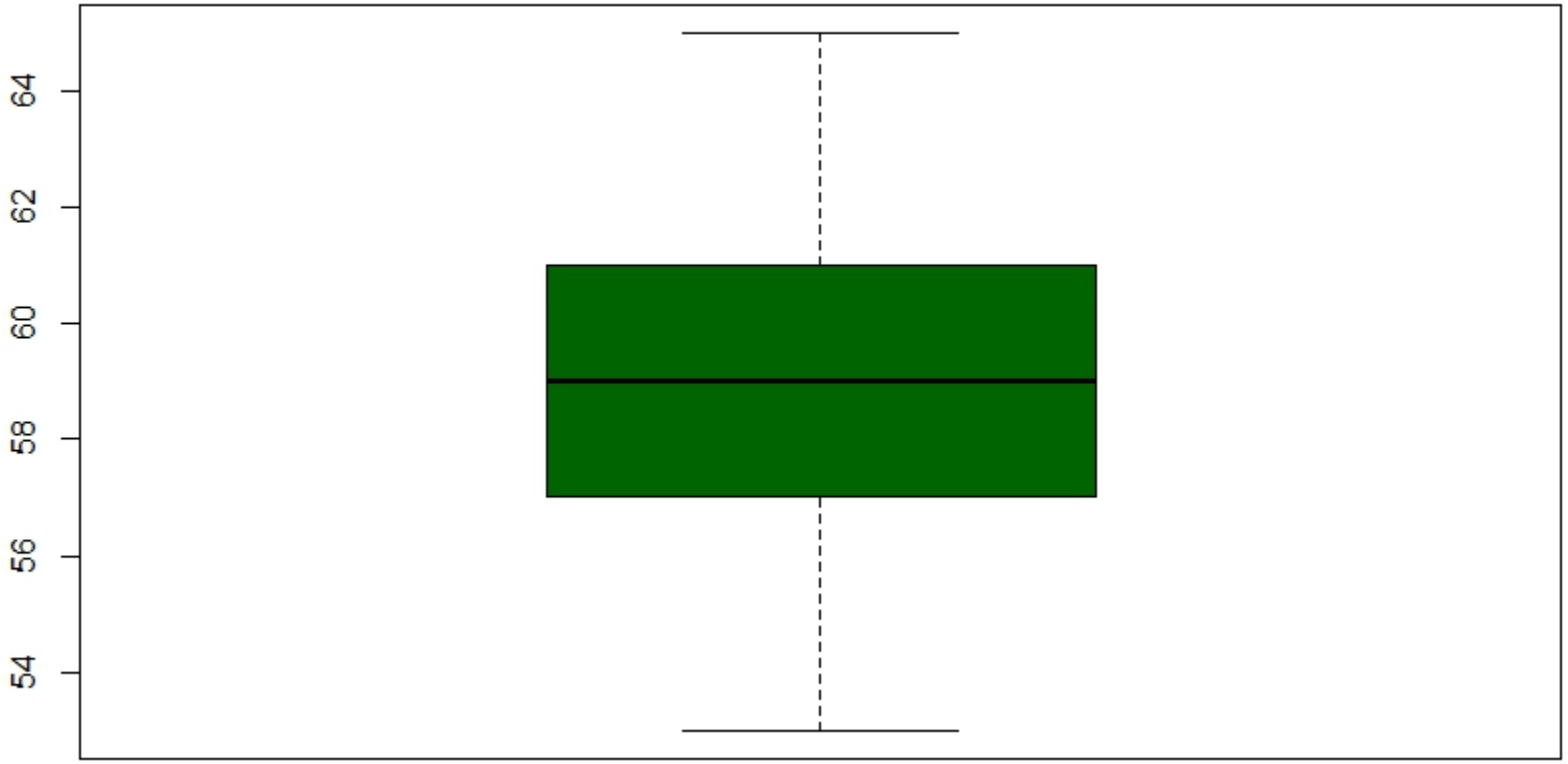
DESCRIPTIVE STATISTICS

Histogram : Variable - CC



DESCRIPTIVE STATISTICS

Box plot : Variable - CC



DATA PREPROCESSING

1. Missing value replenishment
2. Merging data files
3. Appending the data files
4. Transformation or normalization
5. Random Sampling

Missing Value Handling

Example: Suppose a telecom company wants to analyze the performance of its circles based on the following parameters

1. Current Month's Usage
2. Last 3 Month's Usage
3. Average Recharge
4. Projected Growth

The data set is given in next slide. (Missing_Values_Telecom Data)

Missing Value Handling

Example: Circle wise Data

SL No.	Current Month's Usage	Last 3 Month's Usage	Average Recharge	Projected Growth	Circle
1	5.1	3.5	99.4	99.2	A
2	4.9	3	98.6	99.2	A
3		3.2		99.2	A
4	4.6	3.1	98.5	9..2	A
5	5		98.4	99.2	A
6	5.4	3.9	98.3	99.4	A
7	7	3.2	95.3	98.4.	B
8	6.4	3.2	95.5	98.5	B
9	6.9	3.1	95.1	98.5	B
10		2.3	96	98.3	B
11	6.5	2.8	95.4	98.5	B
12	5.7		95.5	98.3	B
13	6.3	3.3		98.6	B
14	6.7	3.3	94.3	97.5	C
15	6.7	3	94.8	97.3	C
16	6.3	2.5	95	98.9	C
17		3	94.8	98	C
18	6.2	3.4	94.6	97.3	C
19	5.9	3	94.9	98.8	C

Missing Value Handling

Example: Read data and variables to R

```
> mydata = Missing_Values_Telecom
> cmusage = mydata[,2]
> l3musage = mydata[,3]
> avrecharge = mydata[,4]
```

Missing Value Handling

Option 1: Discard all records with missing values

```
>newdata = na.omit(mydata)

>write.csv(newdata,"E:/ISI/newdata.csv")
```

SL.No.	Current.Month.s.Usage	Last.3.Month.s.Usage	Average.Recharge	Projected.Growth	Circle
1	5.1	3.5	99.4	99.2	A
2	4.9	3	98.6	99.2	A
4	4.6	3.1	98.5	9..2	A
6	5.4	3.9	98.3	99.4	A
7	7	3.2	95.3	98.4.	B
8	6.4	3.2	95.5	98.5	B
9	6.9	3.1	95.1	98.5	B
11	6.5	2.8	95.4	98.5	B
14	6.7	3.3	94.3	97.5	C
15	6.7	3	94.8	97.3	C
16	6.3	2.5	95	98.9	C
18	6.2	3.4	94.6	97.3	C
19	5.9	3	94.9	98.8	C

Missing Value Handling

Option 2: Replace the missing values with variable mean, median, etc

Replacing the missing values with mean

Compute the means excluding ghe missing values

```
>cmusage_mean = mean(cmusage, na.rm = TRUE)
>l3musage_mean = mean(l3musage_mean, na.rm = TRUE)
> avrecharge_mean = mean(avrecharge, na.rm = TRUE)
```

Replace the missing values with mean

```
> cmusage[is.na(cmusage)]=cmusage_mean
> l3musage[is.na(l3musage)]= l3musage_mean
>avrecharge[is.na(avrecharge)]=avrecharge_mean
```

Missing Value Handling

Option 2: Replace the missing values with variable mean, median, etc

Replacing the missing values with mean

Replace the missing values with mean

```
> cmusage[is.na(cmusage)]=cmusage_mean
> l3musage[is.na(l3musage)]= l3musage_mean
>avrecharge[is.na(avrecharge)]=avrecharge_mean
```

Making the new file

```
> mynewdata = cbind(cmusage, l3musage, avrecharge, mydata[,5],mydata[,6])
> write.csv(mynewdata, "E:/ISI/mynewdata.csv")
```

Missing Value Handling

Option 2: Replace the missing values with variable mean, median, etc

Replacing the missing values with men

SL No	cmusage	l3musage	avrecharge	Proj Growth	Circle
1	5.1	3.5	99.4	11	1
2	4.9	3	98.6	11	1
3	5.975	3.2	96.14117647	11	1
4	4.6	3.1	98.5	1	1
5	5	3.105882353	98.4	11	1
6	5.4	3.9	98.3	12	1
7	7	3.2	95.3	6	2
8	6.4	3.2	95.5	7	2
9	6.9	3.1	95.1	7	2
10	5.975	2.3	96	5	2
11	6.5	2.8	95.4	7	2
12	5.7	3.105882353	95.5	5	2
13	6.3	3.3	96.14117647	8	2
14	6.7	3.3	94.3	3	3
15	6.7	3	94.8	2	3
16	6.3	2.5	95	10	3
17	5.975	3	94.8	4	3
18	6.2	3.4	94.6	2	3
19	5.9	3	94.9	9	3

DATA MERGING

Exercise: The data of 30 customers on credit card usage in INR1000 is given in CC_Usage.txt. Similarly the user profile namely gender (1: male, 2: female) and whether they have done shopping or banking (1: yes , 2: no) with credit card are given in cc_Profile.csv. Can you merge the two files into a single data set?

Read the files

```
>myprofile = CC_Profile
> myusage = CC_Usage
```

Merge the files by “ID” field

```
>mydata = merge(myprofile, myusage, by = “ID”)
```

DATA APPEND

Exercise: The data on user profile of customers whom are included in the previous mailing campaign is compiled into two files namely classification1.csv and classification2.txt. Can you append the second data set with the first one and store the new data set in a new file?

Read the files

```
>class1 = Classification1
> class2 = Classification2
```

Append class1 with class2

```
>mydata = rbind(class1, class2)
```

TRANSFORMATION / NORMALIZATION

z transform:

Transformed data = (Data – Mean) / SD

Exercise : Normalize the variables in the Supply_Chain.csv ?

Read the files

```
>mydata  = Supply_Chain
> mydata = mydata[,2:7]
```

Normalize or standardize the variable

```
>mystddata = scale(mydata)
```

RANDOM SAMPLING

Example: Take a sample of size 60 (10%) randomly from the data given in the file bank-data.csv and save it as a new csv file?

Read the files

```
>mydata  = bank-data

> mysample = mydata[sample(1:nrow(mydata), 60, replace = FALSE),]

>write.csv(mysample,"E:/ISI/mysample.csv")
```

RANDOM SAMPLING

Example: Split randomly the data given in the file bank-data.csv into sets namely training (75%) and test (25%) ?

Read the files

```
>mydata  = bank-data

>sample = sample(2, nrow(mydata), replace = TRUE, prob = c(0.75, 0.25))
> sample1 = mydata[sample ==1, ]
> sample2 = mydata[sample ==2,]
```

NORMALITY TEST

NORMALITY TEST

Normality test

A methodology to check whether the characteristic under study is normally distributed or not

Two Methods :

Normality test - Quantile – Quantile (Q- Q) plot

Plots the ranked samples from the given distribution against a similar number of ranked quantiles taken from a normal distribution

If the sample is normally distributed then the line will be straight in the plot

Normality test – Shapiro – Wilk test

H0: Deviation from bell shape (normality) = 0

H1 : Deviation from bell shape $\neq 0$

If p value ≥ 0.05 (5%), then H0 is not rejected, distribution is normal

NORMALITY TEST

Normality test

Exercise 1 : The processing times of purchase orders is given in PO_Processing.csv. Is the distribution of processing time is normally distributed?

Reading the data and variable

```
> mydata = PO_Processing
> PT = mydata$Processing_Time
```

NORMALITY TEST

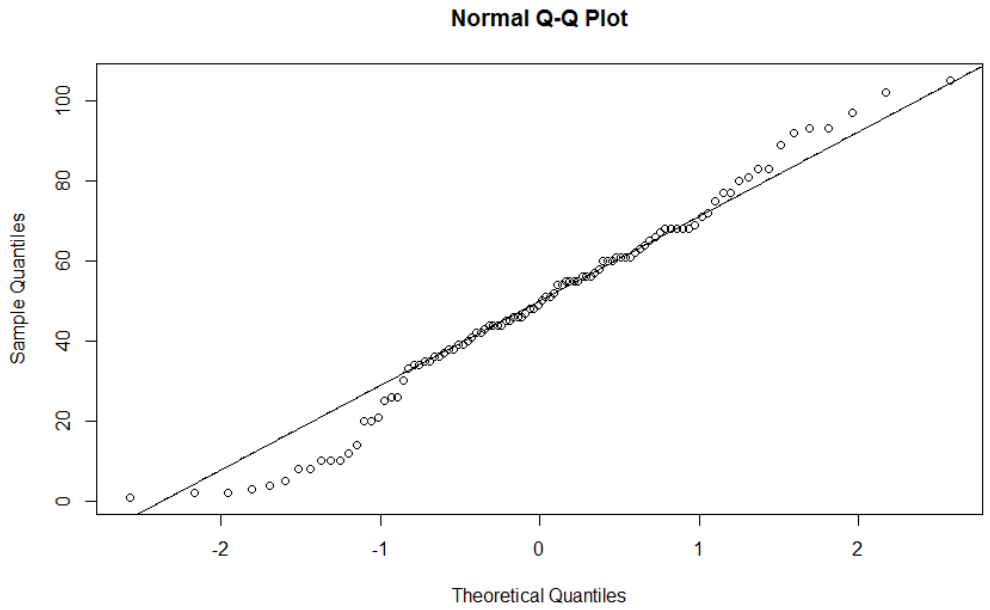
Normality test

Exercise 1 : The processing times of purchase orders is given in PO_Processing.csv. Is the distribution of processing time is normally distributed?

Normality Check using Normal Q – Q plot

> qqnorm(PT)

> qqline(PT)



NORMALITY TEST

Normality test

Exercise 1 : The processing times of purchase orders is given in PO_Processing.csv. Is the distribution of processing time is normally distributed?

Normality Check using Shapiro – Wilk test

```
> shapiro.test(PT)
```

Statistics	Value
W	0.9804
p value	0.1418

TEST *of* HYPOTHESIS

TEST OF HYPOTHESIS

Introduction:

In many situations, it is required to accept or reject a statement or claim about some parameter

Example:

1. The average cycle time is less than 24 hours
2. The % rejection is only 1%

The statement is called the hypothesis

The procedure for decision making about the hypothesis is called hypothesis testing

Advantages

1. Handles uncertainty in decision making
2. Minimizes subjectivity in decision making
3. Helps to validate assumptions or verify conclusions

TEST OF HYPOTHESIS

Commonly used hypothesis tests on mean of normal distribution:

- Checking mean equal to a specified value ($mu = mu_0$)
- Two means are equal or not ($mu_1 = mu_2$)

TEST OF HYPOTHESIS

Null Hypothesis:

A statement about the status quo

One of no difference or no effect

Denoted by H0

Alternative Hypothesis:

One in which some difference or effect is expected

Denoted by H1

TEST OF HYPOTHESIS

Methodology demo: To Test Mean = Specified Value (mu = mu_0)

Suppose we want to test whether mean of a process characteristic is 5 based on the following sample data from the process

4	4	5	5	6
5	4.5	6.5	6	5.5

Calculate the mean of the sample, xbar = 5.15

Compare xbar with specified value 5

or xbar - specified value = xbar - 5 with 0

If xbar - 5 is close to 0

then conclude mean = 5

else mean $\neq$ 5

TEST OF HYPOTHESIS

Methodology demo : To Test Mean = Specified Value (mu = mu_0)

Consider another set of sample data. Check whether mean of the process characteristic is 500

400	400	500	500	600
500	450	650	600	550

Mean of the sample, xbar = 515

xbar - 500 = 515 - 500 = 15

Can we conclude mean $\neq$ 500?

Conclusion:

Difficult to say mean = specified value by looking at xbar - specified value alone

TEST OF HYPOTHESIS

Methodology demo: To Test Mean = Specified Value ($mu = mu_0$)

Test statistic is calculated by dividing (xbar - specified value) by a function of standard deviation

To test Mean = Specified value

Test Statistic t_0 = (xbar - Specified value) / (SD / $\sqrt{n}$)

If test statistic is close to 0, conclude that Mean = Specified value

To check whether test statistic is close to 0, find out p value from the sampling distribution of test statistic

TEST OF HYPOTHESIS

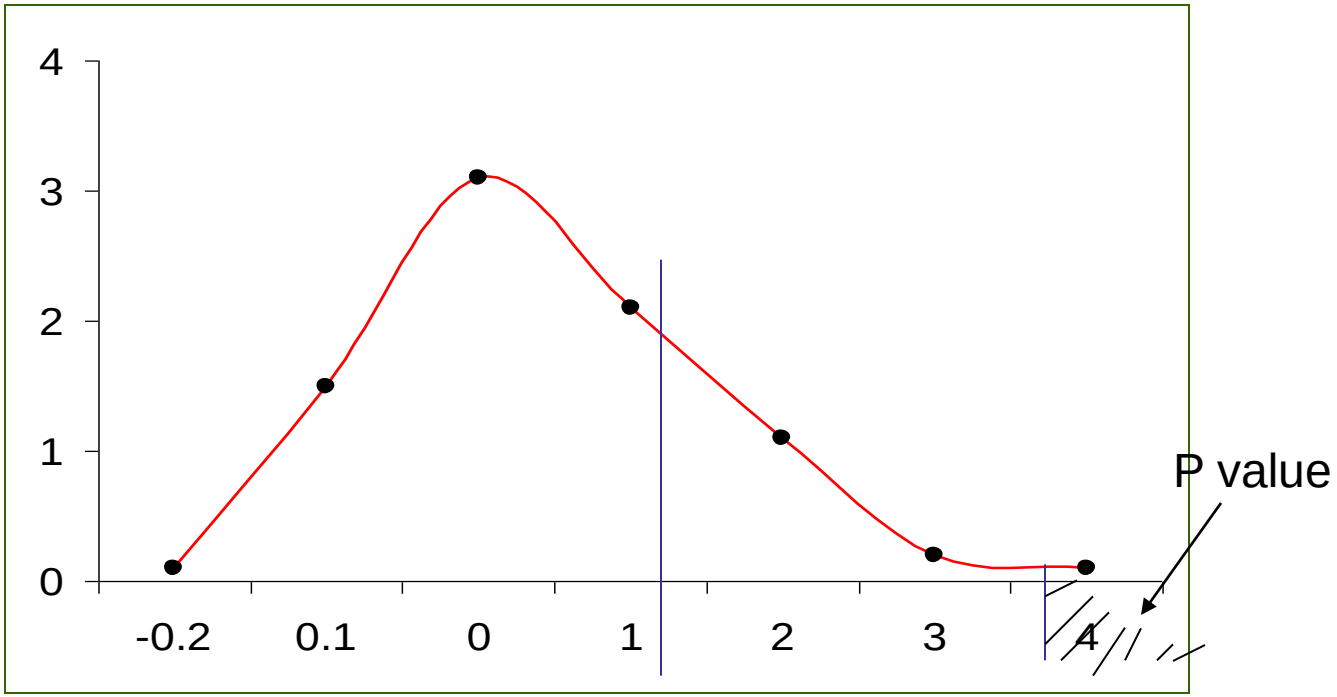
Methodology demo: To Test Mean = Specified Value

P value

The probability that such evidence or result will occur when H0 is true

Based on the reference distribution of test statistic

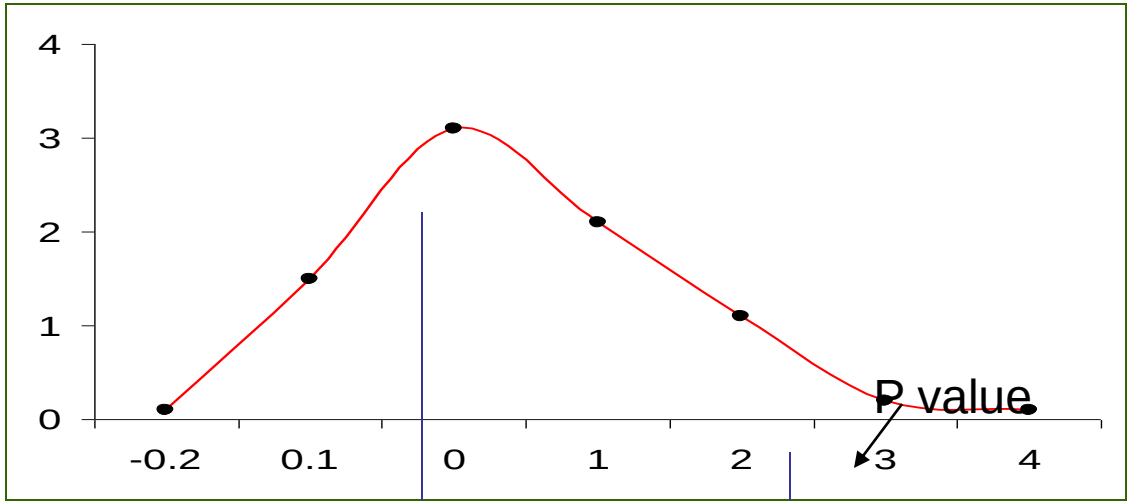
The tail area beyond the value of test statistic in reference distribution



TEST OF HYPOTHESIS

Methodology demo : To Test Mean = Specified Value

P value



If test statistic t_0 is close to 0 then p will be high

If test statistic t_0 is not close to 0 then p will be small

If p is small , $p < 0.05$ (with alpha = 0.05), conclude that $t \neq 0$, then

Mean ≠ Specified Value, H0 rejected

TEST OF HYPOTHESIS

To Test Mean = Specified Value (mu = mu_0)

Example: Suppose we want to test whether mean of the process characteristic is 5 based on the following sample data

4	4	5	5	6
5	4.5	6.5	6	5.5

H0: Mean = 5

H1: Mean $\neq$ 5

Calculate xbar = 5.15

SD = 0.8515

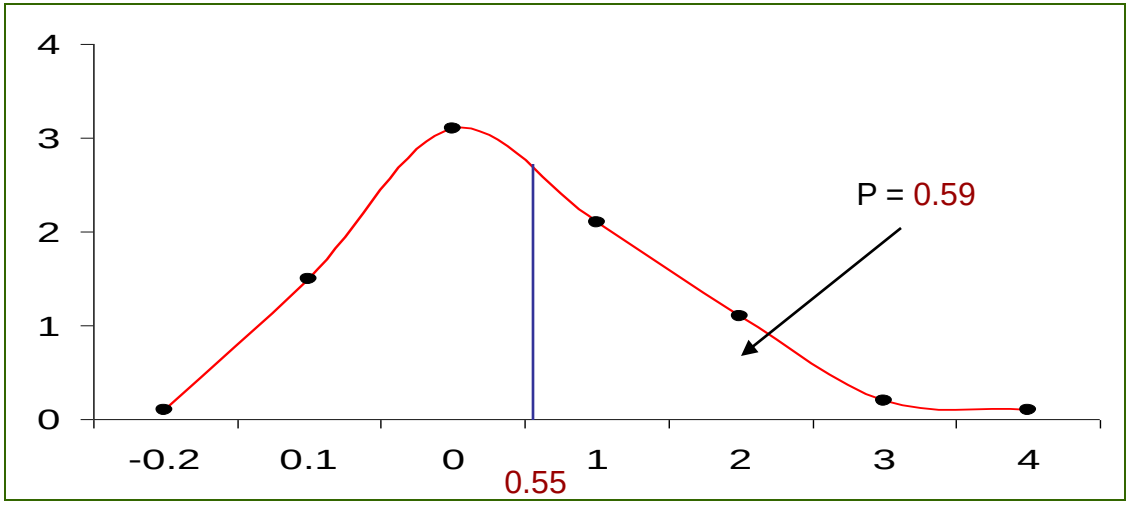
n = 10

Test statistic t_0 = (xbar - 5)/(SD / $\sqrt{n}$) = (5.15 - 5) / (0.8515 / $\sqrt{10}$) = 0.5571

TEST OF HYPOTHESIS

Example: To Test Mean = Specified Value (mu = mu_0)

$t_0 = 0.5571$



P ≥ 0.05, hence Mean = Specified value = 5.

H0: Mean = 5 is not rejected

TEST OF HYPOTHESIS

Hypothesis Testing: Steps

1. Formulate the null hypothesis H0 and the alternative hypothesis H1
2. Select an appropriate statistical test and the corresponding test statistic
3. Choose level of significance alpha (generally taken as 0.05)
4. Collect data and calculate the value of test statistic
5. Determine the probability associated with the test statistic under the null hypothesis using sampling distribution of the test statistic
6. Compare the probability associated with the test statistic with level of significance specified

TEST OF HYPOTHESIS

One sample t test

Exercise 1 : A company claims that on an average it takes only 40 hours or less to process any purchase order. Based on the data given below, can you validate the claim? The data is given in PO_Processing.csv

TEST OF HYPOTHESIS

One sample t test

Exercise 1 : A company claims that on an average it takes only 40 hours or less to process any purchase order. Based on the data given below, can you validate the claim? The data is given in PO_Processing.csv

Reading data to mydata

```
> mydata = PO_Processing$Processing_Time
```

Performing one sample t test

```
> t.test(mydata, alternative = 'greater', mu = 40)
```

Statistics	Value
t	3.7031
df	99
P value	0.0001753

ANALYSIS *of* VARIANCE

ANALYSIS OF VARIANCE

ANOVA

Analysis of Variance is a test of means for two or more populations

Partitions the total variability in the variable under study to different components

H0 = $Mean_1 = Mean_2 = - - - = Mean_k$

Reject H0 if p – value < 0.05

Example:

To study location of shelf on sales revenue

ANALYSIS OF VARIANCE

One Way Anova : Example

An electronics and home appliance chain suspect the location of shelves where television sets are kept will influence the sales revenue. The data on sales revenue in lakhs from the television sets when they are kept at different locations inside the store are given in sales revenue data file. The location is denoted as 1:front, 2: middle & 3: rear. Verify the doubt? The data is given in Sales_Revenue_Anova.csv.

ANALYSIS OF VARIANCE

One Way Anova : Example

Factor: Location(A)

Levels : front, middle, rear

Response: Sales revenue

ANALYSIS OF VARIANCE

One Way Anova : Example

Step 1: Calculate the sum, average and number of response values for each level of the factor (location).

Level 1 Sum(A_1):

Sum of all response values when location is at level 1 (front)

= 1.55 + 2.36 + 1.84 + 1.72

= 7.47

nA_1: Number of response values with location is at level 1 (front)

= 4

ANALYSIS OF VARIANCE

One Way Anova : Example

Step 1: Calculate the sum, average and number of response values for each level of the factor (location).

Level 1 Average:

Sum of all response values when location is at level 1 / number of response values with location is at level 1

= A_1 / nA_1 = 7.47 / 4 = 1.87

ANALYSIS OF VARIANCE

One Way Anova : Example

Step 1: Calculate the sum, average and number of response values for each level of the factor (location).

	Level 1 (front)	Level 2 (middle)	Level 3 (rear)
Sum	A_1: 7.47	A_2: 30.31	A_3: 15.55
Number	nA_1: 4	nA_2: 8	nA_3: 6
Average	1.87	3.79	2.59

ANALYSIS OF VARIANCE

One Way Anova : Example

Step 2: Calculate the grand total (T)

T = Sum of all the response values

= 1.55 + 2.36 + - - - + 2.72 + 2.07 = 53.33

Step 3: Calculate the total number of response values (N)

N = 18

Step 4: Calculate the Correction Factor (CF)

CF = (Grand Total)^2 / Number of Response values

= T^2 / N = $(537.33)^2$ / 18 = 158.0049

ANALYSIS OF VARIANCE

One Way Anova : Example

Step 5: Calculate the Total Sum of Squares (TSS)

TSS = Sum of square of all the response values - CF

$= 1.55^2 + 2.36^2 + - - - + 2.72^2 + 2.07^2 – 158.0049$

$= 15.2182$

ANALYSIS OF VARIANCE

One Way Anova : Example

Step 6: Calculate the between (factor) sum of square

$$SS_A = A_1^2 / nA_1 + A_2^2 / nA_2 + A_3^2 / nA_3 - CF$$

$$= 7.47^2 / 4 + 30.31^2 / 8 + 15.55^2 / 4 – 158.0049$$

$$= 11.0827$$

Step 7: Calculate the within (error) sum of square

SS_e = Total sum of square – between sum of square

$= TSS - SS_A = 15.2182 – 11.0827 = 4.1354$

ANALYSIS OF VARIANCE

One Way Anova : Example

Step 8: Calculate degrees of freedom (df)

Total df = Total Number of response values - 1

= 18 - 1 = 17

Between df

= Number of levels of the factor - 1

= 3 - 1 = 2

Within df = Total df – Between df

= 17 - 2 = 15

ANALYSIS OF VARIANCE

One Way Anova : Example

Anova Table:

Source	df	SS	MS	F	F Crit	P value
Between	2	11.08272	5.541358	20.09949	3.68	0.0000
Within	15	4.135446	0.275696			
Total	17	15.21816				

MS = SS / df

$F = MS_{Between} / MS_{Within}$

F Crit =finv (probability, between df, within df) , probability = 0.05

P value = fdist (F, between df, within df)

ANALYSIS OF VARIANCE

One Way Anova : R Code

Reading data and variables to R

```
> mydata = Sales_Revenue_Anova
> location = mydata$Location
> revenue = mydata$Sales.Revenue
```

Converting location to factor

```
> location = factor(location)
```

Computing ANOVA table

```
> fit = aov(Revenue  ~  location)
> summary(fit)
```

ANALYSIS OF VARIANCE

One Way Anova : Decision Rule

If p value < 0.05, then

The factor has significant effect on the process output or response.

Meaning:

When the factor is changed from 1 level to another level, there will be significant change in the response.

ANALYSIS OF VARIANCE

One Way Anova : Example Result

For factor Location, $p = 0.000 < 0.05$

Conclusion:

Location has significant effect on sales revenue

Meaning:

The sales revenue is not same for different locations like front, middle & rear

ANALYSIS OF VARIANCE

One Way Anova : Example Result

The expected sales revenue for different location under study is equal to level averages.

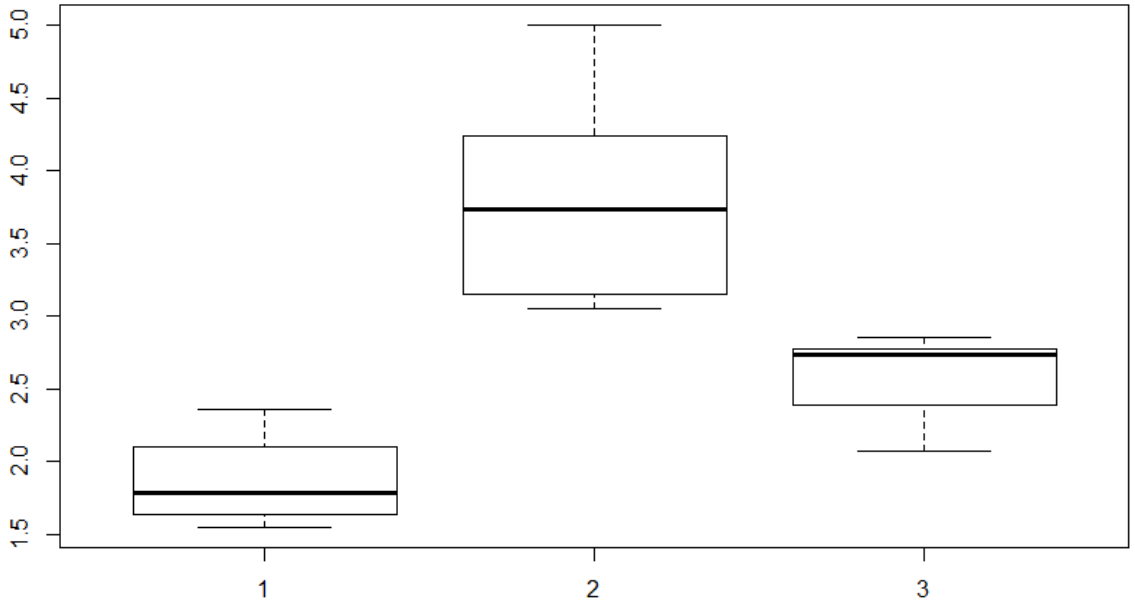
Location	Expected Sales Revenue
Front	1.8675
Middle	3.78875
rear	2.591667

```
> aggregate(Revenue ~ location, FUN = mean)
```

ANALYSIS OF VARIANCE

One Way Anova : Example Result

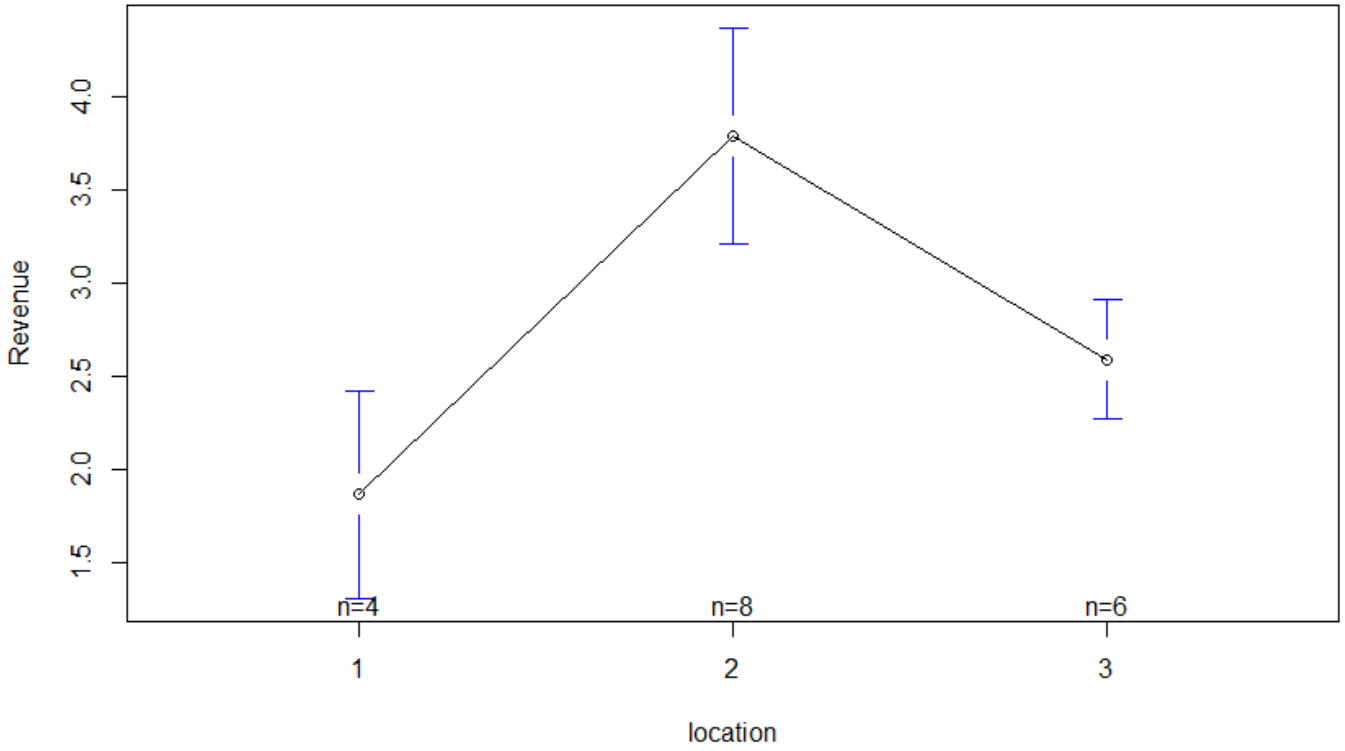
```
> boxplot(Revenue ~ location)
```



ANALYSIS OF VARIANCE

One Way Anova : Example Result

```
> library(gplots)
> plotmeans(Revenue ~ location)
```



ANALYSIS OF VARIANCE

One Way Anova : Tukey's Honestly Significant Difference (HSD) Test

Used to do pair wise comparison between the levels of factors

R code

```
>TukeyHSD(fit)
```

Comparison	Mean difference	Lower	Upper	p value
2 - 1	1.92125	1.086067	2.756433	0.0000
3 - 1	0.724167	-0.15619	1.604527	0.1158
3 - 2	-1.19708	-1.93365	-0.46052	0.0020

ANALYSIS OF VARIANCE

Anova logic:

Two Types of Variations:

1. Variation within the level of a factor
2. Variation between the levels of factor

ANALYSIS OF VARIANCE

Anova logic :

Variation between the level of a factor:

The effect of Factor.

Variation within the levels of a factor:

The inherent variation in the process or Process Error.

	Location		
	Front	Middle	rear
Sales Revenue	1.34 1.89 1.35 2.07 2.41 3.06	3.20 2.81 4.52 4.40 4.75 5.19 3.42 9.80	2.30 1.91 1.40 1.48

ANALYSIS OF VARIANCE

Anova logic :

If the variation between the levels of a factor is significantly higher than the inherent variation

then the factor has significant effect on response

To check whether a factor is significant:

Compare variation between levels with variation within levels

ANALYSIS OF VARIANCE

Anova logic :

Measure of variation between levels: MS of the factor ($MS_{between}$)

Measure of variation within levels: MS Error (MS_{within})

To check whether a factor is significant:

Compare MS of between with MS within

i.e. Calculate $F = MS_{between} / MS_{within}$

If F is very high, then the factor is significant.

ANALYSIS OF VARIANCE

Variation Within levels:

Ideally variation within all the levels should be same

To check whether variation within the levels are same or not

Do Bartlett's test

If p value ≥ 0.05, then variation within the levels are equal, otherwise not

R Code for Bartlett's test

```
> bartlett.test(Revenue, location, data = mydata)
```

ANALYSIS OF VARIANCE

Variation Within levels:

Bartlett's Test result for sales revenue (location of TV sets) example

Bartlett's K^2 Statistic	df	p value
3.8325	2	0.1472

Since p value = 0.1472 > 0.05, the variance within the levels are equal

REGRESSION ANALYSIS

REGRESSION ANALYSIS

Regression

Correlation helps

To check whether two variables are related

If related

Identify the type & degree of relationship

Regression helps

- To identify the exact form of the relationship
- To model output in terms of input or process variables

REGRESSION ANALYSIS

Exercise 1: The data from the pulp drying process is given in the file DC_Simple_Reg.csv. The file contains data on the dry content achieved at different dryer temperature. Develop a prediction model for dry content in terms of dryer temperature.

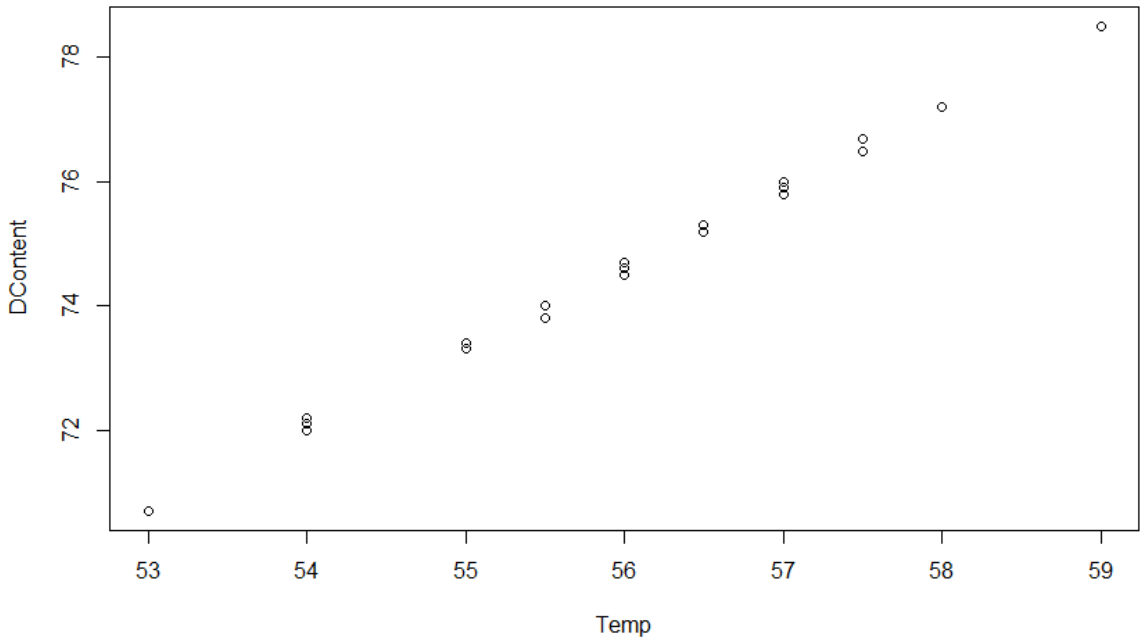
1. Reading the data and variables

```
> mydata = DC_Simple_Reg
> Temp = mydata$Dryer.Temperature
> DContent = mydata$Dry.Content
```

REGRESSION ANALYSIS

2. Constructing Scatter Plot

```
> plot(Temp, DContent)
```



REGRESSION ANALYSIS

3. Computing Correlation Matrix

```
> cor(Temp, DContent)
```

Attribute	Dry Content
Temperature	0.9992

Remark:
Correlation between y & x need to be high (preferably 0.8 to 1 to -0.8 to -1.0)

REGRESSION ANALYSIS

4: Performing Regression

```
> model = lm(DContent ~ Temp)

> summary(model)
```

Statistic	Value	Criteria
Residual standard error	0.07059	
Multiple R-squared	0.9984	> 0.6
Adjusted R-squared	0.9983	> 0.6

Model	df	F	p value
Regression	1	24497	0.000
Residual	40		
Total	41		

Criteria:
P value < 0.05

REGRESSION ANALYSIS

4: Performing Regression

Attribute	Coefficient	Std. Error	t Statistic	p value
Intercept	2.183813	0.463589	4.711	0.00
Temperature	1.293432	0.008264	156.518	0.00

Interpretation

The p value for independent variable need to be < significance level α (generally $\alpha = 0.05$)

Model: Dry Content = 2.183813 + 1.293432 x Temperature

REGRESSION ANALYSIS

5: Regression Anova

```
> anova(model)
```

ANOVA

Source	SS	df	MS	F	p value
Temp	122.057	1	122.057	24497	0.000
Residual	0.199	40	0.005		
Total	122.256	41			

Criteria: P value < 0.05

REGRESSION ANALYSIS

5: Residual Analysis

```
> pred = fitted(model)
> Res = residuals(model)
> write.csv(pred,"D:/Infosys/DataSets/Pred.csv")
> write.csv(Res,"D:/Infosys/DataSets/Res.csv")
```

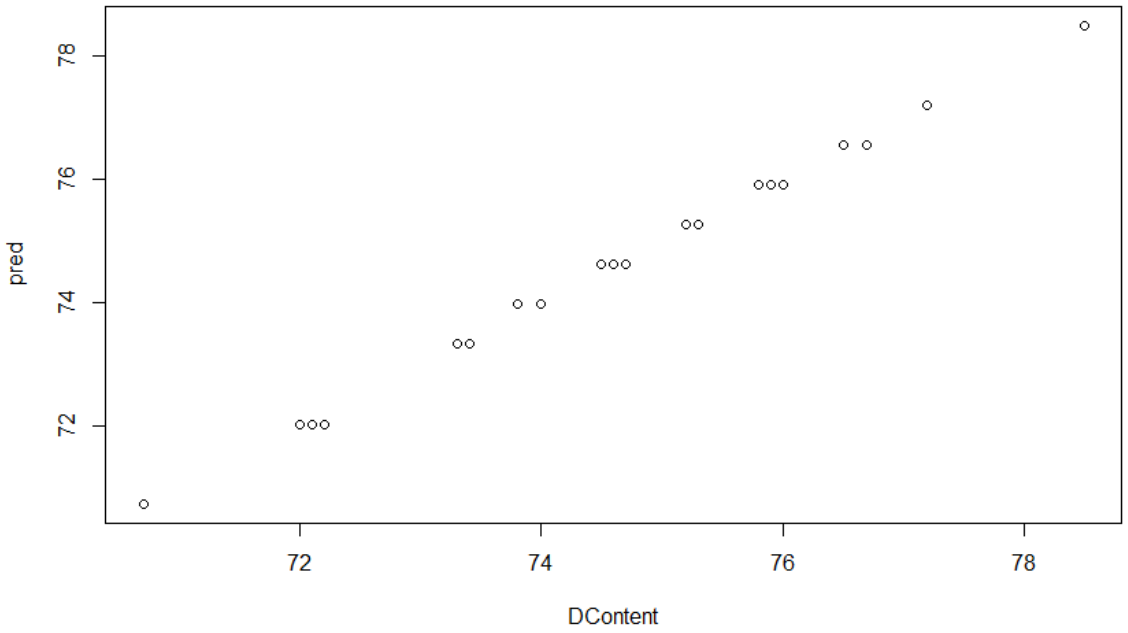
SL No.	Fitted	Residuals	SL No.	Fitted	Residuals
1	73.32259	-0.02259	22	74.61602	-0.01602
2	74.61602	-0.01602	23	75.26274	-0.06274
3	73.96931	0.030693	24	73.96931	0.030693
4	78.49632	0.00368	25	75.90946	-0.00946
5	74.61602	-0.01602	26	75.26274	0.03726
6	73.96931	0.030693	27	73.96931	0.030693
7	75.26274	-0.06274	28	78.49632	0.00368
8	77.20289	-0.00289	29	76.55617	-0.05617
9	75.90946	-0.00946	30	74.61602	-0.11602
10	74.61602	-0.01602	31	75.90946	0.090544
11	73.32259	-0.02259	32	76.55617	-0.05617
12	75.90946	-0.00946	33	76.55617	0.143828
13	75.90946	0.090544	34	75.90946	0.090544
14	74.61602	-0.01602	35	75.90946	-0.10946
15	74.61602	0.083977	36	73.96931	-0.16931
16	74.61602	-0.11602	37	73.32259	-0.02259
17	70.73573	-0.03573	38	74.61602	-0.01602
18	72.02916	-0.02916	39	73.32259	0.077409
19	72.02916	0.070841	40	75.90946	0.090544
20	72.02916	0.170841	41	73.96931	0.030693
21	70.73573	-0.03573	42	75.26274	-0.06274

REGRESSION ANALYSIS

5: Residual Analysis

Scatter Plot: Actual Vs Predicted (fit)

> plot(DContent, pred)



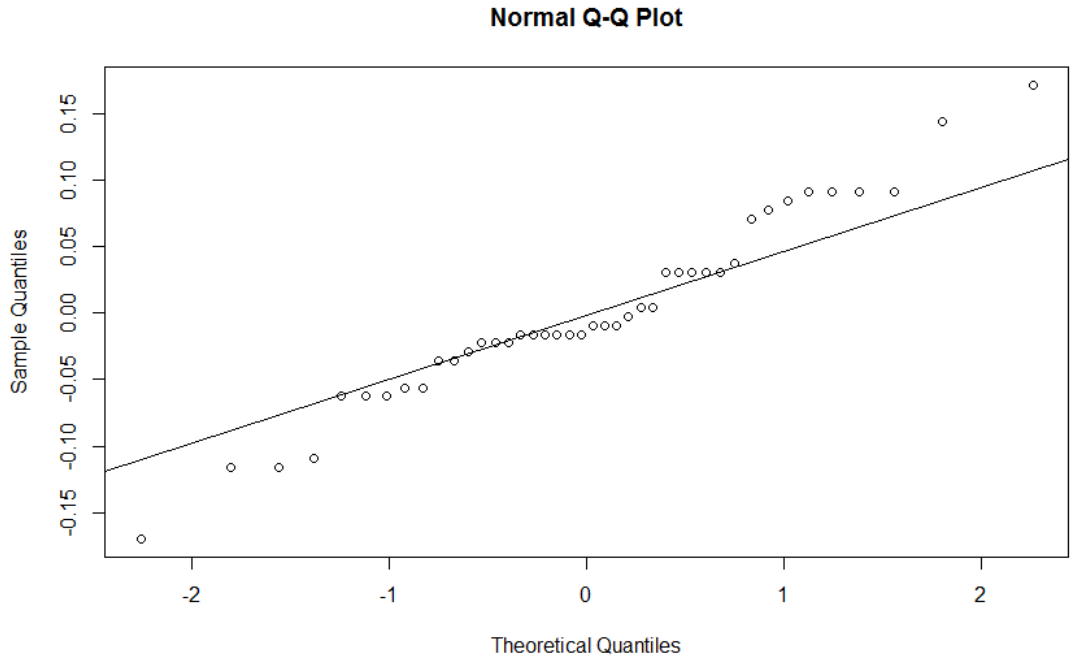
REGRESSION ANALYSIS

5: Residual Analysis

Normality Check on residuals

> qqnorm(Res)

> qqline(Res)



Residuals should be normally distributed or bell shaped

REGRESSION ANALYSIS

5: Residual Analysis

Normality Check on residuals

```
> shapiro.test(Res)
```

Shapiro-Wilk normality Test:	
W	p value
0.9693	0.3132

Residuals should be normally distributed or bell shaped

REGRESSION ANALYSIS

5: Residual Analysis

```
> plot(pred, Res)
> plot(Temp, Res)
```

Residuals should be independent and stable

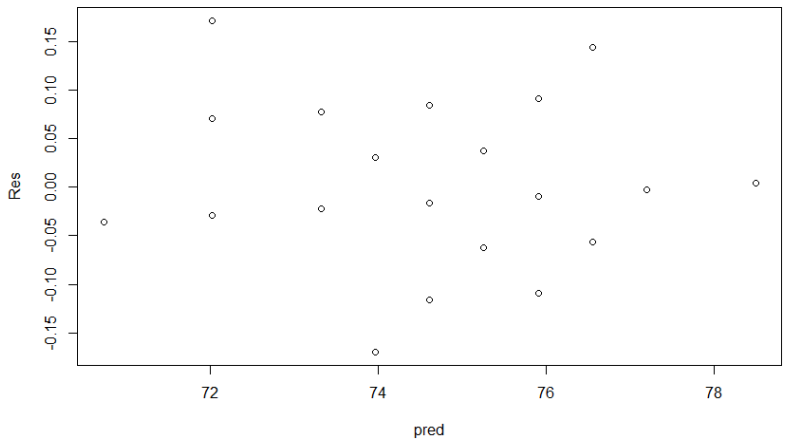
Plot the residuals against fitted value. The points in the graph should be scattered randomly and should not show any trend or pattern. The residuals should not depend in anyway on the fitted value.

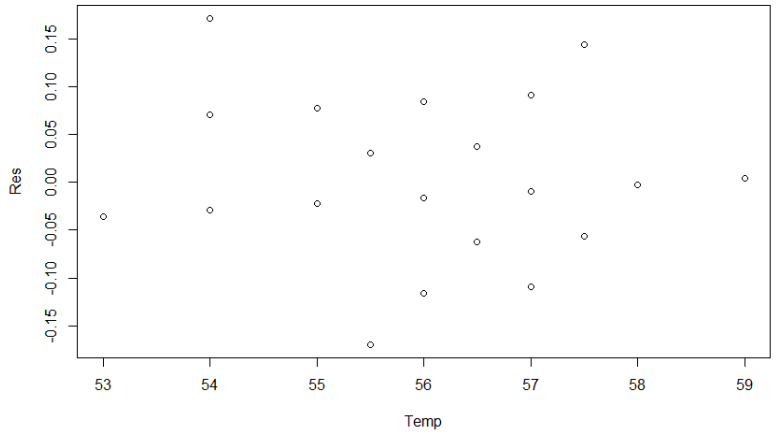
If there is a pattern then a transformation such as log y or $\sqrt{y}$ to be used

Similarly the residuals shall not depend on x. This can be checked by plotting residuals vs x. A pattern in this plot is an indication that the residuals are not independent of x. Instead of x, develop the model with a function of as predictor (Eg: x^2, 1/x, $\sqrt{x}$, log(x), etc.)

REGRESSION ANALYSIS

Residual Analysis





There is no trend or pattern on residuals vs fitted value ,residuals vs observation order or residuals vs x plot. Hence the assumptions of independence and stability of residuals are satisfied.

REGRESSION ANALYSIS

6: Outlier test

Observations with Bonferonni p – value < 0.05 are potential outliers

```
> library(car)
> outlierTest(model)
```

Observation	Studentized Residual	Bonferonni p value
20	2.723093	0.40417

REGRESSION ANALYSIS

7: Leave One Out Cross Validation (LOOCV)

- Split the data into two parts : training data and test data

  Test data consists of only one observation (x_1, y_1)

  Training data consists of the remaining n – 1 observations namely (x_2, y_2) , (x_3, y_3) , - - -, (x_n, y_n)

- Develop the model using n – 1 training data observations and predict the response y_1 of the test data observation

  Compute the residuals and mean square error $MSE_1 = (y_{1actual} – y_{1pred})^2$

- Repeat the process by taking (x_1, y_1) as test data and the remaining n – 1 observations as training data
- Compute MSE_2
- Repeating the procedure n times produces n squared errors MSE_1, MSE_2, - - -, MSE_n
- LOOCV estimate of the test MSE is the average of these n test error estimates

$$CV_{(n)} = \frac{1}{n}\sum_{i=1}^{n} MSE_i$$

REGRESSION ANALYSIS

7: Leave One Out Cross Validation (LOOCV)

```
> library(boot)
> attach(mydata)
> mymodel = glm(Dry.Content ~ Dryer.Temperature)
> valid = cv.glm(mydata, mymodel)
> valid$delta[1]
```

Statistic	Value
Delta	0.005201004

REGRESSION ANALYSIS

Multiple Linear Regression

To model output variable y in terms of two or more variables.

General Form:

$$y = a + b_1x_1 + b_2x_2 + - - - + b_kx_k + \varepsilon$$

Two variable case:

$$y = a + b_1x_1 + b_2x_2 + \varepsilon$$

Where

a: intercept (the predicted value of y when all x's are zero)

b_j: slope (the amount change in y for unit change in x_j keeping all other x's constant, j = 1,2,---,k)

REGRESSION ANALYSIS

Exercise : The effect of temperature and reaction time affects the % yield. The data collected in given in the Mult-Reg_Yield file. Develop a model for % yield in terms of temperature and time?

Step 1: Correlation Analysis

Attribute	Time	Temperature	% Yield
Time	1.00	-0.01	0.90
Temperature	-0.01	1.00	-0.05
% Yield	0.90	-0.05	1.00

Correlation between xs & y should be high

Correlation between xs should be low

REGRESSION ANALYSIS

Step 2: Regression Output

Statistic	Value	Criteria
Adjusted R Square	0.7766	≥ 0.6

Regression ANOVA

Model	SS	df	MS	F	p value
Regression	6797.063	2	3398.531	27.07	0.0000
Residual	1632.08138	13	125.5447		
Total	8429.14438	15			

Criteria: P value < 0.05

REGRESSION ANALYSIS

Step 2: Regression Output

ANOVA

Source	SS	df	MS	F	p value
Time	6777.8	1	6777.8	53.9872	0.000
Temp	19.3	1	19.3	0.1534	0.702
Residual	1632.1	13	125.5		

Criteria: P value < 0.05

REGRESSION ANALYSIS

Step 2: Regression Output – Identify the model

Attribute	Coefficient	Std. Error	t Statistic	p value
Time	0.9061	0.12337	7.344	0.0000
Temperature	-0.0642	0.16391	-0.392	0.702
Intercept	-67.8844	40.58652	-1.67	0.118

Interpretation: Only time is related to % yield as p value < 0.05

REGRESSION ANALYSIS

Step 2: Regression Output – Identify the model

Attribute	Coefficient	Std. Error	t Statistic	p value
Time	0.9065	0.1196	7.580	0.0000
Intercept	-81.6205	19.7906	-4.124	0.00103

Model % Yield= 0.9065 x Time - 81.621

REGRESSION ANALYSIS

Step 3: Residual Analysis

SL No.	Temperature	% Yield	Predicted	Time
1	190	35.0	36.22	130
2	176	81.7	76.10	174
3	205	42.5	39.84	134
4	210	98.3	91.51	191
5	230	52.7	67.94	165
6	192	82.0	94.23	194
7	220	34.5	48.00	143
8	235	95.4	86.98	186
9	240	56.7	44.38	139
10	230	84.4	88.79	188
11	200	94.3	77.01	175
12	218	44.3	59.79	156
13	220	83.3	90.61	190
14	210	91.4	79.73	178
15	208	43.5	38.03	132
16	225	51.7	52.53	148

REGRESSION ANALYSIS

Step 3: Residual Analysis: Outlier detection

SL No.	Temperature	% Yield	Predicted	Time	Residuals	Std Residuals
1	190	35	36.22	130	-1.22	-0.126
2	176	81.7	76.1	174	5.60	0.5358
3	205	42.5	39.84	134	2.66	0.2686
4	210	98.3	91.51	191	6.79	0.6784
5	230	52.7	67.94	165	-15.24	-1.45
6	192	82	94.23	194	-12.23	-1.238
7	220	34.5	48	143	-13.50	-1.322
8	235	95.4	86.98	186	8.42	0.8272
9	240	56.7	44.38	139	12.32	1.2221
10	230	84.4	88.79	188	-4.39	-0.434
11	200	94.3	77.01	175	17.29	1.6575
12	218	44.3	59.79	156	-15.49	-1.479
13	220	83.3	90.61	190	-7.31	-0.727
14	210	91.4	79.73	178	11.67	1.1244
15	208	43.5	38.03	132	5.47	0.5582
16	225	51.7	52.53	148	-0.83	-0.081
				Mean	0.000	
				SD	10.4918	

REGRESSION ANALYSIS

Step 3: Residual Analysis:

Shapiro-Wilk normality Test: Yield data	
W	p value
0.9449	0.4132

REGRESSION ANALYSIS

6: Outlier test

Observations with Bonferonni p – value < 0.05 are potential outliers

```
> library(car)
> outlierTest(mymodel)
```

Observation	Studentized Residual	Bonferonni p value
11	1.781515	NA

REGRESSION ANALYSIS

7: Leave One Out Cross Validation (LOOCV)

```
> library(boot)
> attach(mydata)
> mymodel = glm(X.Yield ~ Time)
> myvalidation = cv.glm(mydata, mymodel)
> myvalidation$delta[1]
```

Statistic	Value
Delta	128.8541

REGRESSION ANALYSIS

Exercise : The effect of temperature, time and kappa number of pulp affects the % conversion of UB pulp to Cl_2 pulp. inspection. The data collected in given in the Mult_Reg_Conversion file. Develop a model for % conversion in terms of exploratory variables?

REGRESSION ANALYSIS

Step 1: Correlation Analysis

	Temperature	Time	Kappa #	% Conversion
Temperature	1.00	-0.96	0.22	0.95
Time	-0.96	1.00	-0.24	-0.91
Kappa #	0.22	-0.24	1.00	0.37
% Conversion	0.95	-0.91	0.37	1.00

Interpretation

High Correlation between % Conversion and Temperature & Time

High Correlation between Temperature & Time - Multicollinearity

REGRESSION ANALYSIS

Measure for Multicollinearity

Variance Inflation Factor (VIF)

Measures the correlation (linear association) between each x variable with other x's

$VIF_i = 1/(1- R_i^2)$

Where R_i is the coefficient for regressing x_i on other x's

Criteria: VIF < 5

REGRESSION ANALYSIS

Regression Output

Statistic	Value	Criteria
Adjusted R Square	0.899	> 0.6

Regression ANOVA

Model	SS	df	MS	F	p value
Regression	1953.419	3	651.140	45.885	0.0000
Residual	170.290	12	14.191		
Total	2123.709	15			

REGRESSION ANALYSIS

Regression Output

	Coeff	Std. Error	t	p value
Constant	-121.27	55.43571	-2.19	0.0492
Temperature	0.12685	0.04218	3.007	0.0109
Time	-19.0217	107.92824	-0.18	0.863
Kappa #	0.34816	0.17702	1.967	0.0728

Variance-inflation factors (VIF)

```
> vif(mymodel)
```

x	VIF
Temperature	12.23
Time	12.33
Kappa #	1.062

REGRESSION ANALYSIS

Tackling Multicollinearity:

1. Remove one or more of highly correlated independent variable
2. Principal Component Regression
3. Partial Least Square Regression
4. Ridge Regression

REGRESSION ANALYSIS

Tackling Multicollinearity:

Method 1: Removing highly correlated variable – Stepwise Regression

Approach

- A null model is developed without any predictor variable x. In null model, the predicted value will be the overall mean of y
- Then predictor variables x's are added to the model sequentially
- After adding each new variable, the method also remove any variable that no longer provide an improvement in the model fit
- Finally the best model is identified as the one which minimizes Akaike information criterion *(*AIC)

$$AIC = \frac{1}{n\hat{\sigma}^2}(RSS + 2d\hat{\sigma}^2$$

REGRESSION ANALYSIS

Tackling Multicollinearity:

Method 1: Removing highly correlated variable – Stepwise Regression

Akaike information criterion (AIC)

$$AIC = \frac{1}{n\hat{\sigma}^2}(RSS + 2d\hat{\sigma}^2)$$

n: number of observations

$\hat{\sigma}^2$: estimate of error or residual variance

d: number of x variables included in the model

RSS: Residual sum of squares

REGRESSION ANALYSIS

Tackling Multicollinearity:

Method 1: Removing highly correlated variable – Stepwise Regression

R code

```
> library(MASS)
> mymodel = lm(X..Conversion ~ Temperature + Time + Kappa.number)
> step =stepAIC(mymodel, direction = "both")
```

Step	x's in the model	AIC
1	Temperature, Time & Kappa Number	45.8
2	Temperature & Kappa Number	43.9

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 1: Stepwise Regression

Attribute	Coefficient	Std. Error	t Statistic	p value
Temperature	0.13396	0.01191	11.250	0.0000
Kappa #	0.35106	0.16955	2.071	0.0589
Intercept	-130.68986	14.14571	-9.239	0.0000

% Conversion = 0.13396 * Temperature + 0.35106 * Kappa # - 130.68986

Variance-inflation factors (VIF)

x	VIF
Temperature	1.0526
Kappa #	1.0526

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 1: Stepwise Regression

```
> pred = predict(mymodel)
> res = residuals(mymodel)
 > cbind(X..Conversion, pred, res)
> mse = mean(res^2)
> rmse = sqrt(mse)
```

Statistic	Value
Mean Square Error (MSE)	10.7
Root Mean Square Error (RMSE)	3.27

REGRESSION ANALYSIS

k fold Cross Validation

Steps

1. Divide the data set into k equal subsets
2. Keep one subset (sample) for model validation
3. Develop the model using all the other k – 1 subsets data put together
4. Predict the responses for the test data and compute residuals
5. Return the test sample back to the original data set and take another subset for model validation
6. Go to step 3 and continue until all the subsets are tested with different models
7. Compute the overall Root Mean Square Residuals. RMSE of validation should not be high compared to the original model developed with all the data points together.

Note: when k = n, then k fold cross validation is same as leave one out cross validation

REGRESSION ANALYSIS

k fold Cross Validation

R code

```
> library(DAAG)
> cv.lm(mymodel, m = 16)
> cv.lm(mymodel, df = mydata, m = 16)
```

m: number of validations required. M = 16 = n, hence equal to leave one out cross validation

Model	MSE	RMSE
Original	10.7	3.27
Cross Validation	19.6	4.43

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 2: Principal Component Regression

1. Perform principal component analysis on x variables
2. Use the principal components as x variables and develop the model

R Code : Principal Component Regression

```
> mydata = mydata[,2:5]
> attach(mydata)
> library(pls)
> mymodel = pcr(X..Conversion ~ ., data = mydata, scale = TRUE)
> summary(mymodel)
> mymodel$loadings
```

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 2: Principal Component Regression

1. Perform principal component analysis on x variables
2. Use the principal components as x variables and develop the model

Cum % Variance	PC1	PC2	PC3
x	68.66	98.61	100
Conversion (y)	90.48	90.62	91.98

Component 1 or 1 & 2 may be sufficient to include in the model

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 2: Principal Component Regression

1. Perform principal component analysis on x variables
2. Use the principal components as x variables and develop the model

Loadings	PC1	PC2	PC3
Temperature	-0.674	0.218	0.705
Time	0.677	-0.2	0.709
Kappa.number	-0.296	-0.955	0

Component 1 is taking care of information in temperature and Time and
Component 2 is mostly representing kappa number

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 2: Principal Component Regression

Principal Component Scores

SL No.	Comp 1	Comp 2	Comp 3
1	-1.079	1.2498	0.1202
2	-1.158	0.9967	0.1236
3	-1.273	0.6625	0.117
4	-1.371	0.2313	0.1563
5	-1.543	-0.362	0.1756
6	-1.889	-1.365	0.1558
7	0.4709	1.1733	-0.133
8	0.3133	0.8148	-0.173
9	0.0021	0.2622	-0.299
10	-0.257	-0.122	-0.428
11	-0.268	-0.763	-0.24
12	-0.432	-1.819	-0.07
13	2.2484	0.6246	-0.022
14	2.4329	0.165	0.2963
15	2.1218	-0.388	0.1699
16	1.6801	-1.362	0.0493

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 2: Principal Component Regression

Identifying the required number of components in the model

```
> pred = predict(mymodel, type = "response", ncomp = 1)
> res = X..Conversion - pred
> mse = mean(res^2)
> prednew = predict(mymodel, type = "response", ncomp = 2)
> resnew = X..Conversion - prednew
> msenew = mean(resnew^2)
```

Statistics	Regression with	
	PC1	PC1 & PC2
MSE	12.64226	12.45593

Since there is not much reduction in MSE by including the second principal component , only PC1 is required for modelling

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 3: Partial Least Square Regression

Principal component regression involves the identification of a linear combinations of predictors that best represents the x variables

The response y is not used to help the determination of principal components

The response does not supervise the identification of principal components

Identifies the best linear combinations which best explains the predictor variables x but may not the ones best for predicting the response y

Partial least square regression is a supervised alternative to principal component regression

Partial least square method identifies the components or directions (linear combinations of x variables) using the response variable y.

Partial least square places highest weight on the variables that are most strongly related the response y

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 3: Partial Least Square Regression

R code

```
> mydata = mydata[,2:5]
> attach(mydata)
> library(pls)
> mymodel = plsr(X..Conversion ~ ., data = mydata, scale = TRUE)
> summary(mymodel)
> mymodel$loading
```

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 3: Partial Least Square Regression

Cum % Variance	PLS1	PLS2	PLS3
x	68.65	96.92	100
Conversion (y)	90.63	90.86	91.98

Loadings	PLS1	PLS2	PLS3
Temperature	0.677	0.344	0.299
Time	-0.679	-0.207	0.607
Kappa.number	0.285	-1.391	0.736

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 3: Partial Least Square Regression

```
> ps = mymodel$scores
> score = ps[,1:2]
```

SL No	PLS1	PLS2
1	1.11324	0.89634
2	1.18502	0.73368
3	1.2913	0.51027
4	1.3792	0.25877
5	1.5361	-0.1142
6	1.85493	-0.7845
7	-0.4425	0.66627
8	-0.2949	0.40157
9	-0.0005	-0.0564
10	0.24599	-0.4059
11	0.24426	-0.6809
12	0.3833	-1.24
13	-2.2314	0.4067
14	-2.4222	0.35105
15	-2.1279	-0.1069
16	-1.7138	-0.8359

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 3: Partial least square regression

Identifying the required number of components in the model

```
> pred = predict(mymodel, data = mydata, scale = TRUE, ncomp = 1)
> res = X..Conversion - pred
> mse = mean(res^2)

> prednew = predict(mymodel, , data = mydata, scale = TRUE ,  ncomp = 2)
> resnew = X..Conversion - prednew
> msenew = mean(resnew^2)
```

Statistics	Regression with	
	PLS1	PLS11 & PLS2
MSE	12.44252	12.13185

Since there is not much reduction in MSE by including the second component , only PLS1 is required for modelling

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 4: Ridge regression

In least square regression, the coefficients β's of x variables are identified by minimizing the residual sum of squares

$$RSS = \sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij})^2$$

In ridge regression, the coefficients β's of x variables are identified by minimizing a slightly different quantity

$$\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{p}\beta_j x_{ij})^2 + \lambda\sum_{j=1}^{p}\beta_j^2 = RSS + \lambda\sum_{j=1}^{p}\beta_j^2$$

Where $\lambda \geq 0$ is a turning parameter and $\lambda\sum_{j=1}^{p}\beta_j^2$ is the shrinkage penalty,

which will be small when $\beta_1, \beta_2, - - -, \beta_p$ are close to zero.

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 4: Ridge regression

Ridge regression seeks coefficient estimates that fit the data well by minimizing the RSS and the tuning parameter λ has the effect of shrinking the estimates β_j towards zero
The value of λ s identified through 10 fold cross validation

10 fold Cross Validation

- Divide the data set into 10 equal parts
- Develop the model using 9 parts and test it with the remaining one part
- Repeat the process 10 times to get an unbiased estimate of MSE

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 4: Ridge regression

R Code

```
> library(glmnet)
> set.seed(1)
> y = mydata[,5]]
> x =mydata[,2:4]
> x = as.matrix(x)
```

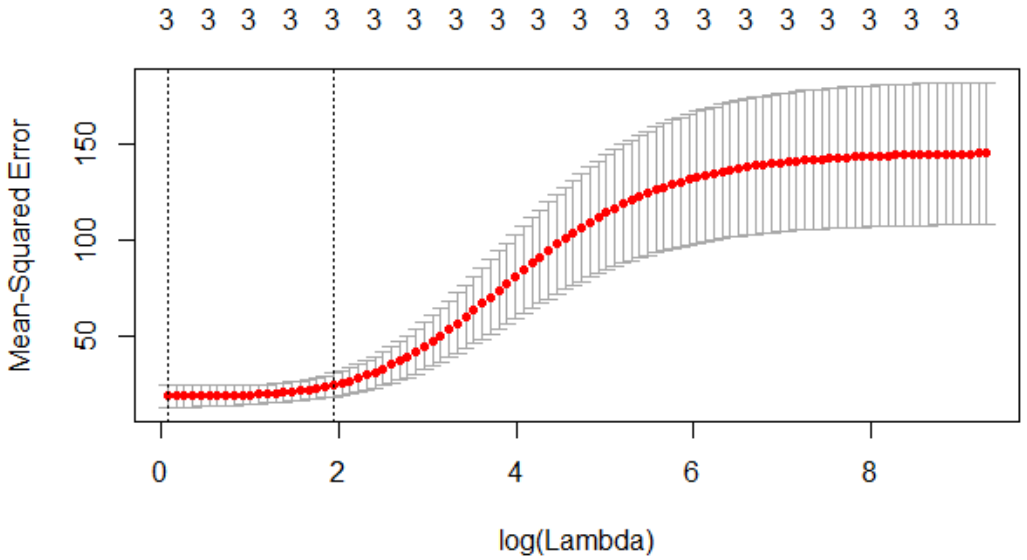
Cross Validation

```
> mymodel = cv.glmnet(x , y, alpha =0)
> plot(mymodel)
```

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 4: Ridge regression



Choose the λ which minimizes the mean square error

> bestlambda = mymodel$lambda.min

Best λ= 1.088771

REGRESSION ANALYSIS

Tackling Multi collinearity:

Method 4: Ridge regression

Develop the model with best λ and identify the coefficients

```
> mynewmodel = glmnet(x, y, alpha = 0)
> predict (mynewmodel, type = "coefficients", s = bestlambda)[1:4,]
```

Variable	Coefficients
(Intercept)	-63.0713
Temperature	0.0823
Time	-117.5048
Kappa.number	0.3268

CORRELATION & REGRESSION

Regression with dummy variables

When x's are not numeric but nominal

Each nominal or categorical variable is converted into dummy variables

Dummy variables takes values 0 or 1

Number of dummy variable for one x variable is equal to number of distinct values of that variable - 1

Example: A study was conducted to measure the effect of gender and income on attitude towards vocation. Data was collected from 30 respondents and is given in Travel_dummy_reg file. Attitude towards vocation is measured on a 9 point scale. Gender is coded as male = 1 and female = 2. Income is coded as low=1, medium = 2 and high = 3. Develop a model for attitude towards vocation in terms of gender and Income?

CORRELATION & REGRESSION

Regression with dummy variables

Variable		Dummy
Gender	Code	gender_Code
Male	1	0
Female	2	1

Variable		Dummy	
Income	Code	Income1	Income 2
Low	1	0	0
Medium	2	1	0
High	3	0	1

CORRELATION & REGRESSION

Regression with dummy variables

Read the fie and variables

```
> mydata =  Travel_dummy_Reg
> mydata = mydata[,2:4]
> gender = mydata$Gender
> Income = mydata$Income
> Attitude = mydata$Attitude
```

Converting categorical x's to factors

```
> gender = factor(gender)
> income = factor(income)
```

CORRELATION & REGRESSION

Regression with dummy variables – Output

> mymodel = lm(attitude ~ genser + income)

> summary (mumodel)

Multiple R^2	0.8603
Adjusted R^2	0.8442
F Statistics	53.37
P value	0.00

	Estimate	Std. Error	t value	p value
(Intercept)	2.4	0.3359	7.145	0.00000
gender2	-1.6	0.3359	-4.763	0.00006
income2	2.8	0.4114	6.806	0.00000
income3	4.8	0.4114	11.668	0.00000

CORRELATION & REGRESSION

Regression with dummy variables – Output

> anova (mumodel)

	Df	Sum Sq	Mean Sq	F	p value
gender	1	19.2	19.2	22.691	0.0001
income	2	116.27	58.133	68.703	0.0000
Residuals	26	22	0.846		

BINARY LOGISTIC REGRESSION

BINARY LOGISTIC REGRESSION

Used to develop models when the output or response variable y is binary

The output variable will be binary, coded as either success or failure

Models probability of success p which lies between 0 and 1

Linear model is not appropriate

$$p = \frac{e^{a+b_1x_1+b_2x_2+---+b_kx_k}}{1+e^{a+b_1x_1+b_2x_2+---+b_kx_k}}$$

p: probability of success
x_i's : independent variables
a, b_1, b_2, ---: coefficients to be estimated

If estimate of $p \geq 0.5$, then classified as success, otherwise as failure

BINARY LOGISTIC REGRESSION

Usage: When the dependant variable (Y variable) is binary

Example: Develop a model to predict the number of visits of family to a vacation resort based on the salient characteristics of the families. The data collected from 30 households is given in Resort_Visit.csv

1. Reading the file and variables

```
> mydata = Resort_Visit
> visit = mydata$Resort_Visit
> income = mydata$Family_Income
> attitude = mydata$Attitude.Towards.Travel
> importance = mydata$Importance_Vacation
> size = mydata$House_Size
> age = mydata$Age._Head
```

2. Converting response variable to discrete

```
> visit = factor(visit)
```

BINARY LOGISTIC REGRESSION

3. Correlation Matrix

> cor(mydata)

	Resort_Visit	Family_Income	Attitude_Travel	Importance_Vacation	House_Size	Age_Head
Resort_Visit	1.00	-0.60	-0.27	-0.42	-0.59	-0.21
Family_Income	-0.60	1.00	0.30	0.23	0.47	0.21
Attitude_Travel	-0.27	0.30	1.00	0.19	0.15	-0.13
Importance_Vacation	-0.42	0.23	0.19	1.00	0.30	0.11
House_Size	-0.59	0.47	0.15	0.30	1.00	0.09
Age_Head	-0.21	0.21	-0.13	0.11	0.09	1.00

Interpretation: Correlation between X variables should be low

BINARY LOGISTIC REGRESSION

4. Converting response variable to discrete

```
> visit = factor(visit)
```

5. Checking relation between Xs and Y

```
> aggregate(income ~visit, FUN = mean)
> aggregate(attitude ~visit, FUN = mean)
> aggregate(importance ~visit, FUN = mean)
> aggregate(size ~visit, FUN = mean)
> aggregate(age ~visit, FUN = mean)
```

Resort_Visit	Mean				
	Family_Income	Attitude_Travel	Importance_Vacation	House_Size	Age_Head
0	58.5200	5.4000	5.8000	4.3333	53.7333
1	41.9133	4.3333	4.0667	2.8000	50.1333

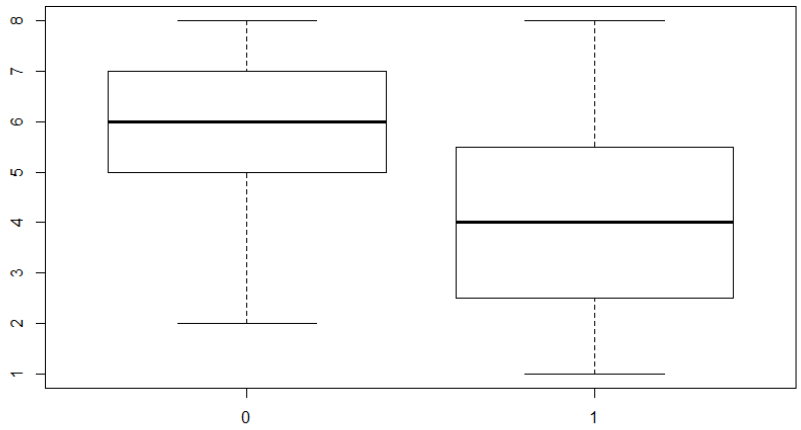
Higher the difference in means, stronger will be the relation to response variable

BINARY LOGISTIC REGRESSION

5. Checking relation between Xs and Y – box plot

```
> boxplot(income ~ visit)
> boxplot(attitude ~ visit)
> boxplot(importance ~ visit)
> boxplot(size ~ visit)
> boxplot(age ~ visit)
```

Income Vs visit



BINARY LOGISTIC REGRESSION

6. Perform Logistic regression

```
> model = glm(visit ~ income + attitude + importance + size + age, family = binomial(logit))
> summary(model)
```

	Estimate	Std. Error	z value	Pr(>|z|)
(Intercept)	15.49503	6.68017	2.32	0.0204
Income	-0.11698	0.06605	-1.771	0.0766
attitude	-0.28129	0.33919	-0.829	0.4069
importance	-0.46157	0.32006	-1.442	0.1493
size	-0.80699	0.49314	-1.636	0.1018
age	-0.07019	0.07199	-0.975	0.3295

BINARY LOGISTIC REGRESSION

6. Perform Logistic regression - Anova

> anova(model,test = 'Chisq')> summary(model)

	Df	Deviance	Resid.Df	Resid.Dev	Pr(>Chi)
NULL	29	41.589			
income	1	12.9813	28	28.608	0.00031
attitude	1	0.4219	27	28.186	0.51598
importance	1	3.8344	26	24.351	0.05021
size	1	3.4398	25	20.911	0.06364
age	1	1.0242	24	19.887	0.31152

Since p value < 0.05 for Income, Importance_Vacation & Size, redo the modelling with important factors only

BINARY LOGISTIC REGRESSION

7. Perform Logistic regression - Modified

	Estimate	Std Error	z value	p value
(Intercept)	8.46599	3.02494	2.799	0.00513
Income	-0.10641	0.05156	-2.064	0.03904
Size	-0.93539	0.47632	-1.964	0.04955

Since p value < 0.05 for both factors, Income & Size, the response variable can be modelled in terms of those two factors

The model is

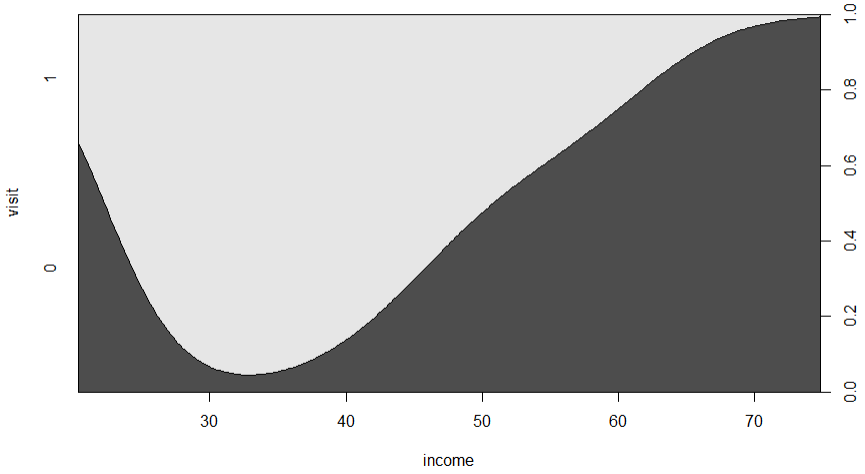
$$y = \frac{e^{8.46599 - 0.10641 Annual\_Income - 0.93539 Size}}{1 + e^{8.46599 - 0.10641 Annual\_Income - 0.93539 Size}}$$

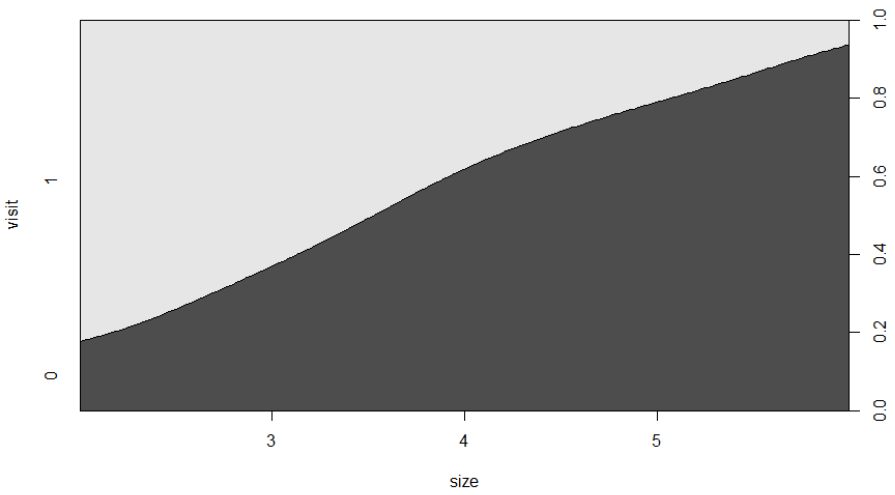
BINARY LOGISTIC REGRESSION

8. Conditional Density plots (Response Vs Factors)

Describing how the conditional distribution of a categorical variable y changes over a numerical variable x

```
> cdplot(visit ~ income)
> cdplot(visit ~ size)
```





BINARY LOGISTIC REGRESSION

9. Fitted Values and residuals

```
> predict(model,type = 'response')
> residuals(model,type = 'deviance')
> predclass = ifelse(predict(model, type ='response')>0.5,"1","0")
```

SL No.	Actual	Fitted	Residuals	Predicted Class	SL No.	Actual	Fitted	Residuals	Predicted Class
1	0	0.970979	-2.66073	1	16	1	0.904132	0.448954	1
2	0	0.059732	-0.35097	0	17	1	0.939523	0.353222	1
3	0	0.021049	-0.20627	0	18	1	0.880611	0.50426	1
4	0	0.202309	-0.67236	0	19	1	0.345537	1.457845	0
5	0	0.292461	-0.83182	0	20	1	0.724535	0.802777	1
6	0	0.014893	-0.17324	0	21	1	0.925508	0.393479	1
7	0	0.677783	-1.50501	1	22	1	0.677559	0.882337	1
8	0	0.038723	-0.28105	0	23	1	0.680103	0.878079	1
9	0	0.109432	-0.48145	0	24	1	0.516151	1.150092	1
10	0	0.030543	-0.24908	0	25	1	0.680326	0.877704	1
11	0	0.017609	-0.1885	0	26	1	0.77062	0.721887	1
12	0	0.050856	-0.32309	0	27	1	0.629425	0.962235	1
13	0	0.04202	-0.29301	0	28	1	0.954395	0.305541	1
14	0	0.601981	-1.35739	1	29	1	0.841493	0.587498	1
15	0	0.499424	-1.17643	0	30	1	0.900286	0.45835	1

BINARY LOGISTIC REGRESSION

10. Model Evaluation

```
> mytable = table(visit, predclass)
> mytable
> prop.table(mytable)
```

	Predicted Count		Total
Actual Count	0	1	
0	12	3	15
1	1	14	15
Total	13	17	30

	Predicted %		Total
Actual %	0	1	
0	40	10	50
1	3	47	50
Total	43	50	100

Statistics	Value
Accuracy %	87
Error %	13

Accuracy of $\geq$ 80 % is good

CLASSIFICATION *and* REGRESSION TREE

CLASSIFICATION AND REGRESSION TREE

Objective

To develop a predictive model to classify dependant or response metric (Y) in terms of independent or exploratory variables(Xs).

When to Use

Xs : Continuous or discrete

Y : Discrete or continuous

CLASSIFICATION AND REGRESSION TREE

Classification Tree

When response Y is discrete

Method = “class”

Regression Tree

When response Y continuous

Method = “anova”

CLASSIFICATION AND REGRESSION TREE

Classifies data (develops a model) based on the training data

Each sample is assumed to belong to a predefined class

Sample data set used for building the model is training set

Usage:

For classifying future or unknown data

CLASSIFICATION AND REGRESSION TREE

Example:

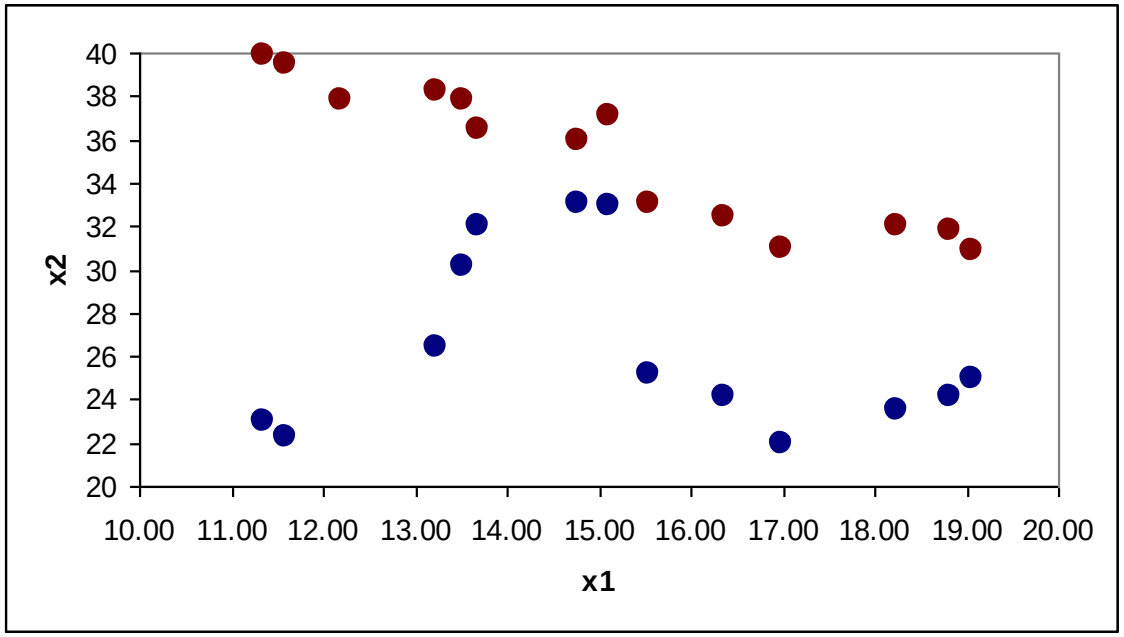
Attribute 1	x1
Attribute 2	x2
Label : y	Y1 (Red) , y2 (Blue)

x1	x2	Y	x1	x2	Y
11.35	23	Blue	11.85	39.9	Red
11.59	22.3	Blue	12.09	39.5	Red
12.19	24.5	Blue	12.69	37.8	Red
13.23	26.4	Blue	13.73	38.2	Red
13.51	30.2	Blue	14.01	37.8	Red
13.68	32	Blue	14.18	36.5	Red
14.78	33.1	Blue	15.28	36	Red
15.11	33	Blue	15.61	37.1	Red
15.55	25.2	Blue	16.05	33.1	Red
16.37	24.1	Blue	16.87	32.4	Red
16.99	22	Blue	17.49	31	Red
18.23	23.5	Blue	18.73	32	Red
18.83	24.1	Blue	19.33	31.8	Red
19.06	25	Blue	19.56	30.9	Red

CLASSIFICATION AND REGRESSION TREE

Example:

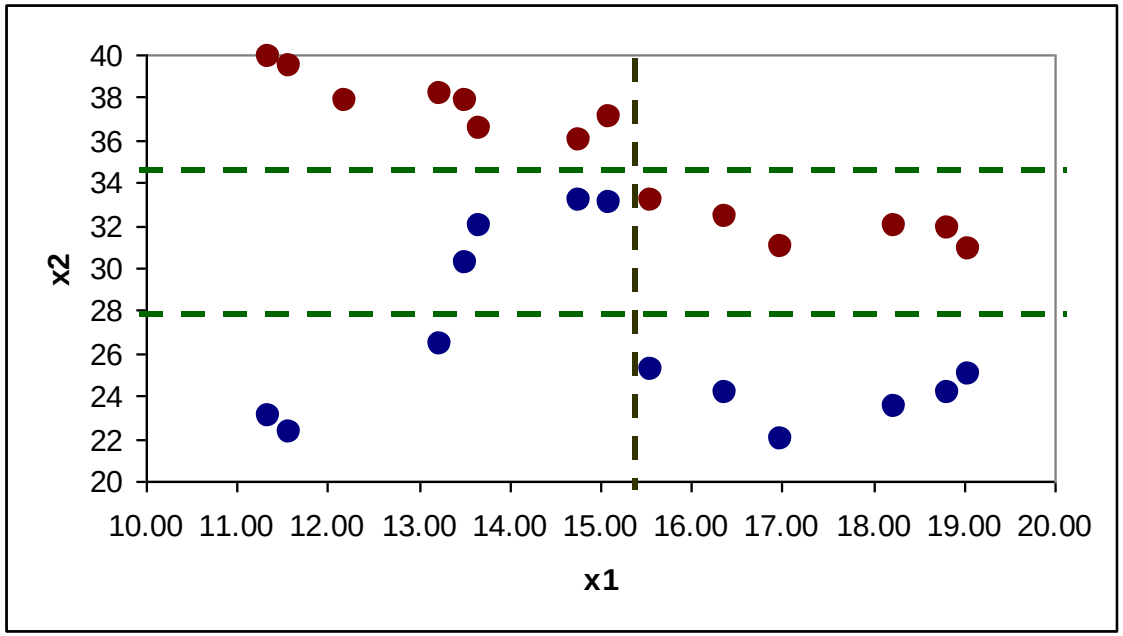
Attribute 1	x1
Attribute 2	x2
Label : y	Y1 (Red) , y2 (Blue)

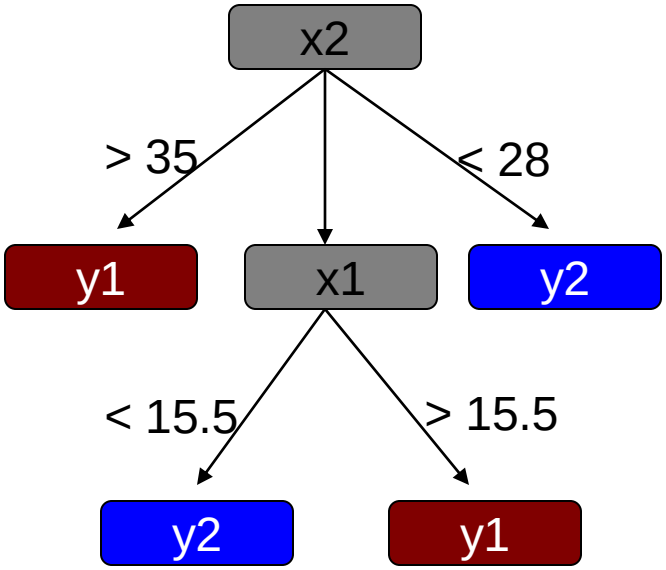


CLASSIFICATION AND REGRESSION TREE

Example:

Attribute 1	x1
Attribute 2	x2
Label : y	y1 (Red) , y2 (Blue)





CLASSIFICATION AND REGRESSION TREE

Example: Rules

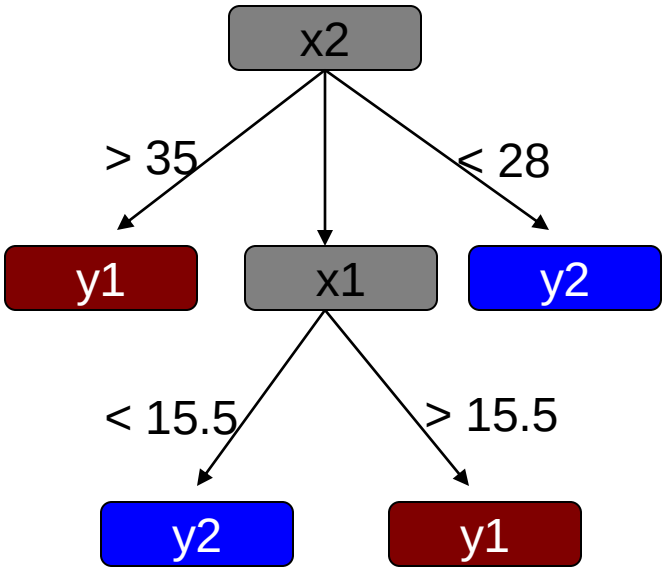
Attribute 1	x1
Attribute 2	x2
Label : y	y1 (Red) , y2 (Blue)

If x2 > 35 then y = y1

If x2 < 28, then y = y2

If 28 > x2 > 35 & x1 > 15.5, then y = y1

If 28 > x2 > 35 & x1 < 15.5, then y = y2



CLASSIFICATION AND REGRESSION TREE

Challenges

How to represent the entire information in the dataset using minimum number of rules?

How to develop the smallest tree?

Solution

Select the variable with maximum information (highest relation with Y) for first split

CLASSIFICATION AND REGRESSION TREE

Example: A marketing company wants to optimize their mailing campaign by sending the brochure mail only to those customers who responded to previous mail campaigns. The profile of customers are given below. Can you develop a rule to identify the profile of customers who are likely to respond (Mail_Respond.csv)

SL No	District	House Type	Income	Previous_Customer	Outcome
1	Suburban	Detached	High	No	No Response
2	Suburban	Detached	High	Yes	No Response
3	Rural	Detached	High	No	Responded
4	Urban	Semi-detached	High	No	Responded
5	Urban	Semi-detached	Low	No	Responded
6	Urban	Semi-detached	Low	Yes	No Response
7	Rural	Semi-detached	Low	Yes	Responded
8	Suburban	Terrace	High	No	No Response
9	Suburban	Semi-detached	Low	No	Responded
10	Urban	Terrace	Low	No	Responded
11	Suburban	Terrace	Low	Yes	Responded
12	Rural	Terrace	High	Yes	Responded
13	Rural	Detached	Low	No	Responded
14	Urban	Terrace	High	Yes	No Response

CLASSIFICATION AND REGRESSION TREE

Example: A marketing company wants to optimize their mailing campaign by sending the brochure mail only to those customers who responded to previous mail campaigns. The profile of customers are given below? Can you develop a rule to identify the profile of customers who are likely to respond?

Number of variables = 4

SL No	Variable Name	Number of values
1	District	3
2	House Type	3
3	Income	2
4	Previous Customer	2

Total Combination of Customer Profiles = 3 x 3 x 2 x 2 = 36

CLASSIFICATION AND REGRESSION TREE

Read file and variables

```
> mydata = Mail_Respond

> house = mydata$House_Type

> district = mydata$District

>  income = mydata$Income

> prev = mydata$Previous_Customer

> outcome = mydata$Outcome
```

CLASSIFICATION AND REGRESSION TREE

Check relationship between the response and predictor variables: Mosaic Plots

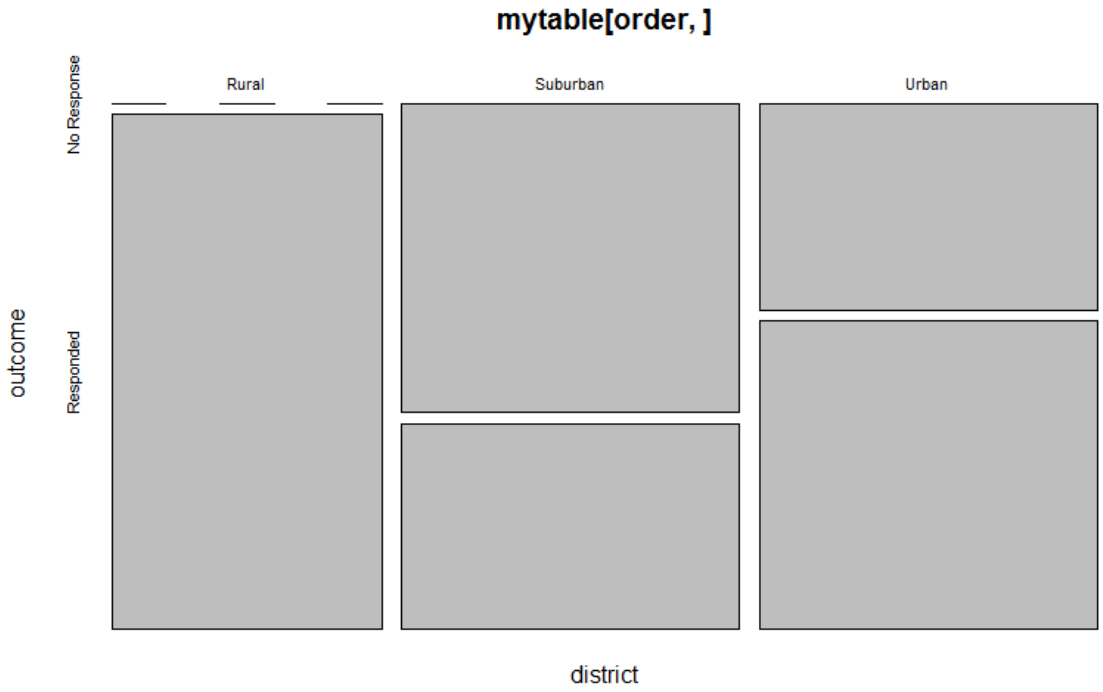
District Vs Outcome

```
> mytable = table(district, outcome)
> mytable
> prop.table(mytable)
> order = order(apply(mytable, 1, sum))
> mosaicplot(mytable[order,],)
```

CLASSIFICATION AND REGRESSION TREE

Check relationship between the response and predictor variables: Mosaic Plots

District Vs Outcome



CLASSIFICATION AND REGRESSION TREE

Develop the model

```
> library(rpart)

> mymodel = rpart( outcome ~ district + house + income + prev, method = “class”,
control = rpart.control(minsplit = 2))
```

Note: When response is categorical, method = “class”, when response is numeric, methos = “anova”

```
>print(mymodel)
```

CLASSIFICATION AND REGRESSION TREE

1) root 14 5 Responded (0.3571429 0.6428571)
  2) dist=Suburban,Urban 10 5 No Response (0.5000000 0.5000000)
    4) income=High 5 1 No Response (0.8000000 0.2000000)
      8) house=Detached,Terrace 4 0 No Response (1.0000000 0.0000000) *
      9) house=Semi-detached 1 0 Responded (0.0000000 1.0000000) *

    5) income=Low 5 1 Responded (0.2000000 0.8000000)
      10) prev=Yes 2 1 No Response (0.5000000 0.5000000)
        20) dist=Urban 1 0 No Response (1.0000000 0.0000000) *
        21) dist=Suburban 1 0 Responded (0.0000000 1.0000000) *

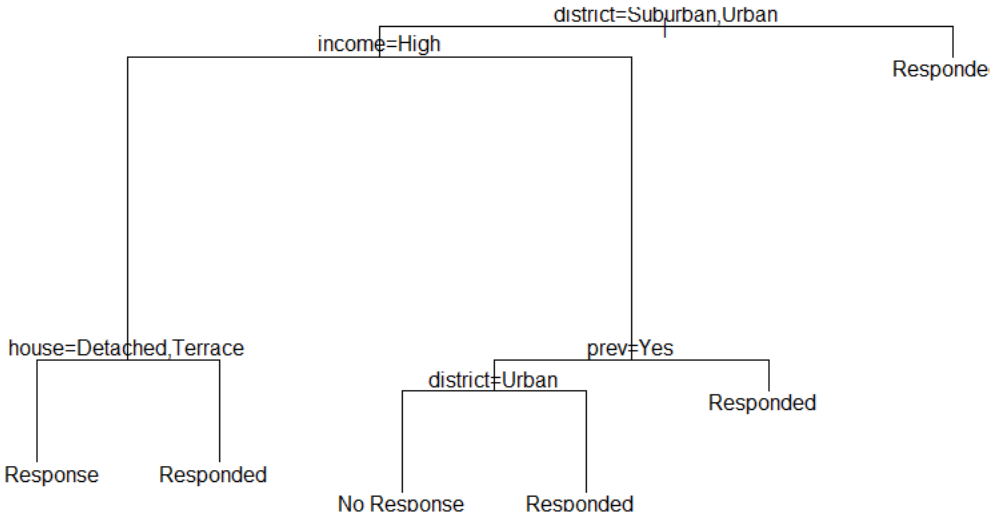
      11) prev=No 3 0 Responded (0.0000000 1.0000000) *

  3) dist=Rural 4 0 Responded (0.0000000 1.0000000) *

CLASSIFICATION AND REGRESSION TREE

Plot the tree

```
> plot(mymodel)
> text(mymodel, pretty = 0)
```



CLASSIFICATION AND REGRESSION TREE

Making predictions

> pred = predict(mymodel, type = “class”)

- mytable = table(outcome, pred)

Confusion Matrix will be

		Predicted	
		Respond	No Respond
Outcome	Respond	9	0
	No Respond	0	5

PRINCIPAL COMPONENTS ANALYSIS

PRINCIPAL COMPONENTS ANALYSIS

- A dimensionality reduction technique
- Reduces the dimensionality of multivariate data without compromising much on the variation in the original data set.
- Achieved by transforming the original varaible into a new set of variables namely prinipal components (PCAs)
- PCAs are uncorrelated and ordered
- Hence the first few of them account for most of the variation in the orignal variables

PRINCIPAL COMPONENTS ANALYSIS

- Desribes the variation in a set of correlated variables $x = (x_1, x_2, - - -, x_q)$ by a set of uncorrelated variables $y = (y_1, y_2, - - -, y_q)$
- Each principal component is a linear combination of the x variables.
- The new variables are derived in decreasing order of importance.
- Hence y_1 account for maximum possible variation in x among all linear combinations of x
- y_2 account for maximum possible of the remaining variation subject to being uncorrelated to y1. and so on.

PRINCIPAL COMPONENTS ANALYSIS

- A dimensionality reduction technique
- Large number of correlated variables can be reduced to a manageable number of uncorrelated or independent factors.
- The emphasis is on the identification of underlying factors that might explain the dimensions associated with large data sets

$$y_i = a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + - - - + a_{iq}x_q$$

Where y_i: estimate of i^{th} principal component, a_i: weight or score coefficient, x_i: i^{th} variable and k: number of variables

The coefficients are selected such that

- the first principal component explains largest portion of the total variation
- the second first principal component accounts for the most of the residual variance, etc.

PRINCIPAL COMPONENTS ANALYSIS

- Helps to understand the variability in large data sets with inter correlated variables using a smaller number of uncorrelated factors.
- Explaining variability of a set of n variables using *m* factors where $m < n$
- The emphasis is on the identification of underlying factors that might explain the dimensions associated with large data

Objectives

- Reduces the complexity of a large set of variables by summarizing them in a smaller set of components or factors
- Tries to improve the interpretation of complex data through logical factors

PRINCIPAL COMPONENTS ANALYSIS

Steps

- Prepare correlation matrix
- Extract a set of principal components using correlation matrix
- Determine the number of principal components
- Interpret results

PRINCIPAL COMPONENTS ANALYSIS

Example: Suppose a researcher wants to determine the underlying benefits consumers seek from the purchase of a toothpaste. A sample of 30 respondents was interviewed. The respondents were asked to indicate their degree of agreement with the following statements using a 7 point scale (1: strongly disagree, 7: strongly agree)

1. It is important to buy a toothpaste that prevents cavities
2. I like a toothpaste that gives shiny teeth
3. A toothpaste should strengthen your gums
4. I prefer toothpaste that freshens breath
5. Prevention of tooth decay is not an important benefit offered by a toothpaste
6. The most important consideration in buying a toothpaste is attractive teeth

PRINCIPAL COMPONENTS ANALYSIS

Step 1: Normalize the data

z transform:

Transformed data = (Data – Mean) / SD

Reading ghe file to R

```
>mydata = mydata[,2:7]
```

Transforming the variables

```
>myzdata = scale(mydata)
> avg = apply(myzdata, 2, mean)
> avg
> s = apply(myzdata, 2, sd)
> s
```

PRINCIPAL COMPONENTS ANALYSIS

Step 2: Check for Correlation

• Variables must be correlated for data reduction

> cor(myzdata)

Correlation Matrix

		x1	x2	x3	x4	x5	x6
Correlation	x1	1.000	-.053	.873	-.086	-.858	.004
	x2	-.053	1.000	-.155	.572	.020	.640
	x3	.873	-.155	1.000	-.248	-.778	-.018
	x4	-.086	.572	-.248	1.000	-.007	.640
	x5	-.858	.020	-.778	-.007	1.000	-.136
	x6	.004	.640	-.018	.640	-.136	1.000

High correlation between x_1, x_3 & x_5

Good correlation between x_2, x_4 & x_6

PRINCIPAL COMPONENTS ANALYSIS

Step 4: Method used: Principle Component Analysis

```
> mymodel = princomp(myzdata)
>summary(mymodel)
```

PRINCIPAL COMPONENTS ANALYSIS

Step 4: Method used: Principle Component Analysis

Used to identify minimum number of components accounting for maximum variance in the data

Eigen Values: Amount of variance attributed to a component

Total Variance = 6 (Sum of all Eigen values)

Prop. variance for PC1= Eigen value of PC1 / Total Variance (2.731/6 = 0.455)

Component	SD	Variance	Proportion of Variance	Cumulative Proportion of Variance
PC 1	1.653	2.732	0.455	0.455
PC 2	1.489	2.217	0.369	0.825
PC 3	0.665	0.442	0.074	0.899
PC 4	0.584	0.341	0.057	0.955
PC 5	0.427	0.182	0.030	0.986
PC 6	0.292	0.085	0.014	1.000
Total		6.000		

PRINCIPAL COMPONENTS ANALYSIS

Step 4: Determine the number of Components

1. Based on Eigen Values: Only components with Eigen value > 1.0 or Eigen value > 0.7 are selected.

2. Based on cumulative % variance: Factors extracted should account for at least 65 % of variance

Component	SD	Variance	Proportion of Variance	Cumulative Proportion of Variance
PC 1	1.653	2.732	0.455	0.455
PC 2	1.489	2.217	0.369	0.825
PC 3	0.665	0.442	0.074	0.899
PC 4	0.584	0.341	0.057	0.955
PC 5	0.427	0.182	0.030	0.986
PC 6	0.292	0.085	0.014	1.000
Total		6.000		

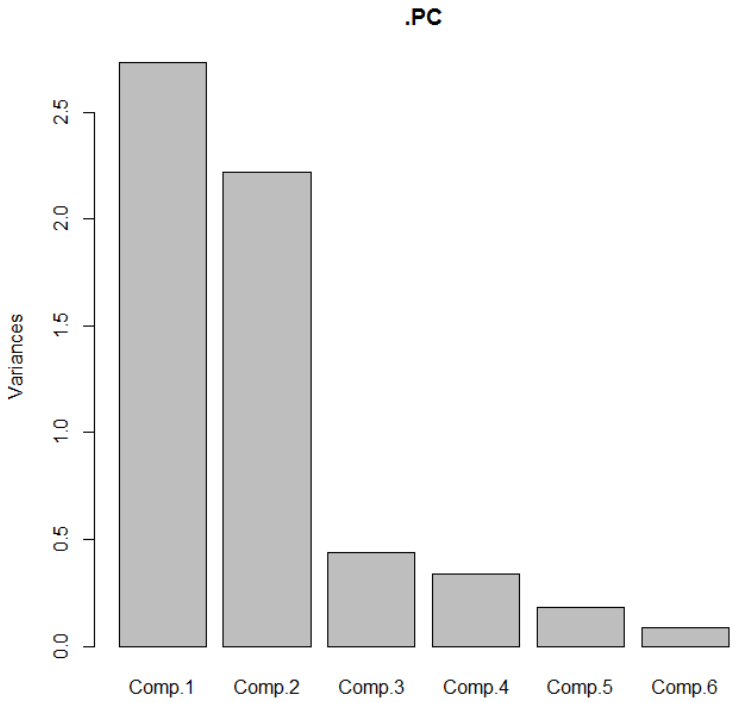
Number of factors selected : 2

PRINCIPAL COMPONENTS ANALYSIS

Step 4: Determine the number of Factors

>plot(mymodel)

3. Based on Scree plot: Plot of the Eigen values against the number of factors in order of extraction. The number of components is identified based on slope change of scree plot



Number of factors selected : 2

PRINCIPAL COMPONENTS ANALYSIS

Step 5: Calculate Component Scores– Eigen Vectors

>loadings(mymodel)

$$y_i = a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + --- + a_{ik}x_k$$

	Component	
	y_1	y_2
x_1	0.562	-0.170
x_2	-0.182	-0.534
x_3	0.566	-0.088
x_4	-0.207	-0.530
x_5	-0.526	0.236
x_6	-0.107	-0.585

PRINCIPAL COMPONENTS ANALYSIS

Step 5: Interpret Components – Eigen Vectors

	Component	
	y_1	y_2
x_1	0.562	-0.170
x_2	-0.182	-0.534
x_3	0.566	-0.088
x_4	-0.207	-0.530
x_5	-0.526	0.236
x_6	-0.107	-0.585

Component 1 is correlated with x_1, x_3 & x_5

Component 2 is correlated with x_2, x_4 & x_6

PRINCIPAL COMPONENTS ANALYSIS

Step 5: Interpret Components

	Component	
	y_1	y_2
Prevention of Cavities	0.562	-0.170
x_2	-0.182	-0.534
Strong Gum	0.566	-0.088
x_4	-0.207	-0.530
Non Prevention of Tooth Decay	-0.526	0.236
x_6	-0.107	-0.585

Interpretation

Component 1 (y_1) represents the health related benefits

PRINCIPAL COMPONENTS ANALYSIS

Step 5: Interpret Components

	Component	
	y_1	y_2
Prevention of Cavities	0.562	-0.170
Shiny Teeth	-0.182	-0.534
Strong Gum	0.566	-0.088
Fresh Breath	-0.207	-0.530
Non Prevention of Tooth Decay	-0.526	0.236
Attractive Teeth	-0.107	-0.585

Interpretation

Component 2 (y_2) represents the social related benefits

PRINCIPAL COMPONENTS ANALYSIS

Step 6: Reduced Data Set

>pc = mymodel$scores

>cbind(pc[,1], pc[,2])

Respondent	PC1	PC2	Respondent	PC1	PC2
1	1.953	-0.071	16	1.412	0.1352
2	-1.6763	0.9852	17	1.261	0.6098
3	2.4298	0.6577	18	2.5041	-0.2372
4	-0.0908	-1.6975	19	-1.2981	1.3974
5	-1.5154	2.7238	20	-1.2777	-1.7423
6	1.6696	0.0148	21	-1.449	1.7912
7	1.0622	1.1536	22	0.9783	-0.2455
8	2.0882	-0.5402	23	-1.4107	0.8217
9	-1.29	1.3543	24	-0.9281	-2.6799
10	-2.7958	-1.6321	25	1.4305	-0.0294
11	2.0398	0.3893	26	-1.0791	-2.2053
12	-1.6682	0.9421	27	1.4698	0.106
13	2.4379	0.6146	28	-1.5875	-1.2162
14	-0.4251	-1.9974	29	-0.8027	-3.2699
15	-1.6509	1.8801	30	-1.7904	1.987

CLUSTER ANALYSIS

CLUSTER ANALYSIS

A technique used to classify objects or cases into relatively homogeneous groups called clusters

Cluster

A collection of data objects similar to one another within the same cluster and dissimilar to the objects in other clusters

Cluster analysis

A procedure for grouping a set of data objects into clusters

CLUSTER ANALYSIS

- A technique used to classify objects or cases into relatively homogeneous groups called clusters

Example: A survey was done to study the consumers attitude towards shopping. The consumers need to be clustered based on their attitude towards shopping. The respondents were asked to express their degree of agreement with the following statements on a 7 point scale (1: strongly disagree, 7: strongly agree).

x1: Shopping is fun

x2: Shopping is bad for your budget

x3: I combine shopping with eating out

x4: I try to get the best buys when shopping

x5: I don’t care about shopping

x6: You can save a lot os money by comparing prices

CLUSTER ANALYSIS

Step 1: Choose Type of clustering - Agglomerative Clustering

- Hierarchical Clustering – characterized by development of a hierarchy or tree like structure
- Starts with each object or record as separate clusters
- Clusters are formed by grouping objects in to bigger and bigger clusters until all objects are in one cluster.
- The objects grouped based on linkage measure
- Commonly used linkage measure is Euclidean distance d,
- Euclidean distance between two records i and j, d_{ij} is defined as

$$d_{ij} = \sqrt{\sum_{k=1}^{q} (x_{ik} - x_{jk})^2}$$

CLUSTER ANALYSIS

Step 1: Choose Type of clustering - Agglomerative Clustering

- The data are not partitioned into a particular number of classes or groups at a single step
- Consists of a series of partitions that may run from a single cluster containing all individuals to n clusters, each contain a single individual
- Produce partitions by a series of successive fusions of the n individuals into groups
- Fusion once made are irreversible, when the algorithm has placed two individuals in the same group they cannot subsequently appear in different groups

CLUSTER ANALYSIS

Types of Linkage

1. Single Linkage:

Based on minimum distance

The first two objects clustered are those having minimum distance between them

$$d_{AB} = \min_{\substack{i \in A \\ j \in B}} (d_{ij})$$

2. Complete Linkage:

Based on maximum distance

The distance between two clusters is calculated as the distance between two furthest points

$$d_{AB} = \max_{\substack{i \in A \\ j \in B}} (d_{ij})$$

Where d_{AB} is the distance between two clusters A and B and d_{ij} is the distance between individuals i and j found from the initial inter – individual distance matrix

CLUSTER ANALYSIS

Types of Linkage

3. Average Linkage:

Based on average distance

The distance between two clusters is defined as the average of the distance between all pairs of points

Preferred method

$$d_{AB} = \frac{1}{n_A n_B} \sum_{i \in A} \sum_{j \in B} d_{ij}$$

Where n_A and n_B are the numbers of individuals in clusters A and B

CLUSTER ANALYSIS

Step 2: Choose Method

Variance method:

Generates clusters with minimum within cluster variance

Uses Ward's Procedure

Ward's Procedure

For each cluster means for all the variables are computed

For each object or record, the Euclidean distance to the cluster mean is computed

CLUSTER ANALYSIS

R Code

Read data to mydata and compute distance

```
> distance = dist(mydata, method = “euclidean”)
```

Generate Clusters

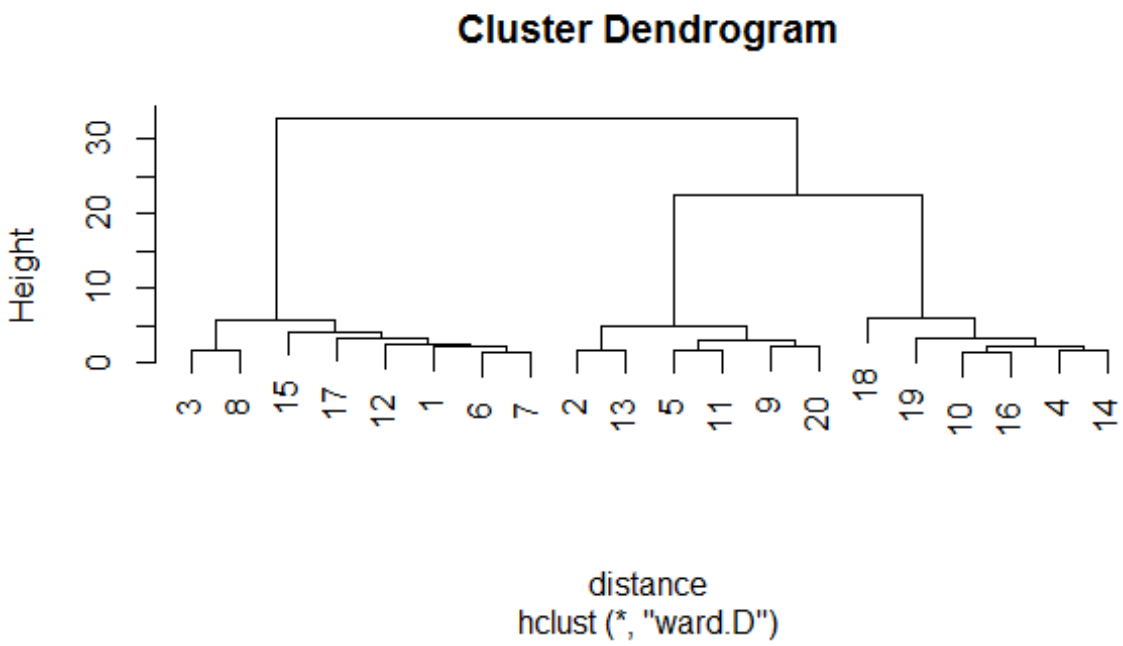
```
>mymodel = hclust(distance, method = “ward”)
```

Plot Dendogram

```
>plot(mymodel)
```

CLUSTER ANALYSIS

Decide on number of clusters: Dendrogram



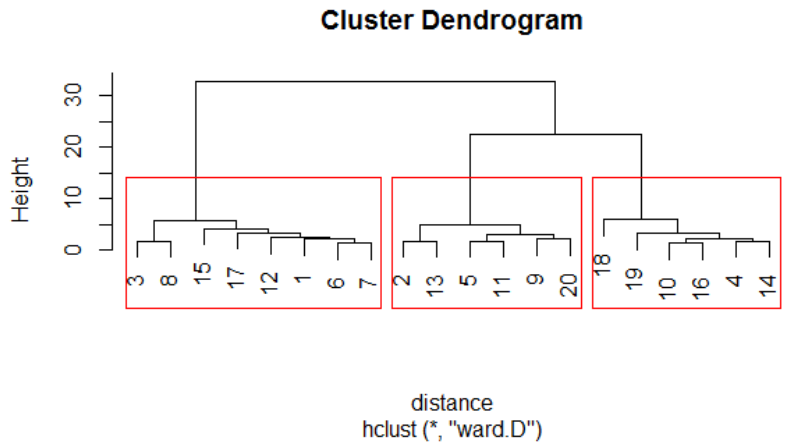
CLUSTER ANALYSIS

Decide on number of clusters: Dendrogram

Stages is given in x axis and distance in y axis

When one move from 3 cluster to 2 cluster the distance increases drastically. So 3 cluster may be appropriate

>groups = cutree(mymodel, k = 3)
> rect.hclust(mymodel, k = 3, border = "red")



CLUSTER ANALYSIS

Identification of cluster membership for each record

> output = cbind(mydata, groups

>write,csv(output, "E:/ISI_Mumbai/output.csv“)

Respondent id	x1	x2	x3	x4	x5	x6	groups
1	6	4	7	3	2	3	1
2	2	3	1	4	5	4	2
3	7	2	6	4	1	3	1
4	4	6	4	5	3	6	3
5	1	3	2	2	6	4	2
6	6	4	6	3	3	4	1
7	5	3	6	3	3	4	1
8	7	3	7	4	1	4	1
9	2	4	3	3	6	3	2
10	3	5	3	6	4	6	3
11	1	3	2	3	5	3	2
12	5	4	5	4	2	4	1
13	2	2	1	5	4	4	2
14	4	6	4	6	4	7	3
15	6	5	4	2	1	4	1
16	3	5	4	6	4	7	3
17	4	4	7	2	2	5	1
18	3	7	2	6	4	3	3
19	4	6	3	7	2	7	3
20	2	3	2	4	7	2	2

CLUSTER ANALYSIS

Cluster Profile

> aggregate(mydata, by = list(groups), FUN = mean)

Variables	Cluster Mean		
	1	2	3
x1 (shopping is fun)	5.750	1.667	3.500
x2 (shopping upsets my budget)	3.625	3.000	5.833
x3 (I combine shopping with eating out)	6.000	1.833	3.333
x4 (I try to get best buys when shopping)	3.125	3.500	6.000
x5 (I don't care about shopping)	1.875	5.500	3.500
x6 (save a lot by comparing prices	3.875	3.333	6.000

Cluster 1: High on x1 & x3 but low on x5
Fun loving and concerned

Cluster 2: Low on x1 & x3 but High on x5
Careless & no fun in shopping (apathetic)

Cluster 3: High on x2 x4 & x6
Concerned about spending money (Economical)

CLUSTER ANALYSIS

k mean clustering

Partitions n individuals in a set of multivariate data into k groups or clusters (G_1, G_2, - - -, G_k)

k is given or a possible range is specified

Common approach is to identify the k groups which minimizes the within – group sum of squares (WGSS)

$$WGSS = \sum_{j=1}^{q}\sum_{l=1}^{k}\sum_{i\in G_i}(x_{ij} - \bar{x}_j^{(1)})^2$$

Where $\bar{x}_j^{(l)} = \frac{1}{n_i}\sum_{i\in G_i} x_{ij}$ is the mean of the individuals in group G_l on variable j

Computing WGSS for each value of k and choose that of value of k which minimize WGSS is almost impossible

One option is to plot WGSS for different values of k and choose the optimum k at which the slope of the curve changes

CLUSTER ANALYSIS

k mean clustering

Computing WGSS for each value of k and choose that of value of k which minimize WGSS is almost impossible

Moreover as number of cluster increases WGSS decreases or WGSS / Total SS will increase

One option is to plot WGSS/ Total SS for different values of k and choose the optimum k at which the the curve flattens or slope changes

CLUSTER ANALYSIS

Example: Cluster the data given in cluster_Analysis_example.csv using k mean method

```
>mynewmodel = kmeans(mydata,3)
> mynewmodel
>cluster = mynewmodel$cluster
>output = cbind(mydata, cluster)
> write.csv(output, "E:/ISI/Applied_Multivariate_Analysis/output.csv")
```

To find optimum k, compute WGSS / Total SS for different values of k

```
> kmeans(mydata,k),  k =.1,2, - - -
```

CLUSTER ANALYSIS

Example: Cluster the data given in cluster_Analysis_example.csv using k mean method

optimum k,

> kmeans(mydata,k, k =.1,2, - - -

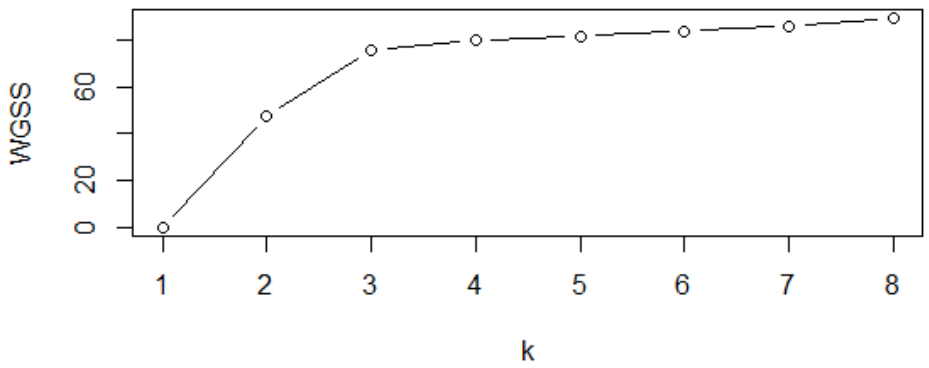
k	WGSS/Total SS
1	0.0
2	47.5
3	75.8
4	79.6
5	81.4
6	83.7
7	85.8
8	89.2

CLUSTER ANALYSIS

Example: Cluster the data given in cluster_Analysis_example.csv using k mean method

optimum k,

```
> plot(k, WGSS, type = "b")
```



The curve flattens after k = 3, hence optimum k is 3

MODELING NONLINEAR RELATIONS

MODELING NONLINEARRELATIONS

The linear regression is fast and powerful tool to model complex phenomena

But makes several assumptions about the data including the assumption of linear relationship exists between predictors and response variable.

When these assumptions are violated, the model breaks down quickly

MODELING NONLINEAR RELATIONS

The linear model $y = x\beta + \varepsilon$ is general model

Can be used to fit any relationship that is linear in the unknown parameter β

Examples:

$$y = \beta_0 + \beta_1 x_1 + --- + \beta_k x_k + \varepsilon$$

$$y = \beta_0 + \beta_1 x_1 + \beta_{11} x_1^2 + \varepsilon$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \beta_{12} x_1 x_2 + \varepsilon$$

In general

$$y = \beta_0 + \beta_1 f(x) + \varepsilon$$

where f(x) can be 1/x, $\sqrt{x}$, log(x), e^x , etc

MODELING NONLINEAR RELATIONS

Detection of non linear relation between predictor x and response variable y

Scatter Plot:

The plotted points are not lying lie in a straight line is an indication of non linear relationship between predictor and dependant variable

Component Residual Plots:

An extension of partial residual plots

Partial residual plots are the plots of residuals of one predictor against dependant variable

Component residual plots(crplots) adds a line indicating where the best fit line lies.

A significant difference between the residual line and the component line indicate that the predictor does not have a linear relationship wit the dependent variable

MODELING NONLINEAR RELATIONS

Example : The data given in Nonlinear_Thrust.csv represent the trust of a jet – turbine engine (y) and 3 predictor variables: x_3 = fuel flow rate, x_4 = pressure, and x_5 = exhaust temperature. Develop a suitable model for thrust in terms of the predictor variables.

Read Data

```
> attach(mydata)
> cor(mydata)
```

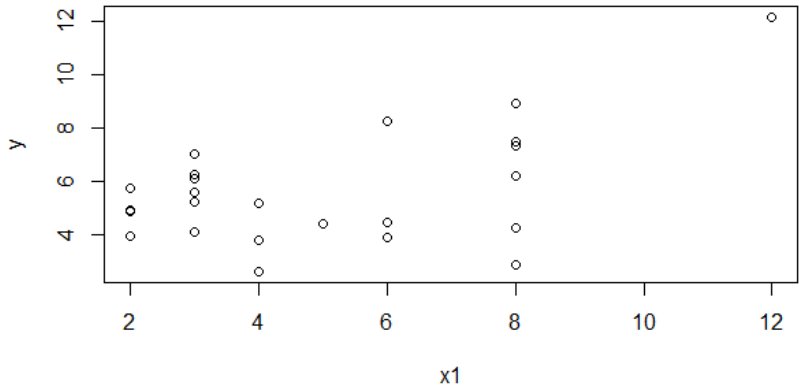
	x1	x2	x3	y
x1	1.00	0.40	-0.20	0.54
x2	0.40	1.00	-0.30	-0.36
x3	-0.20	-0.30	1.00	0.35
y	0.54	-0.36	0.35	1.00

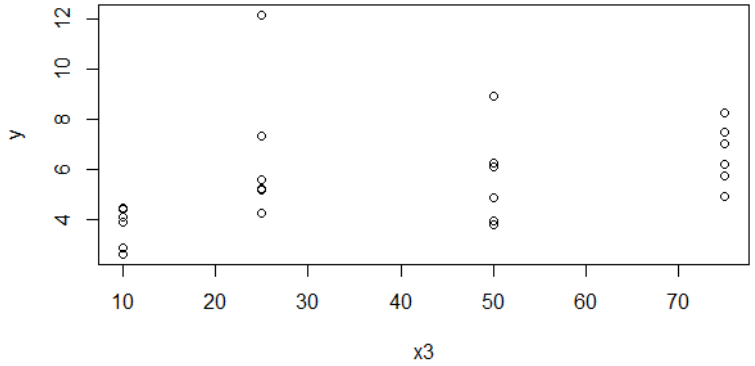
There is no strong correlation between y and x's

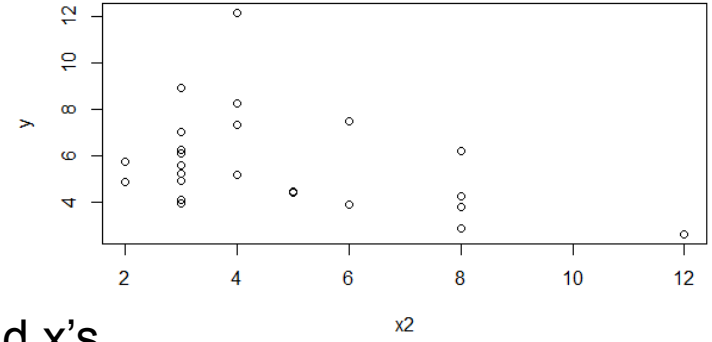
MODELING NONLINEAR RELATIONS

Draw Scatter plots

```
> plot(x1,y)
> plot(x2,y)
> plot(x3,y)
```







There is no strong correlation between y and x's

MODELING NONLINEAR RELATIONS

Develop the model

```
> mymodel = lm(y ~ x1 + x2 + x3, data = mydata)
> summary(mymodel)
```

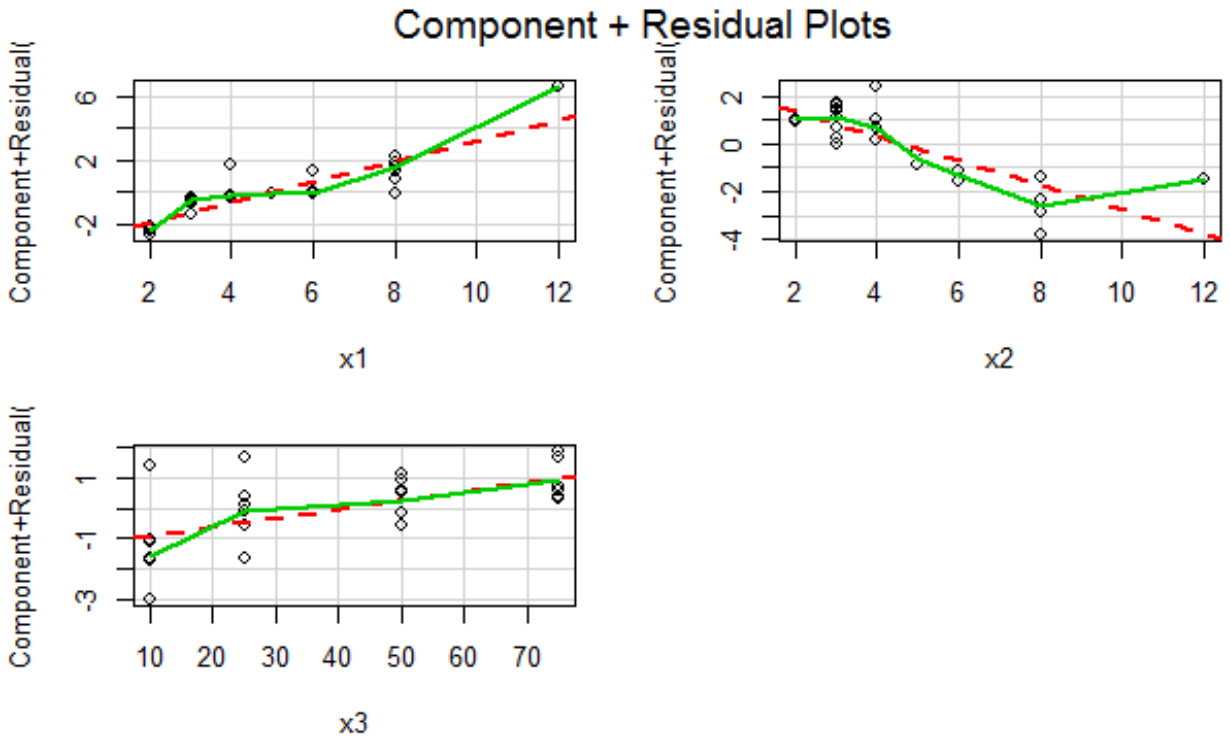
	Estimate	Std. Error	t	p value
(Intercept)	3.58315	0.726839	4.93	0.0001
x1	0.651547	0.0855	7.62	0.0000
x2	-0.509866	0.097132	-5.249	0.0000
x3	0.028888	0.009021	3.202	0.00428

R^2	0.786
Adjusted R^2	0.7563

MODELING NONLINEAR RELATIONS

Develop the model

```
> library(car)
> crPlots(mymodel))
```

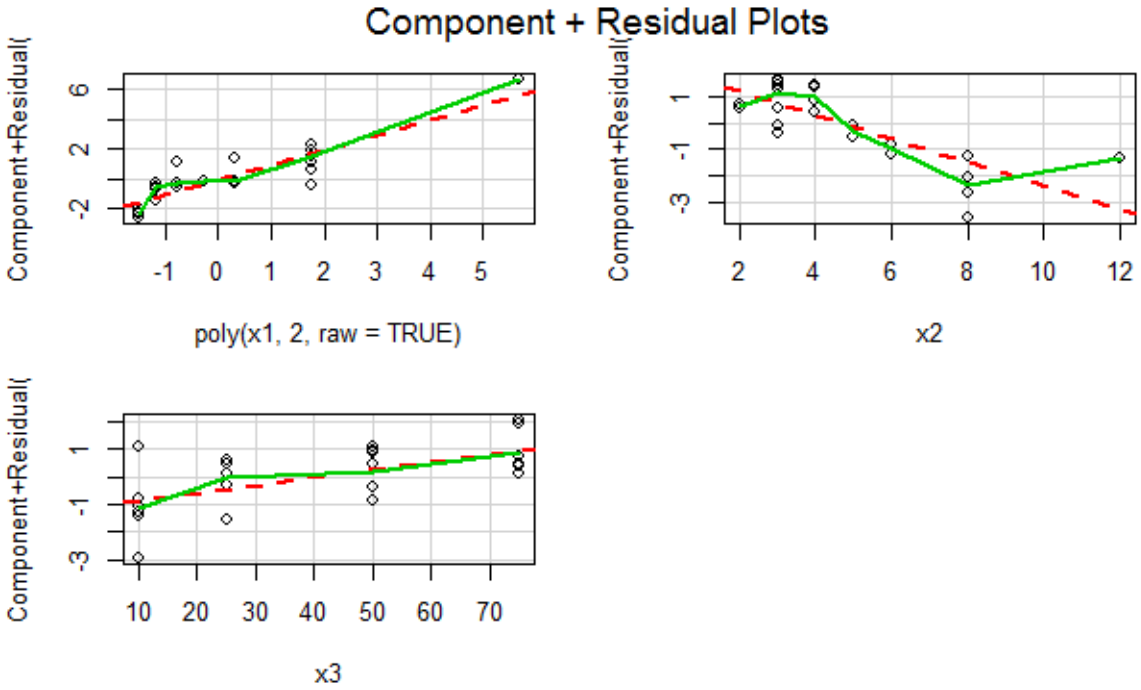


Since the best fit line different from residual line, it is possible improve the model by adding higher order terms

MODELING NONLINEAR RELATIONS

Develop the model

```
> mymodel = lm(y ~ poly(x1, 2, raw = TRUE) + x2 + x3, data = mydata)
> crPlots(mymodel)
```

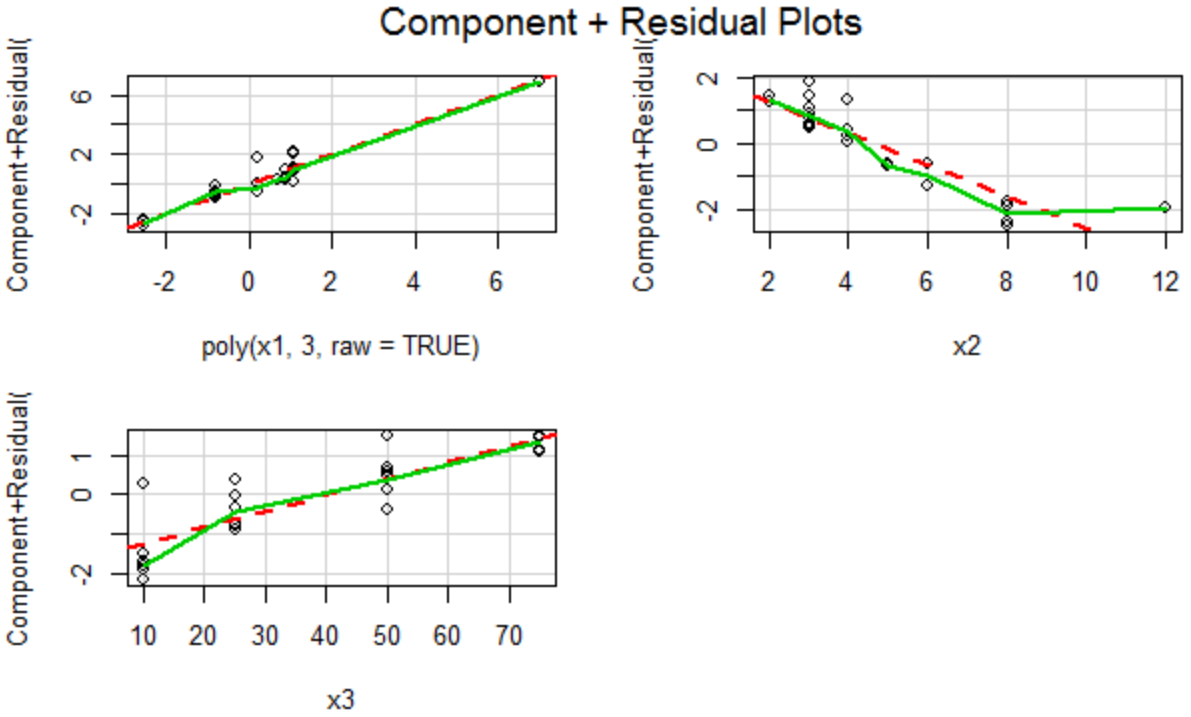


Since the best fit line different from residual line, it is possible improve the model by adding higher order terms

MODELING NONLINEAR RELATIONS

Develop the model

```
> mymodel = lm(y ~ poly(x1, 3, raw = TRUE) + x2 + x3, data = mydata))
> crPlots(mymodel)
```

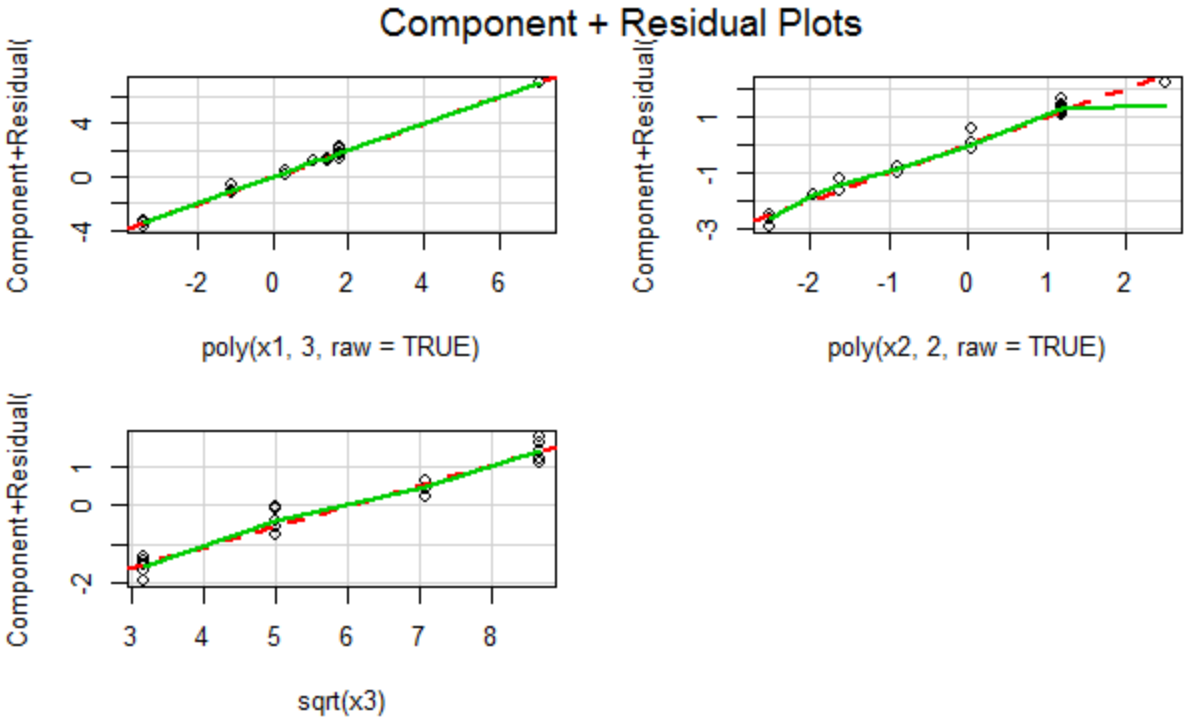


Since the best fit line is more or less overlapping residual line, hence adding square and cube terms of x1 will improve the model. Similarly add additional terms or functions of x2 and x3 to improve the model

MODELING NONLINEAR RELATIONS

Develop the model: Final Model

```
> mymodel = lm(y ~ poly(x1, 3, raw = TRUE) + poly(x2, 2, raw = TRUE) + sqrt(x3), data = mydata))
> crPlots(mymodel)
```



MODELING NONLINEAR RELATIONS

Develop the model: Final Model

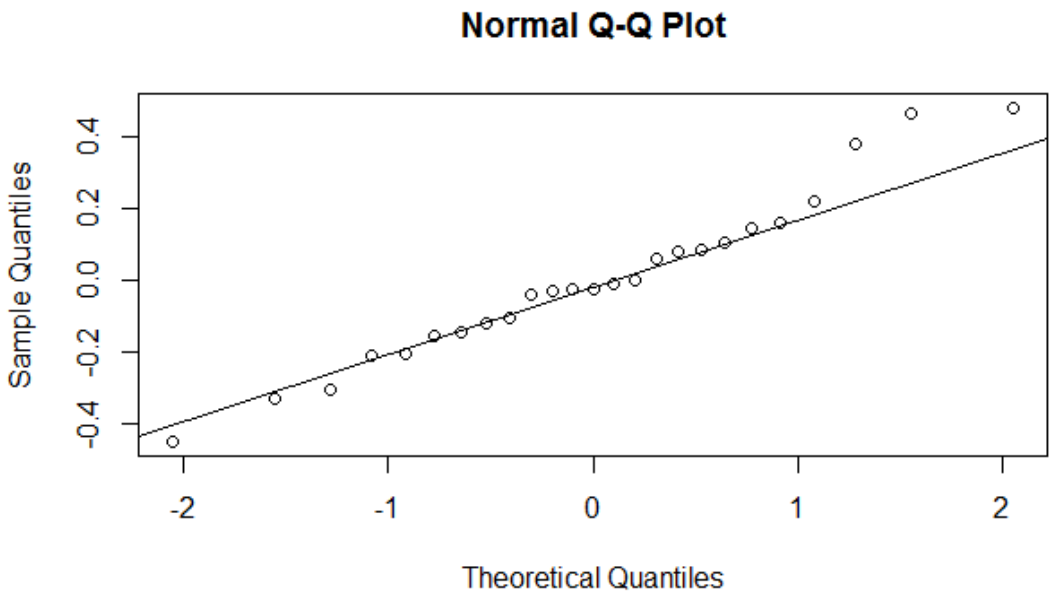
	Estimate	Std. Error	t	p value
(Intercept)	-3.48301	0.705793	-4.935	0.000107
x_1	5.503467	0.36278	15.17	0.0000
x_1^2	-0.77878	0.056814	-13.708	0.0000
x_1^3	0.037516	0.002685	13.971	0.0000
x_2	-1.81437	0.146304	-12.401	0.0000
x_2^2	0.097886	0.010374	9.435	0.0000
$\sqrt{x_3}$	0.527417	0.030664	17.2	0.0000

R^2	0.9881
Adjusted R^2	0.9841

MODELING NONLINEAR RELATIONS

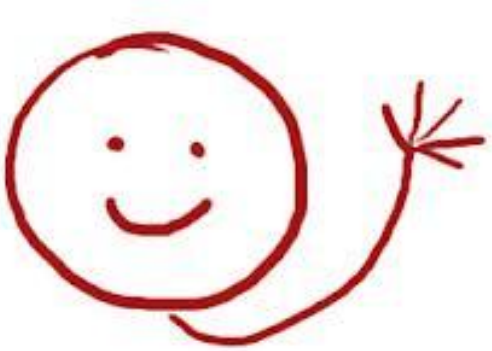
Develop the model: Final Model

```
> res = residuals(mymodel)
> qqnorm(res)
 > qqline(res)
> shapiro.test(res)
```



Shapiro test for Normality	
w	0.9704
p value	0.6569

For other queries mail me at tanujitisi@gmail.com



THANK YOU